BYJU'S

JEE (MAIN) – 2018 TEST PAPER WITH ANSWER
(HELD ON SUNDAY 08TH APRIL, 2018)

PART – B MATHEMATICS

31. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

is equal to

(1) $\frac{-1}{3(1+\tan^3 x)} + C$ (2) $\frac{1}{1+\cot^3 x} + C$

(3) $\frac{-1}{1+\cot^3 x} + C$ (4) $\frac{1}{3(1+\tan^3 x)} + C$

(where C is a constant of integration)

Ans. (1)

32. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the point P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is -

(1) $54\sqrt{3}$ (2) $60\sqrt{3}$

(3) $36\sqrt{5}$ (4) $45\sqrt{5}$

Ans. (4)

33. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is-

(1) 2 (2) 3 (3) $\frac{4}{3}$ (4) $\frac{1}{2}$

Ans. (1)

34. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} = 24$, then $|\vec{u}|^2$ is equal to-

(1) 315 (2) 256

(3) 84 (4) 336

Ans. (4)

35. If $\alpha, \beta \in C$ are the distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to-

(1) 0 (2) 1 (3) 2 (4) –1

Ans. (2)

36. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$ and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve y = (gof) (x) and the lines $x = \alpha$, $x = \beta$ and y = 0 is-

(1) $\frac{1}{2}(\sqrt{3}+1)$ (2) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

(3) $\frac{1}{2}(\sqrt{2}-1)$ (4) $\frac{1}{2}(\sqrt{3}-1)$

Ans. (4)

37. The sum of the co-efficients of all odd degree terms in the expansion of

$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, (x > 1) is-

(1) 0 (2) 1 (3) 2 (4) –1

Ans. (3)

38. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then m is equal to-

(1) 68 (2) 34 (3) 33 (4) 66

Ans. (2)

39. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is-

(1) 4 (2) 2 (3) 3 (4) 9

Ans. (2)

40. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P,Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is-

(1) 50 (2) $100\sqrt{3}$ (3) $50\sqrt{2}$ (4) 100

Ans. (4)

41. Two sets A and B are as under

$A = \{(a, b) \in R \times R : |a - 5| < 1$ and $|b - 5| < 1\}$;

$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \le 36\}$.

Then :-

(1) $A \subset B$

(2) $A \cap B = \phi$ (an empty set)

(3) neither $A \subset B$ nor $B \subset A$

(4) $B \subset A$

Ans. (1)

42. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is-

(1) less than 500

(2) at least 500 but less than 750

(3) at least 750 but less than 1000

(4) at least 1000

Ans. (4)

43. Let $f(x) = x^2 + \frac{1}{x^2}$ and

$g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$,

then the local minimum value of h(x) is :

(1) –3 (2) $-2\sqrt{2}$ (3) $2\sqrt{2}$ (4) 3

Ans. (3)

44. For each $t \in R$, let [t] be the greatest integer less than or equal to t. Then

$$\lim_{x \to 0+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots\ldots + \left[\frac{15}{x}\right]\right)$$

(1) is equal to 15.

(2) is equal to 120.

(3) does not exist (in R).

(4) is equal to 0.

Ans. (2)

45. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}\, dx$ is :

(1) $\frac{\pi}{2}$ (2) 4π (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{8}$

Ans. (3)

46. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$ (3) $\frac{3}{4}$ (4) $\frac{3}{10}$

Ans. (1)

47. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{2}{\sqrt{3}}$

Ans. (3)

48. If sum of all the solutions of the equation

$8 \cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right) = 1$ in $[0, \pi]$ is $k\pi$, then k is equal to :

(1) $\frac{13}{9}$ (2) $\frac{8}{9}$ (3) $\frac{20}{9}$ (4) $\frac{2}{3}$

Ans. (1)

49. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

(1) $2x + 3y = xy$ (2) $3x + 2y = xy$

(3) $3x + 2y = 6xy$ (4) $3x + 2y = 6$

Ans. (2)

50. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2{\cdot}2^2 + 3^2 + 2{\cdot}4^2 + 5^2 + 2{\cdot}6^2 + \ldots\ldots\ldots$

If $B - 2A = 100\lambda$, then λ is equal to :

(1) 248 (2) 464 (3) 496 (4) 232

Ans. (1)

51. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(1) $\frac{7}{2}$ (2) 4 (3) $\frac{9}{2}$ (4) 6

Ans. (3)

52. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

(1) $2\sqrt{10}$ (2) $3\sqrt{\frac{5}{2}}$

(3) $\frac{3\sqrt{5}}{2}$ (4) $\sqrt{10}$

Ans. (2)

53. Let S = {t ∈ R : $f(x) = |x - \pi|\cdot(e^{|x|} - 1)\sin|x|$ is not differentiable at t}. Then the set S is equal to:

(1) {0} (2) {π}

(3) {0, π} (4) ϕ (an empty set)

Ans. (4)

54. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$,

then the ordered pair (A, B) is equal to :

(1) (–4, 3) (2) (–4, 5)

(3) (4, 5) (4) (–4, –5)

Ans. (2)

55. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to :

(1) p (2) q

(3) ~q (4) ~p

Ans. (4)

56. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) 10 (2) – 30

(3) 30 (4) –10

Ans. (1)

57. Let S = {x ∈ R : x ≥ 0 and $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$}. Then S :

(1) contains exactly one element.

(2) contains exactly two elements.

(3) contains exactly four elements.

(4) is an empty set.

Ans. (2)

58. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is :

(1) 185 (2) 85

(3) 95 (4) 195

Ans. (3)

59. Let y = y(x) be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos x = 4x$, $x \in (0, \pi)$.

If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $\frac{-8}{9\sqrt{3}}\pi^2$ (2) $-\frac{8}{9}\pi^2$

(3) $-\frac{4}{9}\pi^2$ (4) $\frac{4}{9\sqrt{3}}\pi^2$

Ans. (2)

60. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 is :

(1) $\frac{1}{3\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{4\sqrt{2}}$

Ans. (1)