

Consortium of Medical Engineering and Dental Colleges of Karnataka (COMEDK)

Undergraduate Entrance Test(UGET)

Maths-2012

1. If the area of the circle $7x^2 + 7y^2 - 7x + 14y + k - 0$ is 12π sq. units, then the value of k is

- a) $\frac{-43}{4}$ b) $\frac{-301}{4}$ c) -16 d) ± 4

Ans: (b)

$$x^2 + y^2 - x + 2y + \frac{K}{7} = 0$$

$$C \equiv \left(\frac{1}{2}, -1\right) \quad r = \sqrt{\frac{1}{4} + 1 - \frac{K}{7}}$$

$$A = 12\pi \quad \Rightarrow 12\pi = \pi \left(\frac{1}{4} + 1 - \frac{K}{7}\right)$$

$$12 = \frac{5}{4} - \frac{K}{7} \Rightarrow 12 = \frac{35 - 4K}{28}$$

$$336 = 35 - 4K$$

$$4K = -301 \quad \therefore K = \frac{-301}{4}$$

2. A man running a race-course notes that the sum of the distances from the two flag posts from him is always 10 metres and the distance between the flag posts is 8 metres. The equations of the path traced by the man is given by

- a) $\frac{x^2}{9} + \frac{y^2}{25} = 1$ b) $\frac{x^2}{9} + \frac{y^2}{16} = 1$ c) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ d) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

Ans: (c)

$$SP + S^1P = 2a \Rightarrow 2a = 10 \Rightarrow a = 5$$

$$2ae = 8$$

$$ae = 4 \Rightarrow b^2 = a^2(1 - e^2)$$

$$25 - 16 = 9$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

3. The number of common tangents to the circles

$$x^2 + y^2 - 2x - 4y + 1 = 0 \text{ and } x^2 + y^2 - 12x - 16y + 91 = 0 \text{ is}$$

- a) 1 b) 2 c) 3 d) 4

Ans: (d)

$$C_1 \equiv (1, 2) \quad r_1 = \sqrt{1 + 4 - 1} = 2$$

$$C_2 \equiv (6, 8) \quad r_2 = \sqrt{36 + 64 - 91} = 3$$

$$C_1C_2 = \sqrt{25 + 36} = \sqrt{61}$$

$$R_1 + r_2 = 5 < \sqrt{61}$$

Circles are far apart \therefore no of common tangents = 4

4. Equation of the chord of the circle $x^2 + y^2 + 4x - 6y - 9 = 0$ bisected at $(0, 1)$ is
 a) $y - 1 = x$ b) $y + 1 = x$ c) $y + 1 = 2x$ d) $y - 1 = 3x$

Ans: (a)

Required T = S_1

$$x(0) + y(1) + 2(x+0) - 3(y+1) - 9 = 0 + 1 + 0 - 6 - 9$$

$$y + 2x - 3y - 3 - 9 = 1 - 15$$

$$2x - 2y - 12 = 1 - 15$$

$$2x - 2y + 2 = 0$$

$$x - y + 1 = 0$$

5. The angle between two asymptotes of the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ is

a) $\tan^{-1} \frac{4}{5}$ b) $2\tan^{-1} \frac{4}{5}$ c) $2\tan^{-1} \frac{5}{4}$ d) $\pi - 2\tan^{-1} \frac{4}{5}$

Ans: (b)

$$\theta = 2\tan^{-1} \left(\frac{b}{a} \right) = 2\tan^{-1} \left(\frac{4}{5} \right)$$

6. The parametric equation of a parabola is $x = t^2 + 1$, $y = 2t + 1$. The Cartesian equation of its directrix is

a) $y = 0$ b) $x = -1$ c) $x = 0$ d) $x - 1 = 0$

Ans: (c)

$$x = t^2 + 1 \quad \rightarrow x - 1 = t^2$$

$$y = 2t + 1 \quad \rightarrow y - 1 = 2t$$

$$(y - 1)^2 = 4t^2 = 4(x - 1)$$

$$(y - 1)^2 = 4(x - 1) \Rightarrow y^2 = 4ax \quad x = -a$$

$$x - 1 = -1 \quad \quad \quad x - h = -a$$

$$x = 0$$

7. If $|\vec{a} \times \vec{b}| = 5$ and $|\vec{a} \cdot \vec{b}| = 3$ then $|\vec{a}|^2 |\vec{b}|^2$ is equal to

a) 16 b) 31 c) 25 d) 34

Ans: (d)

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

$$25 = |\vec{a}|^2 |\vec{b}|^2 - 9$$

$$|\vec{a}|^2 |\vec{b}|^2 = 34$$

8. The direction cosines of the vector $2\vec{i} + \vec{j} - 2\vec{k}$ is equal to

a) $\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$ b) $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$ c) $\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$ d) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$

Ans: (a)

$$\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$$

$$\left| \vec{a} \right| = \sqrt{4 + 1 + 4} = 3$$

9. If $1, \omega, \omega^2$ are the cube roots of unity then $(3 + 3\omega^2 + 5\omega)^6 - (2 + 6\omega^2 + 2\omega)^3$ is equal to
a) 32 b) 64 c) 0 d) -1

Ans: (c)

$$\begin{aligned} & (3 + (1 + \omega^2) + 5\omega)^6 - (2(1 + \omega) + 6\omega^2)^3 \\ &= (3(-\omega) + 5\omega)^6 - (2(-\omega^2) + 6\omega^2)^3 = (2\omega)^6 - (4\omega^2)^3 \\ &= 2^6 - 4^3 = 64 - 64 = 0 \end{aligned}$$

10. If $\int_1^x \frac{dy}{\log_2 \sqrt{e^y - 1}} = \frac{\pi}{6}$ then x is equal to

- a) $\log_e 4$ b) $\log_e 2$ c) 4 d) 2

Ans: (a)

$$\text{put } t = e^y - 1$$

$$\frac{dt}{dy} = e^y \Rightarrow dy = \frac{dt}{t+1}$$

$$2 \int_1^{e^x-1} \frac{dt}{(t+1)2\sqrt{t}} = \frac{\pi}{6} \Rightarrow 2 \int_1^{e^x-1} \frac{d(\sqrt{t})}{(\sqrt{t})^2 + 1} = \frac{\pi}{6}$$

$$\tan^{-1}(\sqrt{t}) \Big|_1^{e^x-1} = \frac{\pi}{12}$$

$$\tan^{-1}(\sqrt{e^x-1}) - \tan^{-1} 1 = \frac{\pi}{12}$$

$$\tan^{-1}(\sqrt{e^x-1}) = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$$

$$\sqrt{e^x-1} = \sqrt{3}$$

$$e^x - 1 = 3$$

$$e^x = 4 \Rightarrow x = \log_e 4$$

11. $\int_{-8}^8 (\sin^{93} x + x^{295}) dx =$

- a) 1 b) -1 c) 0 d) $\frac{8}{3}$

Ans: (c)

'0' (odd function)

12. Area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is given by

- a) 25π sq. units b) 20π sq units c) 4π sq units d) 5π units

Ans: (b)

$$A = \pi ab = \pi \cdot 5 \cdot 4 = 20\pi$$

13. The order of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^5\right]^{\frac{2}{3}} = \frac{d^3y}{dx^3}$
- a) 2 b) 1 c) 3 d) $\frac{2}{3}$

Ans: (c)

Order '3'

14. The solution of $\frac{dy}{dx} - 1 = e^{x-y}$ is
- a) $e^{x-y} + x = c$ b) $e^{-(x-y)} + x = c$ c) $e^{-(x-y)} = x + c$ d) $e^{x-y} = x + c$

Ans: (c)

$$\frac{dy}{dx} - 1 = e^{x-y}$$

$$dy - dx = e^{x-y} \cdot dx$$

$$\frac{-d(x-y)}{e^{x-y}} = dx$$

$$dx + e^{-(x-y)} \cdot d(x-y) = 0$$

$$\text{Integrate : } x - e^{-(x-y)} = C$$

15. If $\sin^{-1}\left(\frac{2p}{1+p^2}\right) - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right) = \tan^{-1}\left(\frac{2x}{1+x^2}\right)$ then the value of x is equal to
- a) $\frac{p+q}{1+pq}$ b) $\frac{p-q}{1-pq}$ c) $\frac{p-q}{pq-1}$ d) $\frac{p-q}{1+pq}$

Ans: (d)

$$\sin^{-1}\left(\frac{2p}{1+p^2}\right) - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$2 \cdot \tan^{-1}p - 2 \tan^{-1}q = \tan^{-1}\frac{2x}{1-x^2} = 2 \tan^{-1}x$$

$$2(\tan^{-1}p - \tan^{-1}q) = \tan^{-1}\frac{2x}{1-x^2} = 2 \tan^{-1}x$$

$$2 \cdot \tan^{-1}\left(\frac{p-q}{1+pq}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2 \tan^{-1}x$$

$$x = \frac{p-q}{1+pq}$$

16. The unit vector in the direction of the vector $\vec{a} + 2\vec{b} - \vec{c}$ is equal to
- a) $\frac{\vec{a} + 2\vec{b} - \vec{c}}{\sqrt{6}}$ b) $\frac{\vec{a} + 2\vec{b} - \vec{c}}{2}$ c) $\frac{\vec{a} + 2\vec{b} - \vec{c}}{4}$ d) $\frac{\vec{a} + 2\vec{b} - \vec{c}}{6}$

Ans: (c)

$$\text{Required} = \frac{\vec{a} + 2\vec{b} - \vec{c}}{\sqrt{1+4+1}} = \frac{\vec{a} + 2\vec{b} - \vec{c}}{\sqrt{6}}$$

17. Identify the false statement.
- a) A non-empty subset H of group G is a subgroup of G if and only if for every $a, b \in H \rightarrow a * b^{-1} \in H$
 - b) The intersection of two subgroups of a group G is again a subgroup
 - c) A group of order three is not abelian
 - d) If in a group F, $(ab)^2 = a^2b^2 \forall a, b \in G$ then G is abelian

Ans: (c)

Is false since every group of order 3 is abelian.

18. If $y = \tan^{-1} \left(\frac{1}{1+x+x^2} \right) + \tan^{-1} \left(\frac{1}{x^2+3x+3} \right) + \tan^{-1} \left(\frac{1}{x^2+5x+7} \right) + \dots +$ upto n terms then

$\frac{dy}{dx}$ at $x = 0$ and $n = 1$ is equal to

- a) $\frac{1}{2}$
- b) $-\frac{1}{2}$
- c) 0
- d) $\frac{1}{3}$

Ans: (b)

$$y = \tan^{-1} \left(\frac{1}{1+x(x+1)} \right) + \tan^{-1} \left(\frac{1}{1+(x+1)(x+2)} \right) + \tan^{-1} \left(\frac{1}{1+(x+2)(x+3)} \right) + \dots + \tan^{-1} \left(\frac{1}{1+(x+n-1)(x+n)} \right)$$

when $n = 1$

$$y = \tan^{-1} \left(\frac{1}{1+x(x+1)} \right) = \tan^{-1} \left(\frac{(x+1) - x}{1+x(x+1)} \right) = \tan^{-1} (x+1) - \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{1+(x+1)^2} - \frac{1}{1+x^2} \therefore \left. \frac{dy}{dx} \right|_{x=0} = \frac{1}{1+1} - \frac{1}{1} = \frac{1}{2} - 1 = -\frac{1}{2}$$

19. If $\cot \alpha \cot \beta = 2$ then $\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)}$ is equal to

- a) 3
- b) $\frac{2}{3}$
- c) $\frac{1}{3}$
- d) $\tan \alpha \tan \beta$

Ans: (c)

$$\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{\cos \alpha \cdot \cos \beta - \sin \alpha \sin \beta}{\cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta} \Rightarrow \frac{\cot \alpha \cdot \cot \beta - 1}{\cot \alpha \cdot \cot \beta + 1} = \frac{2 - 1}{2 + 1} = \frac{1}{3}$$

20. If ω is a cube root of unity, then the value of determinant $\begin{vmatrix} 1 + \omega & \omega^2 & \omega \\ \omega^2 + \omega & -\omega & \omega^2 \\ 1 + \omega^2 & \omega & \omega^2 \end{vmatrix}$ is equal to

- a) $1 + \omega$
- b) $1 - \omega$
- c) 0
- d) ω^2

Ans: All answers wrong

$$\begin{vmatrix} 1 + \omega & \omega^2 & \omega \\ \omega^2 + \omega & -\omega & \omega^2 \\ 1 + \omega^2 & \omega & \omega^2 \end{vmatrix} \xrightarrow{C_1 + C_2} \begin{vmatrix} 0 & \omega^2 & \omega \\ \omega^2 & -\omega & \omega^2 \\ 0 & \omega & \omega^2 \end{vmatrix} = 0 - \omega^2 (\omega^4 - \omega^2) + 0 = -\omega^3 (\omega^3 - \omega)$$

$$= -1 (1 - \omega) = -1 + \omega$$

21. If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a, a) cuts off intercepts α and β on the coordinate axes where $\alpha^2 + \beta^2 = 61$ then the value of 'a' is equal to
 a) 25 b) 36 c) ± 30 d) ± 40

Ans: (c)

$$2y^3 = ax^2 + x^3$$

Diff. w. r. t

$$6y^2 \frac{dy}{dx} = 2ax + 3x^2$$

$$\left. \frac{dy}{dx} \right|_{(a,a)} = \frac{2a^2 + 3a^2}{6a^2} = \frac{5}{6} \therefore x - \text{intercept} = \frac{-a}{5} = \alpha$$

$$y - \text{intercept} = \frac{a}{6} = \beta$$

$$\therefore \alpha^2 + \beta^2 = 61 \Rightarrow \frac{a^2}{25} + \frac{a^2}{36} = 61 \Rightarrow a = \pm 30$$

22. Length of the subtangent at (a, a) on the curve $y^2 = \frac{x^2}{2a+x}$ is equal to

- a) $\frac{18}{5}$ b) $\frac{18a}{5}$ c) $-\frac{18a^2}{5}$ d) $\frac{18a^2}{5}$

Ans: Question is wrong because (a, a) does not satisfy the given equation

23. The function $f(x) = 5 + 36x + 3x^2 - 2x^3$ is increasing in the interval

- a) $(-2, 3)$ b) $(2, 3)$ c) $[2, 3)$ d) $(2, 3]$

Ans: (a)

$$f(x) = 5 + 36x + 3x^2 - 2x^3 \therefore f'(x) = -6x^2 + 6x + 36$$

$$= -6(x^2 - x - 6)$$

$$= -6(x - 3)(x + 2) > 0 \quad [\because \text{for increasing function}]$$

$$\Rightarrow (x - 3)(x + 2) < 0 \Rightarrow [x < 3 \ \& \ x > -2] \ [x > 3 \ \& \ x < -2] \Rightarrow -2 < x < 3$$

24. Divide 20 into two parts such that the product of one part and the cube of the other is maximum. The two parts are

- a) $(12, 8)$ b) $(15, 5)$ c) $(10, 10)$ d) $(2, 18)$

Ans: (b)

$$\text{Given, } x + y = 20 \rightarrow (1)$$

$$P = xy^3 = (20 - y)y^3 \text{ [from (1)]} \Rightarrow P = 20y^3 - y^4$$

$$\frac{dP}{dy} = 60y^2 - 4y^3$$

$$\frac{d^2P}{dy^2} = 120y - 12y^2$$

$$\frac{dP}{dy} = 0 \Rightarrow 60y^2 = 4y^3 \Rightarrow y = 15 \therefore x = 5 \therefore \text{two parts are } (15, 5)$$

[\because correct answer is $(5, 15)$. Among the given answers by not considering the answer $(15, 5)$ can be taken as answer]

25. The number of positive divisors of 4896 is
 a) 32 b) 34 c) 36 d) 38

Ans: (c)

We have, $4896 = 2^5 \times 3^2 \times 17^1$
 $\therefore T(a) = (1 + 5)(1 + 2)(1 + 1) = 36$

26. The last digit of $583! + 7^{291}$ is
 a) 1 b) 2 c) 0 d) 3

Ans: (d)

We have, $583! \equiv 0 \pmod{10}$
 and $7^2 \equiv -1 \pmod{10}$
 $\therefore (7^2)^{145} \cdot 7^1 \equiv (-1)^{145} \cdot 7^1 \pmod{10}$
 $\Rightarrow 7^{291} \equiv -7 \pmod{10} \equiv 3 \pmod{10}$
 $\therefore 583! + 7^{291} \equiv 3 \pmod{10}$
 \Rightarrow last digit is 3

27. $\int x^x(1 + \log x)dx =$
 a) $x^x + C$ b) $x^{-x} + x$ c) $x \log x + x$ d) $\log x + x$

Ans: (a)

$$\int x^x(1 + \log x)dx = \int \frac{d}{dx}(x^x)dx = x^x + C$$

28. If $\int \frac{xe^x}{(1+x)^2} dx = e^x f(x) + x$ then $f(x)$ is equal to
 a) $\frac{1}{(1+x)^2}$ b) $\frac{x}{(1+x)}$ c) $\frac{1}{1+x}$ d) $\frac{x}{(1+x)^2}$

Ans: (c)

$$\int \frac{xe^x}{(1+x)^2} dx = \int \left[\frac{x+1-1}{(1+x)^2} \right] e^x dx$$

$$= \int \left[\frac{1}{1+x} - \frac{1}{(1+x)^2} \right] e^x dx = e^x \left(\frac{1}{1+x} \right) + c = e^x f(x) + c \Rightarrow f(x) = \frac{1}{1+x}$$

29. $\int_0^{\pi/2} \frac{\sin 2t}{\sin^4 t + \cos^4 t} dt =$
 a) π b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

Ans: (d)

$$\int_0^{\pi/2} \frac{\sin 2t}{\sin^4 t + \cos^4 t} dt = \int_0^{\pi/2} \frac{2 \sin t \cdot \cos t}{\sin^4 t + \cos^4 t} dt$$

Divide Nr. & Dr. by $\cos^4 t = \int_0^{\pi/2} \frac{2 \tan t \cdot \sec^2 t}{(\tan^2 t)^2 + 1} dt$

put $\tan^2 t = x = \int_0^{\infty} \frac{1}{1+x^2} dx = \tan^{-1} x \Big|_0^{\infty} = \frac{\pi}{2}$

30. If $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$ then x is equal to
 a) 10 b) 4 c) - 10 d) -4

Ans: (c)

$$4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$$

$$\Rightarrow 4^{\frac{1}{2} \log_3 3} + 9^{2 \log_2 2} = 10^{\log_x 83} \Rightarrow 2 + 81 = 10^{\log_x 83} \Rightarrow 83 = 10^{\log_x 83} \Rightarrow x = 10$$

31. If $p = 3^{\frac{1}{3}} \cdot 3^{\frac{2}{9}} \cdot 3^{\frac{3}{27}} \dots \infty$ then $p^{\frac{4}{3}} =$
 a) $3^{\frac{1}{4}}$ b) 3 c) 9 d) $3^{\frac{3}{4}}$

Ans: (b)

$$P = 3^{\frac{1}{3}} \cdot 3^{\frac{2}{9}} \cdot 3^{\frac{3}{27}} \dots \infty = 3^{\frac{1}{3} \left[1 + \frac{2}{3} + \frac{3}{9} + \dots \infty \right]} = 3^{\frac{1}{3} \left[\frac{a}{1-r} + \frac{dr}{(1-r)^2} \right]} = 3^{\frac{1}{3} \left[\frac{1}{1-\frac{1}{3}} + \frac{1 \cdot \frac{1}{3}}{\left(1-\frac{1}{3}\right)^2} \right]}$$

$$P = 3^{\frac{1}{3} \left[\frac{3}{2} + \frac{3}{4} \right]} = 3^{\frac{3}{4}}$$

$$\Rightarrow P^{\frac{4}{3}} = 3$$

32. If α, β, γ are the roots of the equation $x^3 - 3x^2 + 2x - 1 = 0$ then the value of $(1 - \alpha)(1 - \beta)(1 - \gamma)$ is
 a) 1 b) 2 c) -1 d) -2

Ans: (c)

$$(1 - \alpha)(1 - \beta)(1 - \gamma)$$

$$= 1 - (\alpha + \beta + \gamma) + (\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 1 - \left(\frac{3}{1}\right) + \left(\frac{2}{1}\right) - \left(\frac{1}{1}\right) = -1$$

33. The middle term in the expansion of $(1 + x)^{2n}$ is
 a) $\frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{n} x^n$ b) $\frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{n!} 2^{n-1} x^n$
 c) $\frac{1 \cdot 3 \cdot 5 \dots (2n)}{n!} x^n$ d) $\frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{n!} 2^n x^n$

Ans: (d)

$$(1 + x)^{2n}; \text{ Middle term} = t_{n+1}$$

$$= {}^{2n}C_n \cdot 1^{2n-n} \cdot x^n = \frac{(2n)!}{(2n-n)!n!} \cdot x^n = \frac{2n(2n-1)(2n-2)(2n-3)\dots\dots\dots 4 \cdot 3 \cdot 2 \cdot 1}{(n!)(n!)} \cdot x^n$$

$$= \frac{2^n [n(n-1)(n-2)\dots\dots x \cdot 2 \cdot 1] [(2n-1)(2n-3)\dots\dots 3 \cdot 1]}{(n!)(n!)} x^n = \frac{(2n-1)(2n-3)\dots\dots 3 \cdot 1}{n!} 2^n x^n$$

34. If $p \rightarrow (\sim q \vee r)$ is false then the truth values of p, q, r are
 a) T, T, F b) T, F, T c) F, T, T d) F, F, T

Ans: (a)

Given $P \rightarrow (\sim q \vee r)$ is false
 $\Rightarrow (\sim q \vee r)$ is false and p is true
 $\Rightarrow q$ is true, r is false and p is true

35. If $\frac{2}{9!} + \frac{2}{3!7!} + \frac{1}{5!5!} = \frac{2^a}{b!}$ where $a, b \in \mathbb{N}$ then the ordered pair (a, b) is

- a) (10, 9) b) (10, 7) c) (9, 10) d) (5, 10)

Ans: (c)

$$\begin{aligned} \frac{2}{9!} + \frac{2}{3!7!} + \frac{1}{5!5!} &= \frac{2^a}{b!} \\ &= \frac{1}{9!} \left[2 + \frac{2 \cdot 8 \cdot 9}{3!} + \frac{6 \cdot 7 \cdot 8 \cdot 9}{5!} \right] = \frac{1}{9!} \left[2 + 8 \times 3 + \frac{6 \cdot 7 \cdot 8 \cdot 9}{5 \cdot 4 \cdot 3 \cdot 2} \right] \\ &= \frac{1}{9!} \left[2 + 24 + \frac{126}{5} \right] = \frac{1}{9!} \left[130 + \frac{126}{5} \right] = \frac{256 \times 2}{9! \cdot 5 \times 2} = \frac{512}{10!} \Rightarrow \frac{2^9}{10!} \end{aligned}$$

$a = 9, b = 10$

36. $\tan 10^\circ \tan 20^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ =$

- a) 0 b) -1 c) $\frac{1}{\sqrt{3}}$ d) 1

Ans: (d)

$$\begin{aligned} \tan 10^\circ \tan 20^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ \\ = \tan 10^\circ \cdot \tan 20^\circ \cdot \tan 30^\circ \cdot \tan 40^\circ \cdot \cot 40^\circ \cdot \cot 30^\circ \cdot \cot 20^\circ \cdot \cot 10^\circ = 1 \end{aligned}$$

37. If $\tan \theta = \frac{m}{n}$ then $n \cos 2\theta + m \sin 2\theta$ is equal to

- a) n b) n^2 c) $\frac{n}{m}$ d) $\frac{m^2}{n^2}$

Ans: (a)

$$\begin{aligned} n \cdot \cos 2\theta + m \cdot \sin 2\theta \\ = n \cdot \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} + m \cdot \frac{2 \tan \theta}{1 + \tan^2 \theta} = n \cdot \frac{1 - \frac{m^2}{n^2}}{1 + \frac{m^2}{n^2}} + m \cdot \frac{2 \frac{m}{n}}{1 + \frac{m^2}{n^2}} \\ = n \cdot \frac{n^2 - m^2}{n^2 + m^2} + \frac{2m^2 n}{n^2 + m^2} = \frac{n(n^2 + m^2)}{n^2 + m^2} = n \end{aligned}$$

38. If $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ then $\cos A =$

- a) $\frac{5}{7}$ b) $\frac{1}{5}$ c) $\frac{2}{5}$ d) $\frac{1}{7}$

Ans: (b)

$$\begin{aligned} \frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = k \\ \Rightarrow b+c = 11k, c+a = 12k, a+b = 13k \\ \text{adding them, } 2(a+b+c) = 36k \\ \Rightarrow a+b+c = 18k \Rightarrow a = 7k, b = 6k, c = 5k \\ \therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{36k^2 + 25k^2 - 49k^2}{2(30k^2)} = \frac{12k^2}{60k^2} \Rightarrow \cos A = \frac{1}{5} \end{aligned}$$

Ans: (c)

Squaring $\frac{x}{y} + \frac{y}{x} + 2 = a$

Diff; $\frac{y - xy^1}{y^2} + \frac{xy^1 - y}{x^2} = 0$

$$\frac{1}{y} - \frac{xy^1}{y^2} = \frac{y^1}{x} - \frac{y}{x^2}$$

$$\Rightarrow y^1 \left[\frac{1}{x} + \frac{x}{y^2} \right] = \frac{1}{y} + \frac{y}{x^2}$$

$$\Rightarrow y^1 \frac{y^2 + x^2}{xy^2} = \frac{x^2 + y^2}{x^2y}$$

$$\Rightarrow y^1 = \frac{y}{x} \Rightarrow \frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dx}{dy} = \frac{x}{y} \Rightarrow y \frac{dx}{dy} = x$$

$$y^1 = -\frac{fx}{fy} = -\left[\frac{1 - \frac{y}{x^2}}{-\frac{x}{y^2} + \frac{1}{x}} \right]$$

$$= -\left[\frac{\frac{x^2 - y^2}{x^2y}}{\frac{-x^2 + y^2}{y^2x}} \right] = \frac{y}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x} \Rightarrow y \frac{dx}{dy} = x$$

43. If $x = \frac{1-t}{1+t}$: $y = \frac{2t}{1+t}$ then $\frac{d^2y}{dx^2} =$

a) $\frac{2t}{(1+t)^2}$

b) $\frac{1}{(1+t)^4}$

c) $\frac{2t^2}{(1+t)^2}$

d) 0

Ans: (d)

$$\frac{dy}{dt} = \frac{(1+t)2 - 2t}{(1+t)^2} = \frac{2}{(1+t)^2}$$

$$\frac{dx}{dt} = \frac{-(1+t) - (1-t)}{(1+t)^2} = \frac{-2}{(1+t)^2}$$

$$\therefore \frac{dy}{dx} = -1 \therefore \frac{d^2y}{dx^2} = 0$$

44. In the group $G = \{1, 5, 7, 11\}$ under \otimes_{12} the value of $7 \otimes_{12} 11^{-1}$ is equal to

a) 5

b) 7

c) 11

d) 1

Ans: (a)

Clearly $11^{-1} = 11$ ($\because 11 \otimes_{12} 11 = 1$)

$$\therefore 7 \otimes_{12} 11^{-1} = 7 \otimes_{12} 11 = 5$$

45. Which of the following is a subgroup of the group $G = \{1, 2, 3, 4, 5, 6\}$ under \otimes_7

a) $\{2, 6, 1\}$

b) $\{1, 2, 4\}$

c) $\{5, 4, 2\}$

d) $\{2, 3, 1\}$

Ans: (b)

(c) cant be a subgroup as identity '1' is not present

(d) cant be a subgroup as $2 \otimes_7 3 = 6 \notin \{1, 2, 3\}$

(a) cant be a subgroup as $2 \otimes_7 6 = 5 \notin \{2, 6, 1\}$

\therefore (b) is a subgroup

46. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, $B = (\text{adj } A)$ and $C = 5A$ then $\frac{|C|}{|\text{adj}B|}$ is equal to
- a) 25 b) -1 c) 5 d) -5

Ans: Wrong options

$$\frac{|C|}{|\text{adj}B|} = \frac{|5A|}{|B|^{3-1}} = \frac{5^3|A|}{|B|^2} = \frac{5^3|A|}{|\text{Adj}A|^2} = \frac{5^3|A|}{(|A|^2)^2} = \frac{5^3|A|}{|A|^4} = \frac{5^3}{|A|^3}$$

$$|A| = 3(1) + 3(2) + 4(-2)$$

$$9 - 8 = 1$$

$$\therefore GE = \frac{5^3}{1} = 5^3$$

47. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 16 \\ 22 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $AX = B$ then z is equal to
- a) 1 b) -1 c) -3 d) 3

Ans: (d)

$$GE \Rightarrow x + y + z = 7 \quad \text{----- (1)}$$

$$x + 2y + 3z = 16 \quad \text{----- (2)}$$

$$x + 3y + 4z = 22 \quad \text{----- (3)}$$

$$(2) - (1)$$

$$y + 2z = 9 \quad \text{----- (4)}$$

$$(3) - (2); y + z = 6 \quad \text{----- (5)}$$

$$(4) - (5) z = 3$$

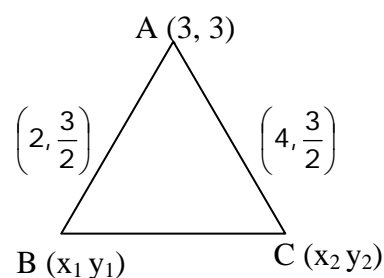
48. If $A = \begin{bmatrix} 1 & \log_b a \\ \log_a b & 1 \end{bmatrix}$ then $|A|$ is equal to
- a) 0 b) $\log_a b$ c) -1 d) $\log_b a$

Ans: (a)

$$|A| = 1 - \log_a b \cdot \log_b a = 1 - 1 = 0$$

49. If a vertex of triangle is $(3, 3)$ and the mid points of two sides through this vertex are $\left(2, \frac{3}{2}\right)$ and $\left(4, \frac{3}{2}\right)$ then the centroid of the triangle is given by
- a) $(1, 3)$ b) $(3, 0)$ c) $(3, 1)$ d) $(0, 3)$

Ans: (c)



$$\left(\frac{x_1+3}{2}, \frac{y_1+3}{2}\right) = \left(2, \frac{3}{2}\right) \quad \left|\quad \left(\frac{x_2+3}{2}, \frac{y_2+3}{2}\right) = \left(4, \frac{3}{2}\right)\right.$$

$$x_1 = 1, y_1 = 0 \quad \left|\quad x_2 + 3 = 8 \quad y_2 = 0\right.$$

$$B \equiv (1, 0) \quad \left|\quad x_2 = 5\right.$$

$$\therefore G = (3, 1) \quad \left|\quad C \equiv (5, 0)\right.$$

50. The image of the point (2, 4) on the line $x + y - 10 = 0$ is
 a) (4, 8) b) (6, 5) c) (6, 8) d) (0, 10)

Ans: (c)

Let the image of (2, 4) = (h, k) Then

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

$$\Rightarrow \frac{h - 2}{1} = \frac{k - 4}{1} = \frac{-2(2 + 4 - 10)}{1^2 + 1^2}$$

$$\Rightarrow h - 2 = +4 \quad \left| \quad k - 4 = 4\right.$$

$$h = 6 \quad \left| \quad k = 8\right.$$

$$\therefore (h, k) = (6, 8)$$

51. If the sum of the slopes of the lines given by $x^2 - 4pxy + 8y^2 = 0$ is three times their product then p has the value

- a) $\frac{1}{4}$ b) 4 c) 3 d) $\frac{3}{4}$

Ans: (d)

Given, $m_1 + m_2 = 3m_1m_2$

$$\Rightarrow \frac{-2h}{b} = \frac{3a}{b} \Rightarrow -2h = 3a \Rightarrow (-4p) = 3 \Rightarrow p = \frac{3}{4}$$

52. $\lim_{x \rightarrow 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}} =$

- a) e^2 b) e c) $\frac{1}{e}$ d) $\frac{5}{3}$

Ans: (a)

$$\lim_{x \rightarrow 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}} = \lim_{x \rightarrow 0} \frac{(1+5x^2)^{\frac{1}{5x^2} \cdot 5}}{(1+3x^2)^{\frac{1}{3x^2} \cdot 3}} = \frac{e^5}{e^3} = e^2.$$

53. If $f(x) = \begin{cases} e^x - 1 & \text{for } x \neq 0 \\ \frac{k+x}{4} & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then $k =$

- a) 5 b) 3 c) 2 d) 0

Ans: (b)

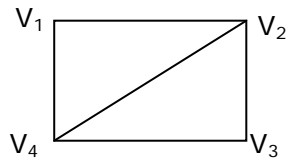
$f(x)$ is continuous at $x = 0$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{e^{3x-1}}{4x} = \frac{k+0}{4} \Rightarrow \lim_{x \rightarrow 0} \frac{e^{3x-1}}{x} = k \Rightarrow \lim_{x \rightarrow 0} 3 \frac{(e^{3x} - 1)}{3x} = k$$

$$\Rightarrow 3.1 = k \Rightarrow k = 3$$

54. The non adjacent vertex in the graph is



- a) V_1V_2 b) V_4V_3 c) V_2V_4 d) V_1V_3

Ans: (d)

55. $\sin \left[2 \cos^{-1} \cot \left(2 \tan^{-1} \frac{1}{2} \right) \right]$ is equal to

- a) $\frac{3\sqrt{7}}{8}$ b) $\frac{5\sqrt{7}}{8}$ c) $\frac{5\sqrt{7}}{2}$ d) $\frac{3\sqrt{7}}{2}$

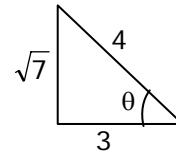
Ans: (a)

$$\sin \left[2 \cos^{-1} \cot \left(\tan^{-1} \frac{1}{1 - \frac{1}{4}} \right) \right] = \sin \left[2 \cos^{-1} \cot \left(\tan^{-1} \frac{4}{3} \right) \right]$$

$$= \sin \left[2 \cos^{-1} \cot \left(\cot^{-1} \frac{3}{4} \right) \right] = \sin \left[2 \cos^{-1} \frac{3}{4} \right]$$

$$= \sin 2\theta \quad \text{where } \cos \theta = \frac{3}{4}$$

$$= 2 \frac{\sqrt{7}}{4} \cdot \frac{3}{4} = \frac{3\sqrt{7}}{8}$$



56. The multiplicative inverse of $\frac{3+4i}{4-5i}$ is

- a) $\left(\frac{-8}{25}, \frac{31}{25} \right)$ b) $\left(\frac{-8}{25}, \frac{-31}{25} \right)$ c) $\left(\frac{8}{25}, \frac{-31}{25} \right)$ d) $\left(\frac{-8}{25}, \frac{31}{5} \right)$

Ans: (b)

$$MI = \frac{4-5i}{3+4i} = \frac{(4-5i)(3-4i)}{9+16} = \frac{-8-31i}{25}$$

$$\text{a) } x = n\pi + \frac{\pi}{4}$$

$$x = n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$$

$$\text{c) } x = n\pi + \frac{\pi}{4}$$

$$\text{b) } x = \frac{n\pi}{4} - \frac{\pi}{4}$$

$$x = n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$$

$$\text{d) } x = n\pi + \frac{\pi}{6}$$

$$x = n\pi + (-1)^n \left(-\frac{\pi}{2}\right)$$

Ans: (a)

$$\tan x - \sin x = 1 - \tan x \sin x$$

$$\frac{\sin x}{\cos x} - \sin x = 1 - \frac{\sin x}{\cos x} \sin x$$

$$\sin x - \sin x \cos x = \cos x - \sin^2 x$$

$$\sin x + \sin^2 x = \cos x + \cos x \sin x$$

$$\sin x (1 + \sin x) = \cos x (1 + \sin x)$$

$$(\sin x - \cos x) (1 + \sin x) = 0$$

$$\sin x - \cos x = 0$$

$$\Rightarrow \tan x = 1$$

$$\Rightarrow \tan x = \tan \frac{\pi}{4}$$

$$x = n\pi + \frac{\pi}{4}$$

$$\sin x = -1 = \sin \left(\frac{-\pi}{2}\right)$$

$$\therefore x = n\pi + (-1)^n \left(\frac{-\pi}{2}\right)$$

OR

$$\tan x + \tan x \sin x = 1 + \sin x$$

$$\tan x (1 + \sin x) = 1 + \sin x$$

$$(1 + \sin x) (\tan x - 1) = 0$$

$$\sin x = -1$$

$$x = n\pi + (-1)^n \left(\frac{-\pi}{2}\right)$$

$$\tan x = 1$$

$$x = n\pi + \frac{\pi}{4}$$

58. The angle between the circles $x^2 + y^2 + 4x + 2y + 1 = 0$ and $x^2 + y^2 - 2x + 6y - 6 = 0$ is

$$\text{a) } \frac{\pi}{6}$$

$$\text{b) } \frac{\pi}{3}$$

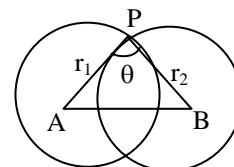
$$\text{c) } \frac{\pi}{2}$$

$$\text{d) } \cos^{-1} \left(\frac{7}{16}\right)$$

Ans: (d)

Using cosine rule in $\triangle APB$

$$AB^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos\theta \quad \text{---- (1)}$$



Here $A \equiv (-2, -1)$, $B \equiv (1, -3)$

$$r_1 = \sqrt{4+1-1} = 2, r_2 = \sqrt{1+9+6} = 4$$

$$AB = \sqrt{9+4} = \sqrt{13}$$

$$(1) \Rightarrow 13 = 4 + 16 - 2(2)(4) \cos\theta$$

$$\Rightarrow 13 = 20 - 16 \cos\theta$$

$$\Rightarrow -7 = -16 \cos\theta$$

$$\Rightarrow \cos\theta = \frac{7}{16} \Rightarrow \theta = \cos^{-1} \frac{7}{16}$$

59. If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ then the angle between \vec{a} and \vec{b} is

a) $\frac{\pi}{3}$

b) $\frac{\pi}{6}$

c) $\frac{\pi}{2}$

d) $\frac{\pi}{4}$

Ans: (b)

$$|\vec{a}| = 2, |\vec{b}| = 7, \vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin\theta$$

$$\Rightarrow \sqrt{9+4+36} = 2 \cdot 7 \cdot \sin\theta$$

$$\Rightarrow \sqrt{49} = 14 \sin\theta \Rightarrow 7 = 14 \sin\theta$$

$$\Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

60. The domain of the function $f(x) = \log(1-x) + \sqrt{x^2-1}$

a) $(-\infty, -1)$

b) $(-\infty, -1]$

c) $(-\infty, 2]$

d) $(-\infty, 0)$

Ans: (a)

$$\log(1-x) \text{ is defined if } 1-x > 0 \Rightarrow x < 1 \Rightarrow x \in (-\infty, 1) \text{ ----- (1)}$$

$$\sqrt{x^2-1} \text{ is defined if } x^2-1 \geq 0 \Rightarrow x^2 \geq 1$$

$$\Rightarrow x \geq 1 \text{ or } x \leq -1 \text{ ----- (2)}$$

\therefore Req'd domain is the intersection of (1) & (2)

i.e. $(-\infty, -1)$