JEE MAINS 2018
QUESTION PAPER & SOLUTIONS
(CODE-B)

PART- A : PHYSICS

ALL THE GRAPHS/DIAGRAMS GIVEN ARE SCHEMATIC AND NOT DRAWN TO SCALE

1. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_C. The values of p_d and p_c are respectively:

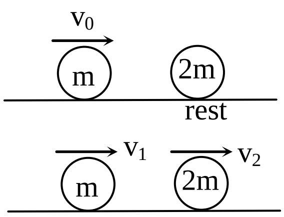
(1) (0, 0) (2) (0, 1) (3) (·89, ·28) (4) (·28, ·89)

1. (3)

L.M.C

$mv_1 + 2m V_2 = mV_0$

$V_1 + 2V_2 = V_0$ ...(1)



$$e = \frac{V_2 - V_1}{V_0} = 1$$

$$V_2 - V_1 = V_0 \quad \text{...(2)}$$

On solving $V_2 = \frac{2v_0}{3}$ and $V_1 = -\frac{V_0}{3}$

$$\text{Final KE of neutron} = \frac{1}{2}mV_1^2 = \frac{1}{2}m\left(\frac{V_0}{3}\right)^2 = \frac{1}{9}\left(\frac{1}{2}mV_0^2\right)$$

$$\text{Loss in KE} = \frac{8}{9}\left(\frac{1}{2}mV_0^2\right)$$

$$\text{Fractional loss} \quad P_d = \frac{8}{9} = 0.89$$

Similarly collision between N and C

$mV_1 + 12m.V_2 = mV_0$

$V_1 + 12V_2 = V_0$ ...(1)

$V_2 - V_1 = V_0$ ...(2)

On Solving $V_2 = \frac{2V_0}{13}$ and $V_1 = -\frac{11V_0}{13}$

$$\text{Final KE} = \frac{1}{2}m\left(\frac{11V_0}{13}\right)^2 = \frac{121}{169}\left(\frac{1}{2}mV_0^2\right)$$

$$\text{Loss in KE} = \frac{48}{169}\left(\frac{1}{2}mV_0^2\right)$$

$$\text{Fractional loss } P_c = \frac{48}{169} = 0.28$$

2. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly:

(1) 2.35×10^2 N/m^2 (2) 4.70×10^2 N/m^2

(3) 2.35×10^3 N/m^2 (4) 4.70×10^3 N/m^2

2. (1)

Change in momentum of one molecule

$\Delta P_1 = 2\text{ mv} \cos 45° = \sqrt{2}\text{ mv}$

Force $F = \frac{\Delta P}{\Delta t} = n \times \Delta P_1$

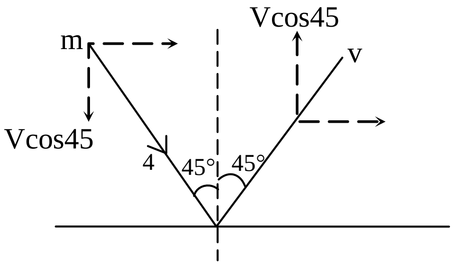
Where $n \rightarrow$ no. of molecules incident per unit time

Pressure $P = \dfrac{\text{Force}}{\text{Area}}$

$P = \dfrac{n \times \sqrt{2}mv}{A}$

$P = \dfrac{10^{23} \times \sqrt{2} \times 3.32 \times 10^{-27} \times 10^{3}}{2 \times 10^{-4}}$

$P = \dfrac{3.32}{1.41} \times 10^{3} = 2.35 \times 10^{3}\ \text{N/m}^2$



3. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\dfrac{dr}{r}\right)$, is:

(1) $\dfrac{mg}{3Ka}$ (2) $\dfrac{mg}{Ka}$ (3) $\dfrac{Ka}{mg}$ (4) $\dfrac{Ka}{3mg}$

3. (1)

Bulk modulus

$K = \left(\dfrac{-dP}{dV/V}\right)$

$\dfrac{dV}{V} = \dfrac{dP}{K}$

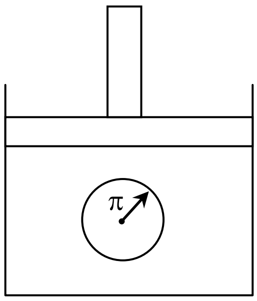
$\dfrac{dV}{V} = \dfrac{mg}{Ka}$ … (1)

$V = \dfrac{4}{3}\pi r^3$

$\dfrac{dV}{V} = 3\dfrac{dr}{r}$ … (2)

From eq. (1) and (2)

$\dfrac{dr}{r} = \dfrac{mg}{3Ka}$



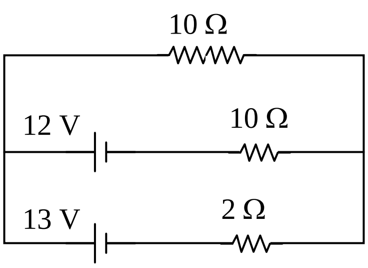
4. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between.

(1) 11.4 V and 11.5 V
(2) 11.7 V and 11.8 V
(3) 11.6 V and 11.7 V
(4)11.5 V and 11.6 V

4. (4)

$E_{eq} = \dfrac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}} = \dfrac{\dfrac{12}{1} + \dfrac{13}{2}}{\dfrac{1}{1} + \dfrac{1}{2}} = \dfrac{37}{3}$

$r_{eq} = \dfrac{1 \times 2}{1 + 2} = \dfrac{2}{3}\,\Omega$



$$i = \frac{E_{eq}}{R + r_{eq}} - \frac{\frac{37}{3}}{10 + \frac{2}{3}} = \frac{37}{32}\,\text{amp}$$

$$\Delta V = iR = \frac{37}{32} \times 10 = \frac{185}{16} = 11.56$$

5. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is:

(1) Zero (2) $-\frac{3}{2}\frac{k}{a^2}$ (3) $-\frac{k}{4a^2}$ (4) $\frac{k}{2a^2}$

5. (1)

$$U = -\frac{k}{2r^2}$$

$$F = -\frac{du}{dr} = +\frac{k}{2}\left(\frac{-2}{r^3}\right)$$

$$F = -\frac{k}{r^3}$$

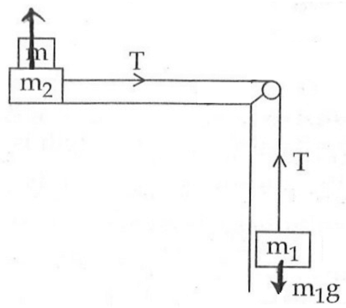
Centripetal force $\frac{mv^2}{r} = \frac{k}{r^3}$

$$mv^2 = \frac{k}{r^2}$$

$$kE = \frac{1}{2}mv^2 = \frac{k}{2r^2}$$

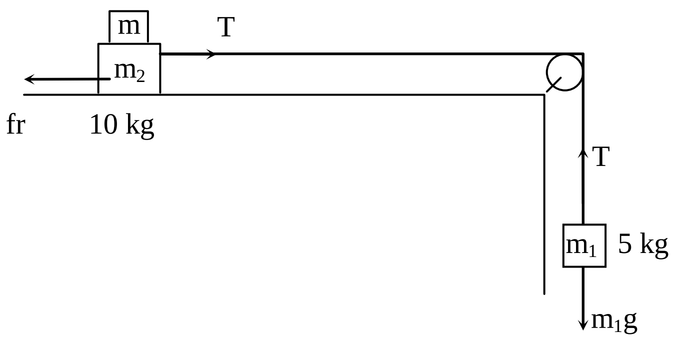
Total Energy E = K + U = O

6. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(1) 43.3 kg (2) 10.3 kg
(3) 18.3 (4) 27.3 kg

6. (4)



at equilibrium

$fr = T = m_1g$

$fr_{max} = \mu\,(m_2 + m)g$

$\mu(10 + m)\,g = 5\,g$

$$10 + m = \frac{5}{0.15}$$

$$10 + m = \frac{100}{3}$$

$m = \frac{70}{3}$ kg = 23.3 kg. The minimum weight in the options is 27.3 kg.

7. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is:

(1) $\nu_L/16$ (2) $\nu_L/25$ (3) $25\,\nu_L$ (4) $16\,\nu_L$

7. (2)

Series limit is

Ly man : $\infty \to 1$

P fund : $\infty \to 5$

$\nu_{Lyman} = RC$

$\nu_{Pfund} = \dfrac{RC}{25}$

8. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. the intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is:

(1) 45° (2) 60° (3) 0° (4) 30°

8. (1)

Unpolarized light passes the A

$\Rightarrow I_{after\ A} = \dfrac{I}{2}$

$I_{after\ B} = \dfrac{I}{2}$ given

$\Rightarrow \angle$ between A and B is 90°

Let A and C have θ

$I_{after\ C} = \dfrac{I}{2}\cos^2\theta$

$I_{after\ B} = \dfrac{I}{2}\cos^2\theta\cos^2(90-\theta) = \dfrac{I}{8}$ $\therefore$ $[\cos\theta\sin\theta]^2 = \dfrac{1}{4}$

$\left[\dfrac{\sin 2\theta}{2}\right]^2 = \dfrac{1}{4}$

$\sin 2\theta = 1 \Rightarrow \theta = 45°$

9. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let A_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n^2 \approx A + B\lambda_n^2$ (2) $\Lambda_n^2 \approx \lambda$ (3) $\Lambda_n \approx A + \dfrac{B}{\lambda_n^2}$ (4) $\Lambda_n \approx A + B\lambda_n$

9. (3)

$m\nu r = \dfrac{nh}{2\pi}$

$\lambda_{de\ Broglie} = \dfrac{h}{p} = \dfrac{2\pi r}{n}$

$\lambda_n = \dfrac{2\pi r_n}{n}$

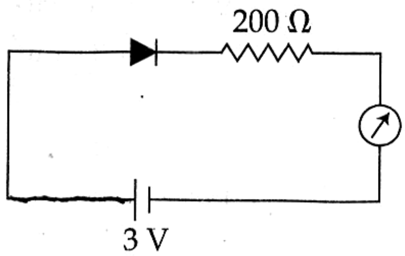
$\lambda_g = \frac{2\pi r_1}{1} = 2\pi r_1$ $\therefore \frac{\lambda_n}{\lambda_g} = \frac{r_n}{nr_1} = n$

$$\frac{1}{\lambda} = R\left[1 - \frac{1}{n^2}\right]$$

$$\Rightarrow \wedge_n = \frac{n^2}{R(n^2-1)} = \frac{1}{R}\left[\frac{\lambda_n^2}{\lambda_n^2 - \lambda_g^2}\right]$$

$$= \frac{1}{R}\left[\frac{1}{1-\left(\frac{\lambda_g}{\lambda_n}\right)^2}\right] \approx \frac{1}{R}\left[1+\left(\frac{\lambda_g}{\lambda_n}\right)^2\right] = A + \frac{B}{\lambda_n^2}$$

10. The reading of the ammeter for a silicon diode in the given circuit is:



(1) 11.5 mA (2) 13.5 mA (3) 0 (4) 15 mA

10. (1)

Knowledge based

Si diode has forward bias resistance

200 Ω at 2 V

400 Ω at 1 V

$\Rightarrow$ here $R_{fb} < 200\ \Omega$ … at 3V

11. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is:

(1) $r_e < r_p < r_\alpha$ (2) $r_e < r_\alpha < r_p$ (3) $r_e > r_p = r_\alpha$ (4) $r_e < r_p = r_\alpha$

11. (4)

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

$K = \frac{1}{2}mv^2$ …same

$$= \frac{p^2}{2m} \Rightarrow p \propto \sqrt{m}$$

B same

$$\therefore r \propto \frac{p}{q} \text{ or } \frac{\sqrt{m}}{q}$$

$q_p = q_e$ $q_\propto = 2q_p$

$m_p = 1836\ m_e$ $m_\alpha = 4\ m_p$

12. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be :

(1) 2.4 n C (2) 0.9 n C (3) 1.2 n C (4) 0.3 n C

12. (3)
Battery remains connected as dielectric is introduced
So E, V unchanged
$q_0 = C_0V$
$q = kC_0V$
Induced charge $q' = q - q_0$
$= C_0V(k-1)$
$= 90 \times 10^{-12} \times 20\left(\frac{5}{3}-1\right) = 1.2$ nc

13. For an RLC circuit driven with voltage of amplitude ν_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by:

(1) $\frac{R}{(\omega_0 C)}$ (2) $\frac{CR}{\omega_0}$ (3) $\frac{\omega_0 L}{R}$ (4) $\frac{\omega_0 R}{L}$

13. (3)
Quantity factor
$Q = \frac{\omega_0 L}{R}$ … from theory

14. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How

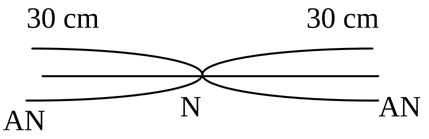
(1) 2×10^5 (2) 2×10^6 (3) 2×10^3 (4) 2×10^4

14. (1)
No of channels = $\frac{\text{carrier frequency} \times 0.1}{\text{channel bandwidth}} = \frac{0.1 \times 10 \times 10^9}{5 \times 10^3} = 2 \times 10^5$

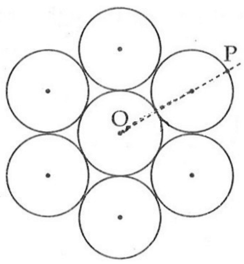
15. a granite rod of 60 cm length is clamed at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 10 kHz (2)7.5 kHz (3) 5 kHz (4) 2.5 kHz

15. (3)
$\nu = \sqrt{\frac{Y}{\rho}} \quad \frac{\lambda}{4}$
$= \frac{\ell}{2} \Rightarrow \lambda = 2\ell$
$\therefore \quad n = \frac{\nu}{\lambda} = \frac{1}{2\ell}\sqrt{\frac{Y}{\rho}}$



16. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is:



(1) $\frac{73}{2}MR^2$ (2) $\frac{181}{2}MR^2$

(3) $\frac{19}{2}MR^2$ (4) $\frac{55}{2}MR^2$

16. (2)

M.I. about origin

$$I_0 = \frac{MR^2}{2} + 6\left[\frac{MR^2}{2} + M(2R)^2\right]$$

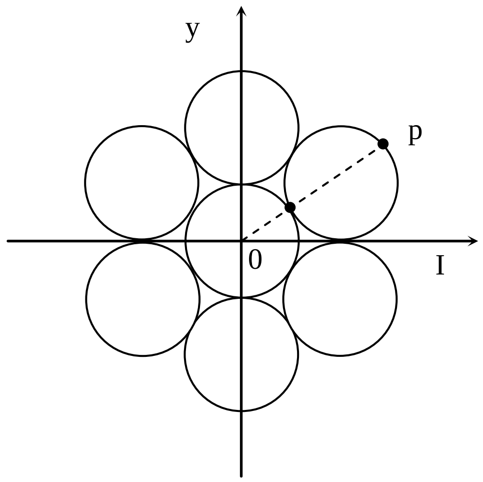
$$I_0 = \frac{MR^2}{2} + 27MR^2$$

$$\boxed{I_0 = \frac{55}{2}MR^2}$$

$$I_p = I_0 + 7M(3R)^2$$

$$I_p = \frac{55}{2}MR^2 + 63MR^2$$

$$\boxed{I_p = \frac{181}{2}MR^2}$$



17. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities + σ, –σ and + σ respectively. The potential of shell B is :

(1) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2 - c^2}{b} + a\right]$

(2) $\frac{\sigma}{\epsilon_0}\left[\frac{b^2 - c^2}{c} + a\right]$

(3) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{a} + c\right]$

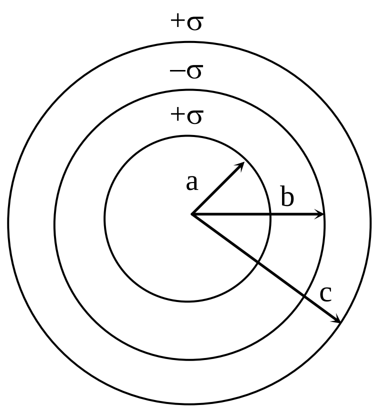
(4) $\frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$

17. (4)

$$V_B = \frac{k(\sigma \times 4\pi a^2)}{b} - \frac{k(\sigma \times 4\pi b^2)}{b} + \frac{k(\sigma \times 4\pi c^2)}{c}$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2}{b} - b + c\right]$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$



18. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 2 Ω (2) 2.5 Ω (3) 1 Ω (4) 1.5 Ω

18. (4)

When switch s is opened

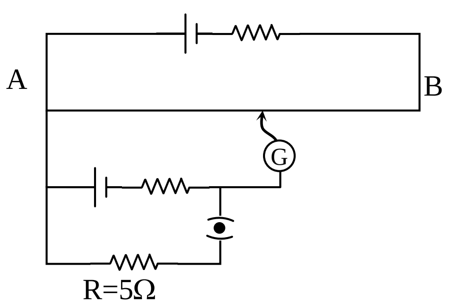
$E = \lambda L_1$ … (I)

Where λ is potential gradient

when switch is closed $E - ir = \lambda L_2$ … (II)

$$\frac{(II)}{(I)} \quad \frac{E - ir}{E} = \frac{L_2}{L_1}$$

$$1 - \frac{ir}{E} = \frac{L_2}{L_1}$$



Replace $i = \frac{E}{R+r}$

$1 - \frac{r}{R+r} = \frac{L_2}{L_1}$

$\frac{R}{R+r} = \frac{L_2}{L_1}$

$r = R\left(\frac{L_1}{L_2} - 1\right) = 5\left(\frac{52}{40} - 1\right) = 5 \times \frac{12}{40} = 1.5\ \Omega$

19. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos\left[k(2z - ct\right]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is non–magnetic. If $\in_{r_1}$ and $\in_{r_2}$ refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{4}$ (2) $\frac{\in_{r_1}}{\in_{r_2}} = \frac{1}{2}$ (3) $\frac{\in_{r_1}}{\in_{r_2}} = 4$ (4) $\frac{\in_{r_1}}{\in_{r_2}} = 2$

19. (1)

$E_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{\cancel{z}}{c} - t\right)\right]$

$E_2 = E_{02}\hat{x}\cos\left[k\left(2\cancel{z} - ct\right)\right] = E_{02}\hat{x}\cos\left[\frac{2\pi}{\lambda} \times c\left(\frac{2\cancel{z}}{c} - t\right)\right]$

$= E_{02}\hat{x}\cos\left[2\pi\nu\left(\frac{2\cancel{z}}{c} - t\right)\right]$

Velocity in new medium

$V = \frac{c}{2}$

$\frac{1}{\sqrt{\mu_0 \in_2}} = \frac{1}{2} \times \frac{1}{\sqrt{\mu_0 \in_1}}$

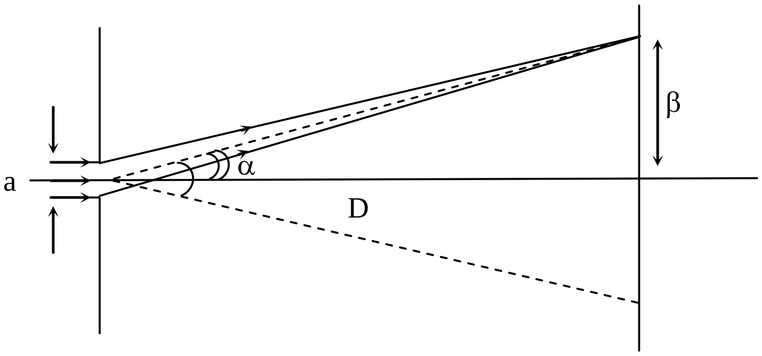
$\boxed{\frac{\in_1}{\in_2} = \frac{1}{4}}$ {relative permittivity $\in_r = \frac{\in}{\in_0}$ }

20. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?
(i.e., distance between the cenrtres of each slit.)

(1) 75 μm (2) 100 μm (3) 25 μm (4) 50 μm

20. (3)

$2\alpha = 60°$
$a = 1\mu m$
$D = 50$ cm



$$\frac{\text{Cond. for minima}}{\text{Path diff } \Delta x = a\sin\theta = n\lambda}$$

a = 1μm and θ = 30° and n = 1

$$\boxed{\lambda = 0.5\mu m}$$

If same setup is used for YDSE

Fringe width $\beta = \frac{\lambda D}{d} = 1\,\text{cm}$

$$d = \frac{0.5\times10^{-6}\times0.5}{0.01} = 25\mu m$$

$$\boxed{d = 25\,\mu m}$$

21. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1})

(1) 2.2 N/m (2) 5.5 N/m (3) 6.4 N/m (4) 7.1 N/m

21. (4)

Frequency $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$k = m\,(2\pi f)^2$

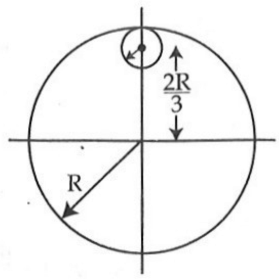
Mass of 1 atom

$m = \frac{108}{6.02\times10^{23}} = 18 \times 10^{-23}$ gm $= 18 \times 10^{-26}$ kg

$k = 18 \times 10^{-26}\,(2\pi \times 10^{12})^2 = 4\pi^2 \times 18 \times 10^{-2}$

$k = 7.2$ N/m ($\pi^2 = 10$)

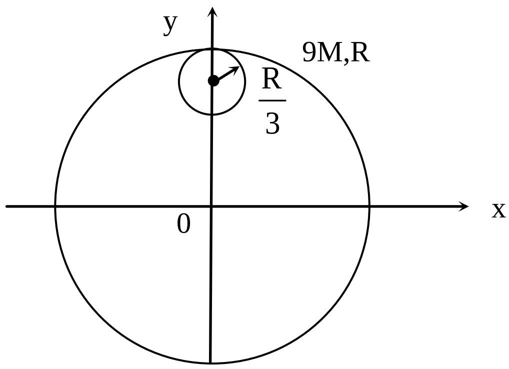
22. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is :



(1) $10\,MR^2$ (2) $\frac{37}{9}MR^2$

(3) $4\,MR^2$ (4) $\frac{40}{9}MR^2$

22. (3)



Mass ∝ Area, $M \propto R^2$

Mass of portion removed

$\frac{M_1}{M_0} = \frac{1}{9}$, $M_1 = \frac{M_0}{9} = M$

$$I_0 = \frac{9MR^2}{2} - \left[\frac{M\left(\frac{R}{3}\right)^2}{2} + M\left(\frac{2R}{3}\right)^2\right]$$

$$I_0 = \frac{9MR^2}{2} - \left[\frac{MR^2}{18} + \frac{4MR^2}{9}\right]$$

$$I_0 = \frac{9MR^2}{2} - \left[\frac{9MR^2}{18}\right] = \frac{9MR^2}{2} - \frac{MR^2}{2}$$

$$\boxed{I_0 = 4MR^2}$$

23. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is :

(1) $\frac{v_0}{2}$ (2) $\frac{v_0}{\sqrt{2}}$ (3) $\frac{v_0}{4}$ (4) $\sqrt{2}\, v_0$

23. (4)

L.M.C.

$mv_1 + m V_2 = m V_0$

$V_1 + V_2 = V_0$ …. (I)

Initial kE = $\frac{1}{2}mV_0^2$

final kE = $\frac{1}{2}\ mV_1^2 + \frac{1}{2}mV_2^2$

final kE = $\frac{3}{2}$ initial kE

$\frac{1}{2}m\left(V_1^2 + V_2^2\right) = \frac{3}{2}\left(\frac{1}{2}mV_0^2\right)$

$V_1^2 + V_2^2 = \frac{3}{2}V_0^2$ … (II)

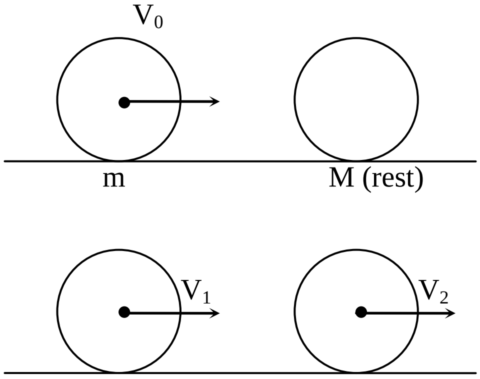
$(I)^2 - (II)$

$2V_1V_2 = -\frac{1}{2}V_0^2$ … (III)

$\left(V_1 - V_2\right)^2 = (V_1 + V_2)^2 - 4V_1V_2$

$\left(V_1 - V_2\right)^2 = V_0^2 + V_0^2 = 2V_0^2$

$V_{rel} = \left|V_1 - V_2\right| = V_0\sqrt{2}$



24. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is :

(1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) 2 (4) $\sqrt{3}$

24. (1)

Dipole moment $\mu = niA$

$\mu = i \times \pi R^2$

If dipole moment is doubled keeping current const.

$\boxed{R_2 = R_1\sqrt{2}}$

Magnetic Field at center of loop

$B = \frac{\mu_0 i}{2R}$

$\frac{B_1}{B_2} = \frac{R_2}{R_1} = \frac{\sqrt{2}}{1}$

25. The density of the material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is :

(1) 4.5 % (2) 6 % (3) 2.5 % (4) 3.5 %

25. (1)

$\rho = \dfrac{m}{V} = \dfrac{m}{\ell^3}$

$\dfrac{\Delta\rho}{\rho}\times 100 \;=\; \dfrac{\Delta m}{m}\times 100 + 3\dfrac{\Delta \ell}{\ell}\times 100$

$= 1.5 + 3 \times 1 = 1.5 + 3 = 4.5$

26. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances?

(1) 550 Ω (2) 910 Ω (3) 990 Ω (4) 505 Ω

26. (1)

Let balancing length is L

$\dfrac{R_1}{R_2} = \dfrac{L}{100-L}$ … (1)

If R_1 and R_2 are interchanged balancing length is (L – 10)

$\dfrac{R_2}{R_1} = \dfrac{L-10}{110-L}$ …(2)

From eq. (1) and (2)

$\dfrac{L}{100-L} = \dfrac{100-L}{L-10}$

$\Rightarrow L^2 - 10L = 110 \times 100 + L^2 - 210\,L$

$\Rightarrow 200\,L = 110 \times 100$

L = 55 cm

$\dfrac{R_1}{R_2} = \dfrac{55}{45} = \dfrac{11}{9}$

$R_1 + R_2 = 1000\ \Omega$

On solving

$R_1 = 550\ \Omega$ and $R_2 = 450\ \Omega$

27. In an a.c. circuit, the instantaneous e.m.f. and current are given by

e = 100 sin 30 t

$i = 20 \sin\left(30\,t - \dfrac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattles current are, respectively :

(1) $\dfrac{50}{\sqrt{2}}, 0$ (2) 50, 0 (3) 50, 10 (4) $\dfrac{1000}{\sqrt{2}}, 10$

27. (4)

e = 100 sin 30 t $\quad\therefore\quad e_{rms} = \dfrac{100}{\sqrt{2}}$

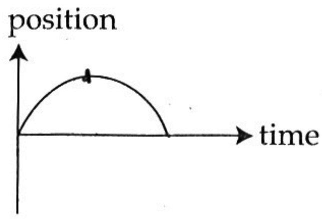
$i = 20 \sin\left(30t - \dfrac{\pi}{4}\right) \quad\therefore\quad i_{rms} = \dfrac{20}{\sqrt{2}}$

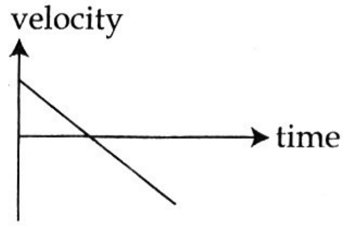
$P = e_{rms}\cdot I_{rms}\cdot \cos\dfrac{\pi}{4} = \dfrac{100}{\sqrt{2}}\times\dfrac{20}{\sqrt{2}}\times\dfrac{1}{\sqrt{2}} = \dfrac{2000}{2\sqrt{2}} = \dfrac{1000}{\sqrt{2}}$

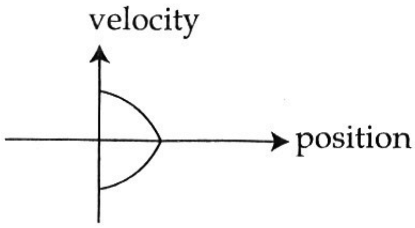
Wattless current $I = I_{rms}\sin\dfrac{\pi}{4}$

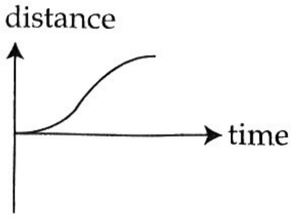
$= \dfrac{20}{\sqrt{2}}\times\dfrac{1}{\sqrt{2}} = \dfrac{20}{2} = 10$

28. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(1) 

(2) 

(3) 

(4) 

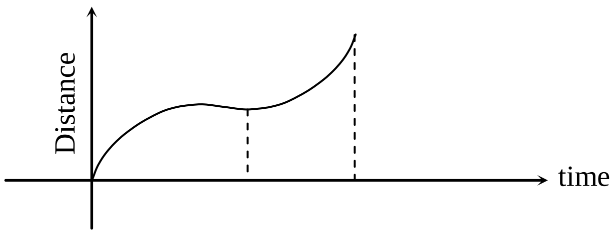
28. (4)

If V *Vs* t is a straight line with –ve slope acc = –ve const.

Displacement *Vs* time is a parabola opening downward

Only incorrect option is (4)

Correct distance *Vs* time graph is



29. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 189 K (b) −2.7 kJ
(2) (a) 195 K (b) 2.7 kJ
(3) (a) 189 K (b) 2.7 kJ
(4) (a) 195 K (b) −2.7 kJ

29. (1)

$\gamma = \frac{5}{3}$ for monoatomic gas.

$T_1 = 27°\ C = 273 + 27 = 300\ K$

$\frac{V_2}{V_1} = 2$

$T\ V\ \gamma - 1 = \text{const.}$

$T_2\ V_2^{\gamma-1} = T_1\ V_1^{\gamma-1}$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{1}{2}\right)^{\frac{5}{3}-1} = \left(\frac{1}{2}\right)^{\frac{2}{3}} = \left(\frac{1}{4}\right)^{\frac{1}{3}} = 0.63$$

$T_2 = 300 \times 0.63$ $\quad\therefore\ T_2 = T_1 \times 0.63$

$= 189\ K$

$\Delta U = \frac{f}{2} nR\Delta T$

$\Delta U = \frac{-3}{2} \times 2 \times 8.3 \times 111$

$\boxed{\Delta U = -2.76\ \text{kJ.}}$

30. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then :

(1) $T \propto R^{(n+1)/2}$ (2) $T \propto R^{n/2}$ (3) $T \propto R^{3/2}$ for any n (4) $T \propto R^{\frac{n}{2}+1}$

30. (1)

$$\frac{mv^2}{R} = K \cdot \frac{1}{R^n}$$

$$v^2 = K \cdot \frac{R}{mR^n} = K \cdot \frac{1}{m\ R^{n-1}}$$

$$v = K' \cdot \frac{1}{R^{\frac{(n-1)}{2}}} \qquad \left[K' = \sqrt{\frac{k}{m}}\right]$$

$$T = \frac{2\pi R}{v} = \frac{2\pi R \times R^{\frac{n-1}{2}}}{K'} = \frac{2\pi}{K'} \cdot R^{\frac{n+1}{2}}$$

$$\therefore \quad T \propto R^{\frac{n+1}{2}}$$

PART- B : MATHEMATICS

31. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$, then the value of c is :

(1) 85 (2) 95 (3) 195 (4) 185

31. (2)

Equation of tangent at (1, 7) to $x^2 = y - 6$ is $2x - y = -5$.

It touches circle $x^2 + y^2 + 16x + 12y + c = 0$.

Hence length of perpendicular from centre (−8, −6) to tangent equals radius of circle.

$$\therefore \quad \left|\frac{-16+6+5}{\sqrt{(2)^2+(-1)^2}}\right| = \sqrt{64+36-c} \quad \Rightarrow c = 95$$

32. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 is :

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{4\sqrt{2}}$ (4) $\frac{1}{3\sqrt{2}}$

32. (4)

$$\begin{vmatrix} \ell & m & n \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \ell + m \leftarrow \text{drs of line } L_1$$

$$\begin{vmatrix} \ell & m & n \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\ell - 5m - 7n \leftarrow \text{drs of line } L_2$$

$$\begin{vmatrix} \ell & m & n \\ 1 & 1 & 0 \\ 3 & 5 & -7 \end{vmatrix} = -7\ell - 7m - 8n \leftarrow \text{Normal plane containing line } L_1 \text{ and } L_2$$

For one point of line L_1

$$2x - 2y + 3z - 2 = 0$$
$$x - y + z = 1 = 0$$
$$\left.\begin{array}{l} -2y + 3z = 2 \\ -y + z = -1 \end{array}\right\} \text{Solving } (0, 5, 4)$$

So, equation of plane is $-7(x - 0) + 7(y - 5) - 8(z - 4) = 0$

$$7x - 7y + 8z + 3 = 0$$

$$\text{Distance} = \left|\frac{7\times0-7\times0+8\times0+3}{\sqrt{7^2+7^2+8^2}}\right| = \frac{1}{3\sqrt{2}}$$

33. If $\alpha, \beta \in C$ are the distinct roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to :

(1) 1 (2) 2 (3) -1 (4) 0

33. (1)

$$x^2 - x + 1 = 0$$

$$x = \frac{1\pm\sqrt{1-4\times1\times1}}{2\times1} = \frac{1\pm i\sqrt{3}}{2}$$
$$= \frac{1+i\sqrt{3}}{2}, \quad \frac{1-i\sqrt{3}}{2}$$
$$= -\frac{-1-i\sqrt{3}}{2}, \quad -\frac{-1+i\sqrt{3}}{2}$$
$$= -\omega^2, -\omega$$

$$\alpha^{101} + \beta^{107} = (-\omega^2)^{101} + (-\omega)^{107}$$
$$= -[\omega^{202} + \omega^{107}]$$
$$= -[(\omega^3)^{67}\,\omega + (\omega^3)^{35}\,\omega^2]$$
$$= -[\omega + \omega^2] = 1$$

34. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is :

(1) $60\sqrt{3}$ (2) $36\sqrt{5}$ (3) $45\sqrt{5}$ (4) $54\sqrt{3}$

34. (3)

$$4x^2 - y^2 = 36 \quad \Rightarrow \frac{x^2}{9} - \frac{y^2}{36} = 1 \quad \Rightarrow a^2 = 9, \quad b^2 = 36$$

From T(0, 3) tangents are drawn to hyperbola at P and Q.

Hence equation of Chord of contact PQ is

$$\frac{x(0)}{9} - \frac{y(3)}{36} = 1 \qquad \Rightarrow y = -12$$

$$\therefore \qquad \frac{x^2}{9} - \frac{144}{36} = 1 \qquad \Rightarrow x^2 = 45 \qquad \Rightarrow x = \pm\, 3\sqrt{5}$$

Hence $P \equiv \left(3\sqrt{5},\ -12\right)$ and $Q \equiv \left(-3\sqrt{5},\ -12\right)$

$$\text{Hence A } (\Delta PQT) \text{ is } \frac{1}{2}\begin{vmatrix} 3\sqrt{5} & -12 & 1 \\ -3\sqrt{5} & -12 & 1 \\ 0 & 3 & 1 \end{vmatrix} = 45\sqrt{5}$$

35. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(1) 4 (2) $\frac{9}{2}$ (3) 6 (4) $\frac{7}{2}$

35. (2)

$y^2 = 6x$

$\frac{dy}{dx} = \frac{3}{y}$

$9x^2 + by^2 = 16$

$18x + 2by\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{9x}{by}$

Let intersection point (x_1, y_1)

$$\left(\frac{dy}{dx}\right)_{(x_1, y_1)} \cdot \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 1$$

$$\Rightarrow \frac{3}{y_1} \times \left(-\frac{9x_1}{by_1}\right) = -1$$

$27x_1 = by_1^2$ [As (x_1, y_1) lies on $y^2 = 6x \Rightarrow y_1^2 = 6x_1$]

$27x_1 = b\,(6x_1)$

$b = \frac{9}{2}$

36. If the system of linear equations :

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) − 30 (2) 30 (3) − 10 (4) 10

36. (4)

Non–zero solution $\Rightarrow \Delta = \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$\Rightarrow 1.\,(-3k + 8) - k\,(-5) + 3\,(12 - 2k) = 0$

$k = 11$

$x + 11y + 3z = 0$

$3x + 11y - 2z = 0$

$$\frac{x}{-22-33} = \frac{y}{-(-2-9)} = \frac{z}{11-33}$$

$$\frac{x}{-55} = \frac{y}{11} = \frac{z}{-22}$$

$$\frac{x}{5} = \frac{y}{-1} = \frac{z}{2} = L \quad \text{(Let)}$$

$$\frac{xz}{y^2} = \frac{(5L)(2L)}{(-L)^2} = 10$$

37. Let S = {$x \in R : x \geq 0$ and $2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0$}. Then S :

(1) contains exactly two elements. (2) contains exactly four elements.

(3) is an empty set. (4) contains exactly one element.

37. (1)

$$2\left|\sqrt{x}-3\right|+\sqrt{x}\left(\sqrt{x}-6\right)+6=0$$

$$2\sqrt{x}-6+x-6\sqrt{x}+6=0 \text{ if } \sqrt{x}>3$$

$$x-4\sqrt{x}=0 \quad \Rightarrow \quad \sqrt{x}\left(\sqrt{x}-4\right)=0 \quad \Rightarrow \sqrt{x}=0,4 \Rightarrow \sqrt{x}=4$$

Also, $2\left(3-\sqrt{x}\right)+\sqrt{x}\left(\sqrt{x}-6\right)+6=0$ if $\sqrt{x}<3$

$$6-2\sqrt{x}+x-6\sqrt{x}+6=0$$

$$x-8\sqrt{x}+12=0 \qquad \Rightarrow \quad \left(\sqrt{x}\right)^2-6\sqrt{x}-2\sqrt{x}+12=0$$

$$\therefore \quad \left(\sqrt{x}-2\right)\left(\sqrt{x}-6\right)=0 \qquad \Rightarrow \quad \sqrt{x}=2,6 \Rightarrow \sqrt{x}=2$$

38. If sum of all the solutions of the equation, $8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0, \pi]$ is $k\pi$, then k is equal to :

(1) $\frac{8}{9}$ (2) $\frac{20}{9}$ (3) $\frac{2}{3}$ (4) $\frac{13}{9}$

38. (4)

$$8\cos x\left[\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right]=1$$

$$8\cos x\left[\frac{\cos\left(\frac{\pi}{3}\right)+\cos 2x-1}{2}\right]=1$$

$$4\cos x\left(\cos 2x-\frac{1}{2}\right)=1$$

$$4\cos x\left(2\cos^2 x-\frac{3}{2}\right)=1$$

$$8\cos^3 x-6\cos x-1=0$$

$$2(4\cos^3 x-3\cos x)-1=0$$

$$2\cos 3x-1=0$$

$$\cos 3x=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$3x=2n\pi\pm\frac{\pi}{3}$$

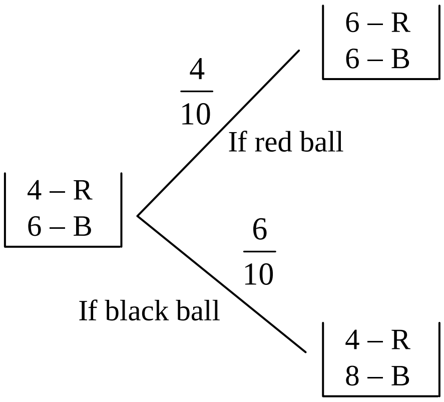
$$x=\frac{2n\pi}{3}\pm\frac{\pi}{9}=\frac{7\pi}{9},\frac{5\pi}{9},\frac{\pi}{9} \text{ in } [0, \pi]$$

$$\therefore \quad k=\frac{13}{9}$$

39. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

(1) $\frac{1}{5}$ (2) $\frac{3}{4}$ (3) $\frac{3}{10}$ (4) $\frac{2}{5}$

39. (4)



From total Probability Theorem

$$P(R) = \frac{4}{10}\times\frac{1}{2}+\frac{6}{10}\times\frac{4}{12}$$

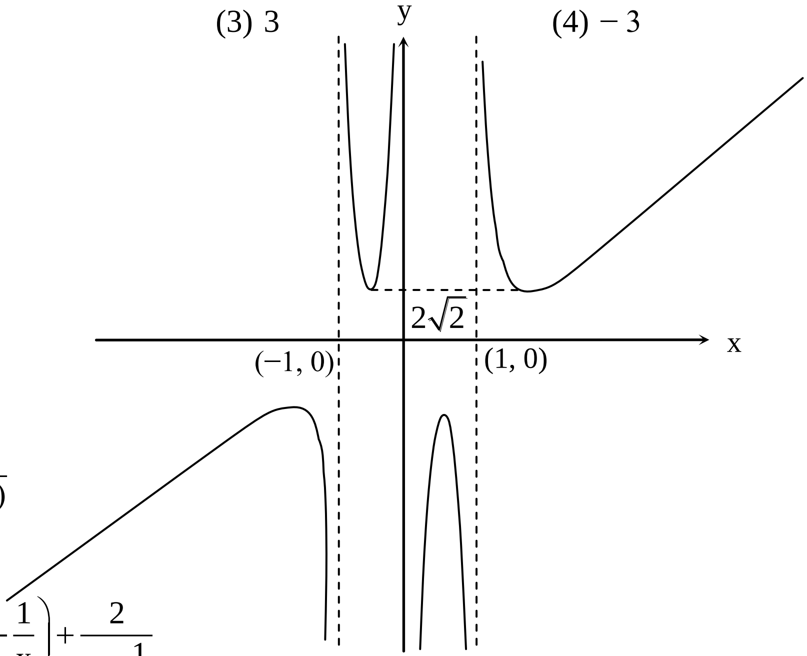
$$= \frac{1}{5}+\frac{1}{5}$$

$$= \frac{2}{5}$$

40. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x-\frac{1}{x}$, $x \in \mathbf{R} - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of h(x) is :

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$ (3) 3 (4) − 3

40. (2)



$$h(x) = \frac{f(x)}{g(x)} = \frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}$$

$$= \frac{x^4+1}{(x^2-1)x}$$

$$= \frac{x^4+1}{x^3-x}$$

$$= \frac{x^4+1}{x(x+1)(x-1)}$$

$$h(x) = \frac{f(x)}{g(x)} = \frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}} = \left(x-\frac{1}{x}\right)+\frac{2}{x-\frac{1}{x}}$$

When $x-\frac{1}{x}<0 \Rightarrow \left(x-\frac{1}{x}\right)+\frac{2}{x-\frac{1}{x}} \le -2\sqrt{2}$

$x-\frac{1}{x}>0 \Rightarrow \left(x-\frac{1}{x}\right)+\frac{2}{x-\frac{1}{x}} \ge 2\sqrt{2}$

Minimum value $2\sqrt{2}$

41. Two sets A and B are as under :

$A = \{(a, b) \in \mathbf{R} \times \mathbf{R} : |a - 5| < 1 \text{ and } |b - 5| < 1\}$'

$B = \{(a, b) \in \mathbf{R} \times \mathbf{R} : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$. Then :

(1) $A \cap B = \phi$ (an empty set) (2) neither $A \subset B$ nor $B \subset A$

(3) $B \subset A$ (4) $A \subset B$

41. (4)

$A = \{(a, b) \in R \times R : |a-5| < 1 \text{ and } |b-5| < 1\}$

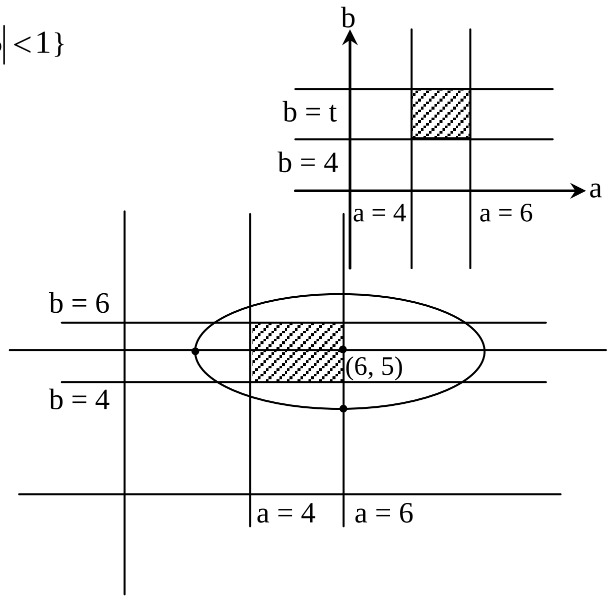
$\Rightarrow -1 < a - 5 < 1$

$\boxed{4 < a < 6}$ $\boxed{4 < b < 6}$

$B = \{(a, b) \in R \times R$

$4(a-6)^2 + 9(b-5)^2 \leq 36$

$$\frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \leq 1$$



42. The Boolean expression : $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to :

(1) q (2) ~q (3) ~p (4) p

42. (3)

p (1)	**q (2)**	**~p (3)**	**$(p \vee q)$ (4)**	**$\sim(p \vee q)$ (5)**	**$\sim p \wedge q$ (6)**	**$\sim(p \vee q) \vee (\sim p \wedge q)$**
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

Entries in column (3) and (7) are identical.

43. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is :

(1) 3 (2) $\frac{4}{3}$ (3) $\frac{1}{2}$ (4) 2

43. (4)

Tangent and normal are drawn at P (16, 16) on $y^2 = 16x$

Hence equation of tangent is $x - 2y - 16$ and equation of normal is $2x + y = 48$

$\therefore$ $A \equiv (-16, 0)$ and $B \equiv (24, 0)$

$\ell(PB) = \sqrt{64 + 256} = 8\sqrt{5}$

$\therefore$ $\ell(PM) = 4\sqrt{5}$

Also $CP^2 = CB^2 = CA^2$

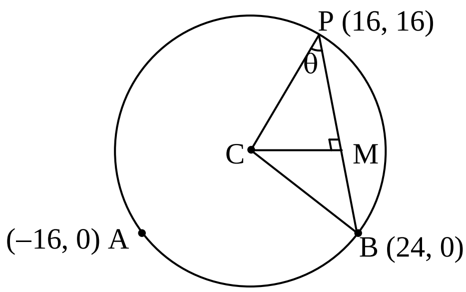
Let $C \equiv (h, k)$

$\therefore$ $(h + 16)^2 + k^2 = (h - 24)^2 + k^2 \Rightarrow h = 4$

Also $(h - 16)^2 + (k - 16)^2 = (h + 16)^2 + k^2$

$\therefore$ $144 + (-32k + 256) = 400 \Rightarrow k = 0$

$\therefore$ $C \equiv (4, 0)$



Hence $CP = \sqrt{144+256} = 20$

In Δ CPM, $\cos\theta = \dfrac{PM}{CP}$

$\therefore \quad \cos\theta = \dfrac{4\sqrt{5}}{20} = \dfrac{\sqrt{5}}{5} = \dfrac{1}{\sqrt{5}} \Rightarrow \sin\theta = \dfrac{2}{\sqrt{5}}$

$\therefore \quad \tan\theta = 2$

44. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to :

(1) (−4, 5) (2) (4, 5) (3) (−4, −5) (4) (−4, 3)

44. (1)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix} \quad = \quad (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$= \quad (5x-4)\begin{vmatrix} 0 & x+4 & 0 \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix}$$

$= \quad (5x - 4)(-1)(x + 4)(x - 4 - 2x)$

$= \quad (-1)(5x - 4)(x + 4)(-x - 4)$

$= \quad (5x - 4)(x + 4)(x + 4)$

$= \quad (-4 + 5x)[x - (-4)]^2$

Hence, A = − 4, B = 5

45. The sum of the coefficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is :

(1) 1 (2) 2 (3) −1 (4) 0

45. (2)

$$\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$$

$$= \left[x^5 + {}^5c_1\, x^4\sqrt{x^3-1} + {}^5c_2\; x^3.\left(x^3-1\right) + {}^5c_3\; x^2\left(x^3-1\right)^{3/2} + {}^5c_4\; x\left(x^3-1\right)^2 + {}^5c_5\; x^0\left(x^3-1\right)^{5/2}\right] +$$

$$\left[x^5 - {}^5c_1\, x^4\sqrt{x^3-1} + {}^5c_2\; x^3.\left(x^3-1\right) - {}^5c_3\; x^2\left(x^3-1\right)^{3/2} + {}^5c_4\; x\left(x^3-1\right)^2 - {}^5c_5\; x^0\left(x^3-1\right)^{5/2}\right]$$

$$= 2\left[x^5 + {}^5c_2\; x^3\left(x^3-1\right) + {}^5c_4\; x\left(x^3-1\right)^2\right]$$

Sum of Coefficient of all odd degree

$$= 2\left[1 - {}^5c_2 + {}^5c_4 + {}^5c_4\right]$$

$= 2$

46. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that, $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 =$ 140 m, then m is equal to :

(1) 34 (2) 33 (3) 66 (4) 68

46. (1)

$a_1, a_2, a_3, \ldots, a_{49}$ are in A.P.

Let first term be A and common difference be D.

$a_9 + a_{43} = 66 \Rightarrow 2A + 50D = 66 \Rightarrow A + 25D = 33$

$\therefore\ a_{26} = 33 \quad \ldots (i)$

$$\sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow a_1 + a_5 + a_9 + \ldots + a_{49} = 416$$

$\therefore\ 13A + 312D = 416 \Rightarrow A + 24D = 32$

$\therefore\ a_{25} = 32 \quad \ldots (ii)$

From (i) and (ii), $a_{26} - a_{25} = D = 1$

Also $A + 25D = 33 \Rightarrow A = 8$

$\therefore\ a_1^2 + a_2^2 + a_3^2 + \ldots + a_{17}^2$

$= 8^2 + 9^2 + 10^2 + \ldots + 24^2$

$$\therefore \sum_{r=1}^{24} r^2 - \sum_{r=1}^{7} r^2 = 140\text{ m}$$

$$\therefore \frac{(24)(25)(49)}{6} - \frac{(7)(8)(15)}{6} = 140\text{ m}$$

$\therefore\ 4900 - 140 = 140\text{ m} \Rightarrow m = 34$

47. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

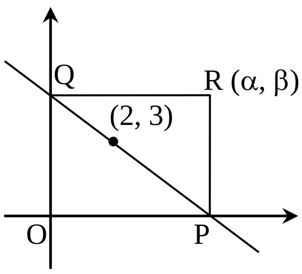
(1) $3x + 2y = xy$ (2) $3x + 2y = 6xy$ (3) $3x + 2y = 6$ (4) $2x + 3y = xy$

47. (1)

line $y - 3 = m(x - 2)$

$y - 3 = mx - 2m$

$P \equiv \left(\frac{2m-3}{m}, 0\right)$, $Q \equiv (0, 3-2m)$



Let R (α, β)

So, $\alpha = \frac{2m-3}{m}$ and $\beta = 3 - 2m$

$\therefore\ m = \frac{3}{2-\alpha}$ and $m = \frac{3-\beta}{2}$

$\therefore\ \frac{3}{2-\alpha} = \frac{3-\beta}{2} \Rightarrow 6 = 6 - 2\beta - 3\alpha + \alpha\beta$

Locus of R (α, β) is $3x + 2y = xy$

48. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is :

(1) 4π (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{2}$

48. (2)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}\,dx \qquad \ldots\ldots\ (1)$$

$$= \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+\frac{1}{2^x}}\,dx \qquad \int_{-a}^{a} f(x)\,dx = \int_{-a}^{a} f(-x)\,dx$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x \times 2^x}{1+2^x}\,dx \qquad \ldots\ldots\ (2)$$

(1) + (2)

$$2I = \int_{-\pi/2}^{\pi/2} \sin^2 x\,dx \qquad \text{[Even Function]}$$

$$I = \int_{0}^{\pi/2} \sin^2 x\,dx$$

$$I = \frac{1}{2}\int_{0}^{\pi/2} (1-\cos 2x)\,dx = \pi/4$$

49. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$ and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is :

(1) $\frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$ (2) $\frac{1}{2}\left(\sqrt{2}-1\right)$ (3) $\frac{1}{2}\left(\sqrt{3}-1\right)$ (4) $\frac{1}{2}\left(\sqrt{3}+1\right)$

49. (3)

$18x^2 - 9\pi x + \pi^2 = 0$

$$\Rightarrow \quad x = \frac{\pi}{6}, \frac{\pi}{3} \ \left(\text{so, } \alpha = \frac{\pi}{6},\ \beta = \frac{\pi}{3}\right)$$

$$y = (gof)(x) = g(f(x)) = g\left(\sqrt{x}\right) = \cos x$$

$$A = \int_{\pi/6}^{\pi/3} \cos x\,dx = \sin x\Big|_{\pi/6}^{\pi/3} = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$$

50. For each $t \in \mathbf{R}$, let $[t]$ be the greatest integer less than or equal to t. Then

$$\lim_{x\to 0+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{15}{x}\right]\right)$$

(1) is equal to 120 (2) does not exist (in **R**) (3) is equal to 0 (4) is equal to 15

50. (1)

$$\lim_{x\to 0+} x\left(\frac{1}{x}-\left\{\frac{1}{x}\right\}+\frac{2}{x}-\left\{\frac{2}{x}\right\}+\ldots+\frac{15}{x}-\left\{\frac{15}{x}\right\}\right)$$

$$= \lim_{x\to 0+}\left(1+2+\ldots+15 - x\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+\ldots+\left\{\frac{15}{x}\right\}\right)\right)$$

$$= 120 - \lim_{x\to 0+} x\underbrace{\left(\left\{\frac{1}{x}\right\}+\left\{\frac{2}{x}\right\}+\ldots+\left\{\frac{15}{x}\right\}\right)}_{\text{finite}}$$

$= 120$

51. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is :

(1) 2 (2) 3 (3) 9 (4) 4

51. (1)

$$\text{Variance} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 = \frac{1}{9}(45) - \left(\frac{1}{9}\times 9\right)^2 = 5 - 1 = 4$$

$\therefore$ S. D. $= \sqrt{4} = 2$

52. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$ is equal to :

(1) $\frac{1}{1+\cot^3 x} + C$ (2) $\frac{-1}{1+\cot^3 x} + C$

(3) $\frac{1}{3(1+\tan^3 x)} + C$ (4) $\frac{-1}{3(1+\tan^3 x)} + C$

(where C is a constant of integration)

52. (4)

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

$$= \int \frac{\tan^2 x . \sec^4 x . \sec^2 x}{(\tan^5 x + \tan^2 x + \tan^3 x + 1)^2} dx \qquad \text{divide by } \cos^{10} x$$

$$= \int \frac{t^2 (1+t^2)^2}{(t^5 + t^2 + t^3 + 1)^2} dt \qquad \tan x = t$$

$$= \int \frac{t^2 (1+t^2)^2}{(t^3 + 1)^2 (t^2 + 1)^2} dt$$

$$= \int \frac{t^2}{(t^3+1)^2} dt \qquad t^3 + 1 = y$$

$$= \int \frac{1}{y^2} . \frac{dy}{3} = \frac{1}{3}\left(-\frac{1}{y}\right) + c = -\frac{1}{3} . \left(\frac{1}{\tan^3 x + 1}\right) + c$$

53. Let S = {t ∈ **R** : $f(x) = |x - \pi| \cdot (e^{|x|} - 1) \sin|x|$ is not differentiable at t}. Then the set S is equal to:

(1) $\{\pi\}$ (2) $\{0, \pi\}$ (3) ϕ (an empty set) (4) $\{0\}$

53. (3)

$f(x) = |x - \pi|\left(e^{|x|} - 1\right)\sin|x|$

Obviously differentiable at x = 0

Check at $x = \pi$

R.H.D. $= \lim_{h\to 0} \frac{f(\pi + h) - f(\pi)}{h}$

$$= \lim_{h\to 0} \frac{|\pi+h-\pi|\left(e^{|\pi+h|}-1\right)\sin|\pi+h|-0}{h}$$

$$= \lim_{h\to 0} \frac{h.\left(e^{\pi+h}-1\right).\sin(\pi+h)}{h} = \lim_{h\to 0} -\sinh.\left(e^{\pi+h}-1\right) = 0$$

$$\text{L.H.D.} = \lim_{h\to 0} \frac{f(\pi-h)-f(\pi)}{-h}$$

$$= \lim_{h\to 0} \frac{|\pi-h-\pi|\left(e^{|\pi-h|}-1\right)\sin|\pi-h|}{-h} = \lim_{h\to 0} \frac{h.\left(e^{\pi-h}-1\right).\sin h}{-h} = 0$$

So, differentiable at $x = \pi$

54. Let $y = y(x)$ be the solution of the differential equation $\sin x\frac{dy}{dx} + y\ \cos x = 4x$, $x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

(1) $-\frac{8}{9}\pi^2$ (2) $-\frac{4}{9}\pi^2$ (3) $\frac{4}{9\sqrt{3}}\pi^2$ (4) $\frac{-8}{9\sqrt{3}}\pi^2$

54. (1)

$\frac{dy}{dx} + y \cot x = \frac{4x}{\sin x}$

I.F. $= e^{\int \cot x dx} = e^{\log_e|\sin x|} = \sin x$ as $x \in (0, \pi)$

$y\cdot\sin x = c + \int 4x\ dx$

$y \sin x = c + 2x^2$

As $y\left(\frac{\pi}{2}\right) = 0 \Rightarrow 0 \cdot \sin\frac{\pi}{2} = c + 2\left(\frac{\pi}{2}\right)^2 \Rightarrow c = -\frac{\pi^2}{2}$

So, $y \sin x = 2x^2 - \frac{\pi^2}{2}$

Put $x = \frac{\pi}{6}$

$y\cdot\sin\frac{\pi}{6} = 2\left(\frac{\pi}{6}\right)^2 - \frac{\pi^2}{2}$

$y\left(\frac{1}{2}\right) = \frac{\pi^2}{18} - \frac{\pi^2}{2} = \frac{\pi^2 - 9\pi^2}{18}$

$y = \frac{-8}{9}\pi^2$

55. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} = 24$, then $|\vec{u}|^2$ is equal to :

(1) 256 (2) 84 (3) 336 (4) 315

55. (3)

$\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$

As coplanar $\begin{vmatrix} u_1 & u_2 & u_3 \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 0$

$4u_1 - 2u_2 + 2u_3 = 0$

$2u_1 - u_2 + u_3 = 0$ .... (1)

$\vec{u}.\vec{a} = 0$

$\Rightarrow$ $2u_1 + 3u_2 - u_3 = 0$ .... (2)

$\vec{u}.\vec{b} = 0$

$\Rightarrow$ $u_2 + u_3 = 24$ .... (3)

Solving $u_1 = -4$, $u_2 = 8$, $u_3 = 16$

$|\vec{u}|^2 = 336$

56. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, x + y + z = 7 is :

(1) $\frac{1}{3}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2}{3}$

56. (2)

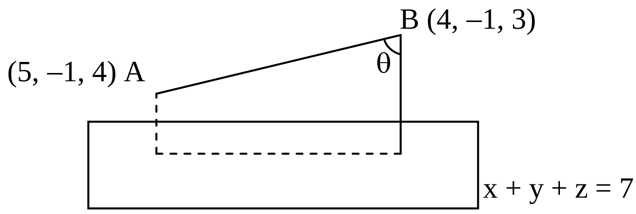
drs of AB = (1, 0, 1)

Let θ be angle between line AB and normal of plane.

So, $\cos\theta = \frac{1\times1+0\times1+1\times1}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$

So, projection of line $AB = |\overrightarrow{AB}|\sin\theta$

$= \sqrt{2}\times\sqrt{1-\frac{2}{3}} = \sqrt{\frac{2}{3}}$



57. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is :

(1) $100\sqrt{3}$ (2) $50\sqrt{2}$ (3) 100 (4) 50

57. (3)

Let TW = tower = h

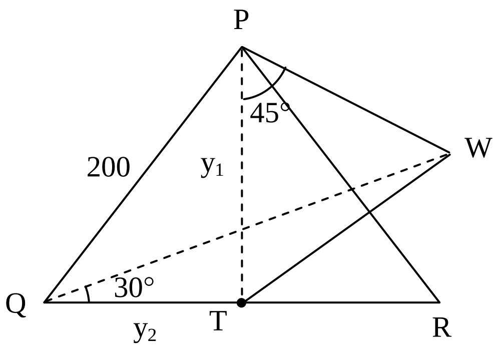
ΔPTW, $\tan 45° = \frac{h}{y_1}$ $\Rightarrow h = y_1$

Similarly, $\frac{h}{y_2} = \tan 30°$ $\Rightarrow y_2 = \sqrt{3}h$

ΔPQT, $PQ^2 = QT^2 + PT^2$

$40000 = 3h^2 + h^2$

$h = 100$



58. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) at least 500 but less than 750 (2) at least 750 but less than 1000

(3) at least 1000 (4) less than 500

58. (3)

We have 6 novels and 3 dictionaries. We can select 4 novels and 1 dictionary in ${}^6C_4 \times {}^3C_1 = \frac{6!}{4!2!}\times3$

$= \frac{6\times5\times3}{2}$ = 45 ways

Now 4 novels and 1 dictionary are to be arranged so that dictionary is always in middle. So remaining 4 novels can be arranged in 4! ways.

Hence total arrangements possible are

$45 \times 24 = 1080$ ways

59. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series.
$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$
If $B - 2A = 100\lambda$, then λ is equal to :
(1) 464 (2) 496 (3) 232 (4) 248

59. (4)

$B = (1^2 + 3^2 + 5^2 + \ldots\ldots + 39^2) + 2\,[2^2 + 4^2 + \ldots\ldots + 40^2]$

$= (1^2 + 2^2 + 3^2 + \ldots\ldots + 40^2) + \quad [2^2 + 4^2 + \ldots\ldots + 40^2]$

$= \quad (1^2 + 2^2 + \ldots\ldots + 40^2) + 4\,[1^2 + 2^2 + \ldots\ldots + 20^2]$

$A = \quad 1^2 + 2^2 + \ldots\ldots + 20^2 + 4\,[1^2 + 2^2 + \ldots\ldots + 10^2]$

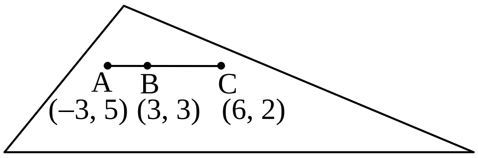
[Using $1^2 + 2^2 + \ldots\ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$]

$B - 2A = 24800$
So, $\lambda = 248$

60. Let the orthocentre and centroid of a triangle be A(−3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

(1) $3\sqrt{\dfrac{5}{2}}$ (2) $\dfrac{3\sqrt{5}}{2}$ (3) $\sqrt{10}$ (4) $2\sqrt{10}$

60. (1)



As we know centroid divides line joining circumcentre and orthocentre internally 1 : 2.

So, C (6, 2)

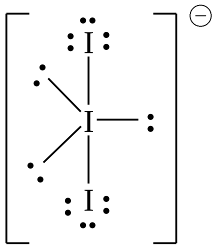
$$AC = \sqrt{(6+3)^2 + (2-5)^2} = \sqrt{90} = 3\sqrt{10}$$

$$r = \frac{AC}{2} = \frac{3\sqrt{10}}{2} = 3\sqrt{\frac{5}{2}}$$

PART- C : CHEMISTRY

61. Total number of lone pair of electron in I_3^- ion is :
(1) 9 (2) 12 (3) 3 (4) 6

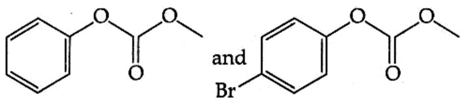
61. (1)

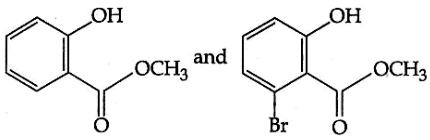


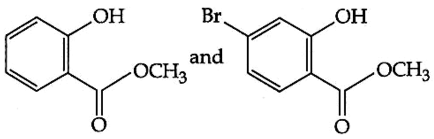
62. Which of the following salts is the most basic in aqueous solution ?
(1) $FeCl_3$ (2) $Pb(CH_3COO)_2$ (3) $Al(CN)_3$ (4) CH_3COOK

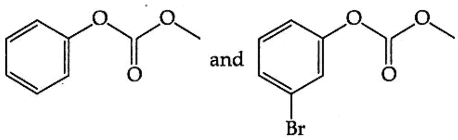
62. (4)
CH_3COOK is most basic among the given options.

63. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively :

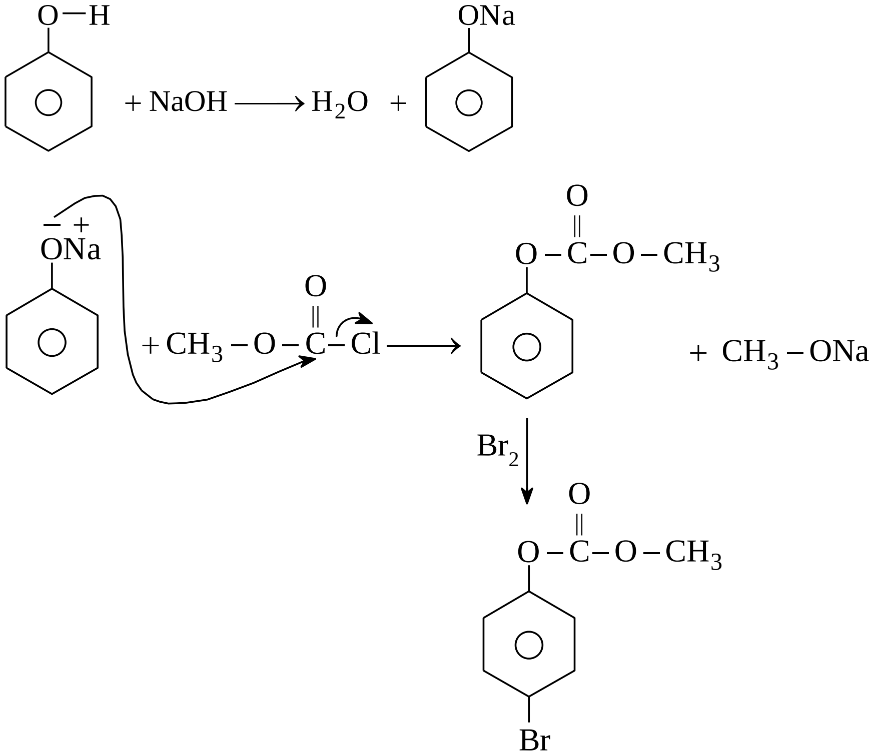
(1) 

(2) 

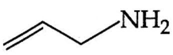
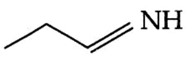
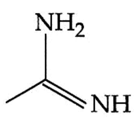
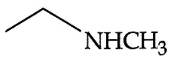
(3) 

(4) 

63. (1)



64. The increasing order of basicity of the following compounds is :

(a)  (b)  (c)  (d) 

(1) (b) < (a) < (d) < (c)
(2) (d) < (b) < (a) < (c)
(3) (a) < (b) < (c) < (d)
(4) (b) < (a) < (c) < (d)

64. (1)
Basic Nature : (b) < (a) < (d) < (c)

65. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination ?

	Base	Acid	End Point
(1)	Weak	Strong	Yellow to pinkish red
(2)	Strong	Strong	Pink to colourless
(3)	Weak	Strong	Colourless to pink
(4)	Strong	Strong	Pinkish red to yellow

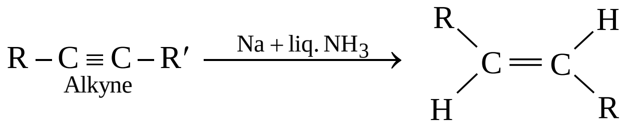
65. (1)
Fact

66. The trans–alkenes are formed by the reduction of alkynes with :

(1) Na/liq.NH_3 (2) Sn–HCl

(3) H_2–Pd/C, $BaSO_4$ (4) $NaBH_4$

66. (1)



Trans-alkene

67. The ratio of mass percent of C and H of an organic compound ($C_xH_YO_Z$) is 6 : 1. If one molecule of the above compound ($C_xH_YO_Z$) contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_xH_YO_Z$ is :

(1) $C_3H_4O_2$ (2) $C_2H_4O_3$ (3) $C_3H_6O_3$ (4) C_2H_4O

67. (2)

$C_XH_YO_Z$

$$\frac{w_C}{w_H}=\frac{12X}{Y}=\frac{6}{1}$$

$$\Rightarrow \frac{X}{Y}=\frac{1}{2}$$

$$C_XH_Y+\left(X+\frac{Y}{4}\right)O_2 \rightarrow XCO_2+\frac{Y}{2}H_2O$$

X : Y : Z = 2 : 4 : 3

68. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively :

(1) H_2O and ($H_2O + O_2$) (2) H_2O and ($H_2O + OH^-$)

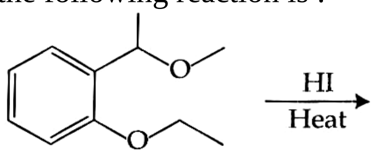
(3) ($H_2O + O_2$) and H_2O (4) ($H_2O + O_2$) and ($H_2O + OH^-$)

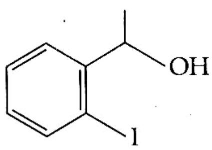
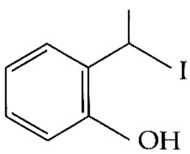
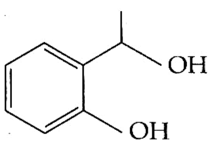
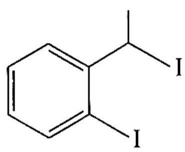
68. (1)

$$\underset{(OA)}{H_2\overset{-1}{O}_2}+[Fe(CN)_6]^{4-} \xrightarrow{H^+} [Fe(CN)_6]^{3-} + H_2\overset{-2}{O}$$

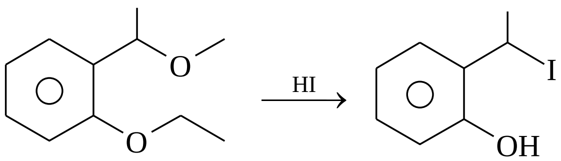
$$\underset{(RA)}{H_2\overset{-1}{O}_2}+[Fe(CN)_6]^{3-} \longrightarrow [Fe(CN)_6]^{4-} + \overset{0}{O}_2$$

69. The major product formed in the following reaction is :



(1)  (2)  (3)  (4) 

69. (2)



70. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane ?
(Atomic weight of B = 10.8 u)
(1) 3.2 hours (2) 1.6 hours (3) 6.4 hours (4) 0.8 hours

70. (1)

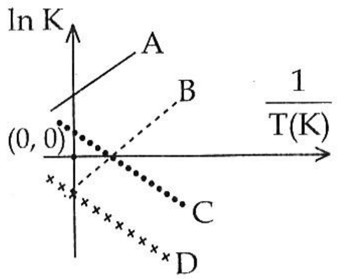
$$B_2H_6 \quad + \quad 3O_2 \longrightarrow B_2O_3 \quad + \quad 3H\ O$$

$$\frac{27.6}{27.6} = 1\,\text{mol} \quad 3\text{ mol}$$

$$\frac{It}{F} = \frac{\theta}{F} = \frac{w}{E} = n_{O_2} \times 4$$

t = 965 × 12 sec = 3.2 hr

71. Which of the following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction ?
(1) C and D
(2) A and D
(3) A and B
(4) B and C



71. (3)

$$\log_{10} K_{eq} \quad = \quad \text{constant} - \left(\frac{\Delta H}{2.303\,R}\right)\left(\frac{1}{T}\right)$$

Given : ΔH = –ve i.e. slope = + ve

72. At 518°C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was 1.00 Torr s^{-1} when 5% had reacted and 0.5 Torr s^{-1} when 33% had reacted. The order of the reaction is :
(1) 1 (2) 0 (3) 2 (4) 3

72. (3)

Rate of a reaction = k $[P]^n$

$$R_1 \quad = \quad 1\text{ torr / sec} = k\left[365 \times \frac{95}{100}\right]^n$$

$$R_2 \quad = \quad 0.5\text{ torr / sec} = k\left[365 \times \frac{67}{100}\right]^n$$

$$\frac{1}{0.5} = \left(\frac{95}{67}\right)^n$$

$$2 = \left(\frac{96}{67}\right)^n = (1.43)^n \qquad \Rightarrow \mathbf{n = 2}$$

73. Glucose on prolonged heating with HI gives :
(1) Hexanoic acid (2) 6–iodohexanal (3) n–Hexane (4) 1–Hexene

73. (3)

$$\text{Glucose} + HI \xrightarrow{\Delta} n - \text{Hexane}.$$

74. Consider the following reaction and statements :

$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$

(I) Two isomers are produced if the reactant complex ion is a cis–isomer.

(II) Two isomers are produced if the reactant complex ion is a trans–isomer.

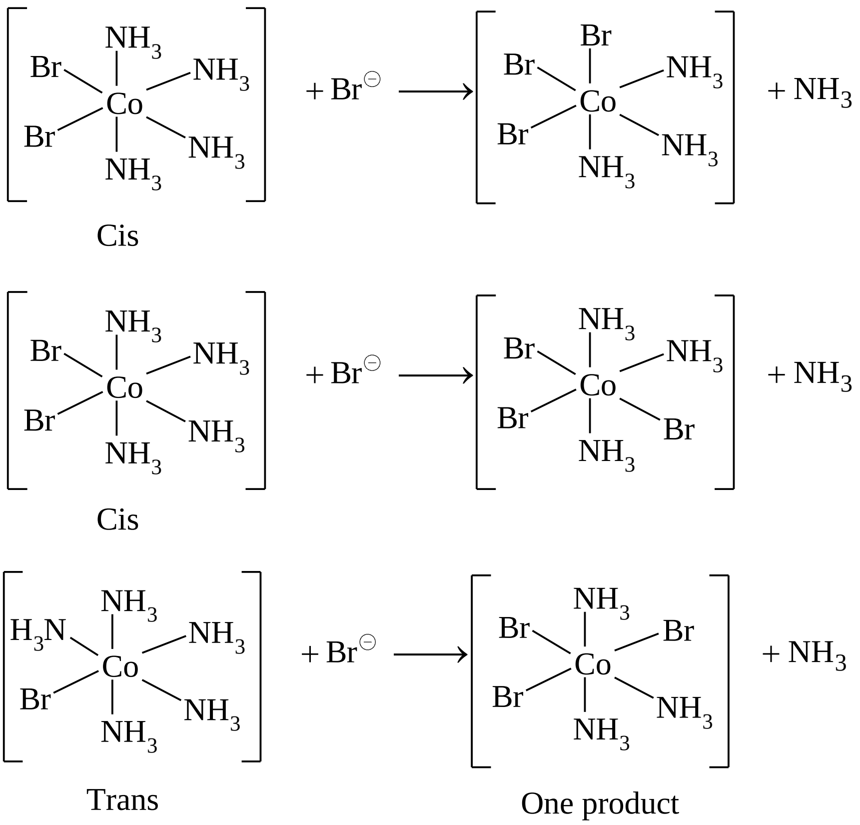
(III) Only one isomer is produced if the reactant complex ion is a trans–isomer

(IV) Only one isomer is produced if the reactant complex ion is a cis–isomer.

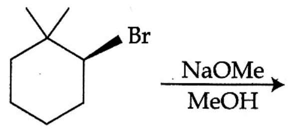
The correct statements are :

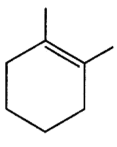
(1) (III) and (IV) (2) (II) and (IV) (3) (I) and (II) (4) (I) and (III)

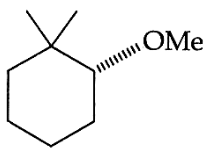
74. (4)

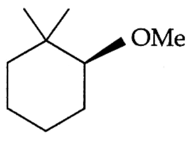


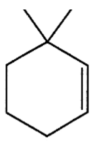
75. The major product of the following reaction is :



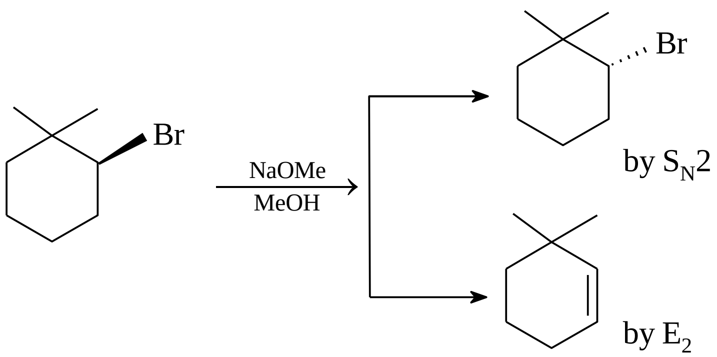
(1) 

(2) 

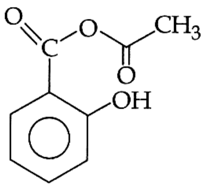
(3) 

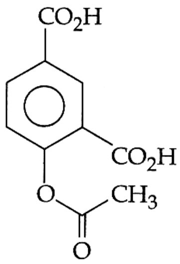
(4) 

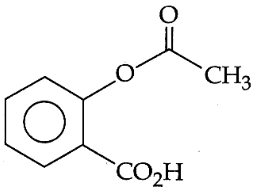
75. (4)

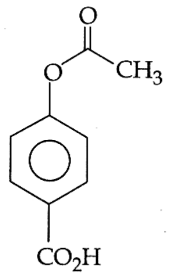


76. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces :

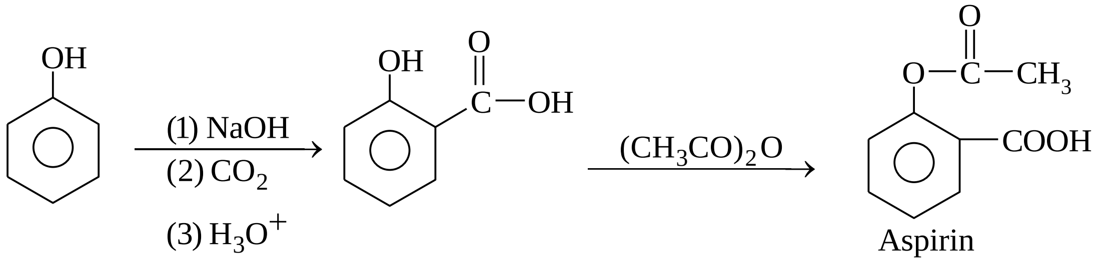
(1) 

(2) 

(3) 

(4) 

76. (3)



77. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+} ?

(1) 1.1×10^{-9} M (2) 1.0×10^{-10} M (3) 5×10^{-9} M (4) 2×10^{-9} M

77. (1)

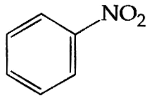
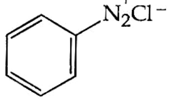
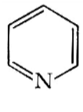
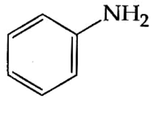
$$\left(\frac{MV}{V+50}\right)\left(\frac{1\times 50}{V+50}\right) = 10^{-10} \qquad \ldots(1)$$

V = 450 mL as total volume = 500 mL

$$\left(\frac{M\times 450}{500}\right)\left(\frac{50}{500}\right) = 10^{-10}$$

$$M = \frac{50}{45}\times 10\times 10^{-10} = \frac{50}{45}\times 10^{-9} = 1.1 \times 10^{-9}\ M$$

78. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation ?

(1) 
(2) 
(3) 
(4) 

78. (4)

Factual.

79. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is :

(1) Al (2) Fe (3) Zn (4) Ca

79. (1)

Alumina is used in chromatography as an adsorbent.

80. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is :

(1) 6×10^{-21} (2) 5×10^{-19} (3) 5×10^{-8} (4) 3×10^{-20}

80 (4)

$K_1 = 10^{-7}$, $K_2 = 1.2 \times 10^{-13}$

$$K_1K_2 = \frac{[H^+]^2[S^{-2}]}{[H_2S]}$$

$$10^{-7} \times 1.2 \times 10^{-13} = \frac{(0.2)^2(S^{-2})}{(0.1)}$$

$$[S^{-2}] = \frac{1.2\times10^{-13}\times10^{-7}\times0.1}{4\times10^{-2}} = \frac{12}{4}\times10^{-20} = 3 \times 10^{-20}$$

81. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$ to :

(1) $[3Ca_3(PO_4)_2 \cdot CaF_2]$ (2) $[3\{Ca(OH)_2\} \cdot CaF_2]$

(3) $[CaF_2]$ (4) $[3(CaF_2) \cdot Ca(OH)_2]$

81. (1)

Factual

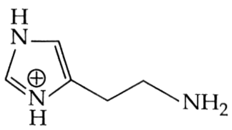
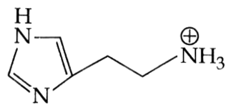
82. The compound that **does not** produce nitrogen gas by the thermal decomposition is :

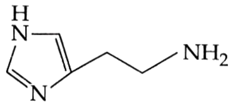
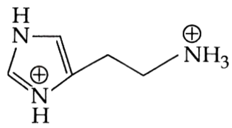
(1) NH_4NO_2 (2) $(NH_4)_2SO_4$ (3) $Ba(N_3)_2$ (4) $(NH_4)_2Cr_2O_7$

82. (2)

$$(NH_4)_2SO_4 \longrightarrow NH_3 \uparrow$$

83. The predominant form of histamine present in human blood is (pK_a, Histidine = 6.0)

(1)  (2) 

(3)  (4) 

83. (2)

Blood is slightly basic is nature (pH 7.35) at this pH terminal NH_2 will get due to more basic nature.

84. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O_2)(NH_3)]$ respectively are :

(1) +3, 0, and +6 (2) +3, 0, and +4 (3) +3, +4, and +6 (4) +3, +2, and +4

84. (1)

85. Which type of 'defect' has the presence of cations in the interstitial sites ?

(1) Frenkel defect (2) Metal deficiency defect

(3) Schottky defect (4) Vacancy defect

85. (1)

86. The combustion of benzene (I) gives $CO_2(g)$ and $H_2O(\ell)$. Given that heat of combustion of benzene at constant volume is – 3263.9 kJ mol^{-1} at 25°C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be :

(R = 8.314 JK^{-1} mol^{-1})

(1) 3260 (2) – 3267.6 (3) 4152.6 (4) – 452.46

86. (2)

$$C_6H_{6(\ell)} + 7.5O_{2(g)} \longrightarrow 6CO_{2(g)} + 3H_2O_{(\ell)}$$

$\Delta n_g = 6 - 7.5 = -1.5$

$$\Delta H = \Delta U + (\Delta_{ng})RT$$

$$= -3263.9 + \frac{(-1.5)\times 8.314\times 298}{1000} = -3267.6 \text{ kJ/mol}$$

87. Which of the following are Lewis acids ?

(1) PH_3 ans $SiCl_4$ (2) BCl_3 and $AlCl_3$ (3) PH_3 and BCl_3 (4) $AlCl_3$ and $SiCl_4$

87. (2)

Factual.

88. Which of the following compounds contain(s) no covalent bond(s) ?

KCl, PH_3, O_2, B_2H_6, H_2SO_4

(1) KCl (2) KCl, B_2H_6 (3) KCl, B_2H_6, PH_3 (4) KCl, H_2SO_4

88. (1)

KCl contains Ionic bond.

89. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_4Cl_2]Cl.2H_2O$ (2) $[Co(H_2O)_3Cl_3].3H_2O$

(3) $[Co(H_2O)_6]Cl_3$ (4) $[Co(H_2O)_5Cl]Cl_2.H_2O$

89. (2)

90. According to molecular orbital theory, which of the following will **not** be a viable molecule ?

(1) H_2^- (2) H_2^{2-} (3) He_2^{2+} (4) He_2^+

90. (2)

Bond order of H_2^{2-} = 0