JEE MAINS 2018
QUESTION PAPER & SOLUTIONS
(CODE-C)

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions. The maximum marks are **360.**
3. There are ***three*** parts in the question paper A, B, C consisting of **Chemistry, Mathematics** and **Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction No. 4 above.
6. For writing particulars/marking responses on ***Side-1*** and ***Side-2*** of the Answer Sheet use ***only Black Ball Point Pen*** provided in the examination hall.
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.

PART–A : CHEMISTRY

1. Which of the following salts is the most basic in aqueous solution?

(1) CH_3COOK (2) $FeCl_3$

(3) $Pb(CH_3COO)_2$ (4) $Al(CN)_3$

Answer (1)

Sol. $CH_3COOK + H_2O \longrightarrow CH_3COOH + \underset{\text{Basic}}{KOH}$

$FeCl_3$ – Acidic solution

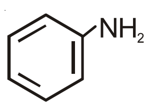
$Al(CN)_3$ – Salt of weak acid and weak base

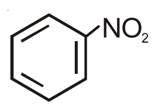
$Pb(CH_3COO)_2$ – Salt of weak acid and weak base

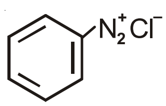
CH_3COOK is salt of weak acid and strong base.

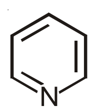
Hence solution of CH_3COOK is basic.

2. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. Kjeldahl method is not applicable for compounds containing nitrogen in nitro, and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions. Hence only aniline can be used for estimation of nitrogen by Kjeldahl's method.

3. Which of the following are Lewis acids?

(1) $AlCl_3$ and $SiCl_4$ (2) PH_3 and $SiCl_4$

(3) BCl_3 and $AlCl_3$ (4) PH_3 and BCl_3

Answer (3)*

Sol. BCl_3 – electron deficient, incomplete octet

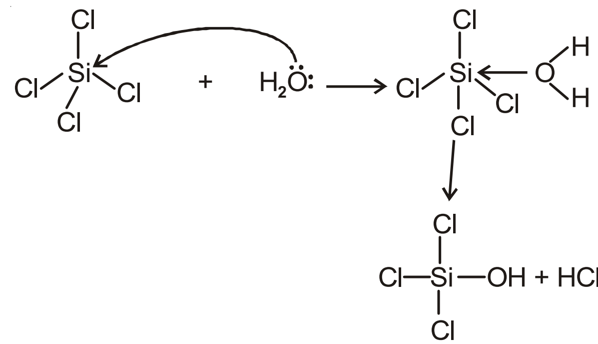
$AlCl_3$ – electron deficient, incomplete octet

Ans-(3) BCl_3 and $AlCl_3$

$SiCl_4$ can accept lone pair of electron in *d*-orbital of silicon hence it can act as Lewis acid.

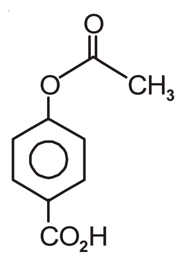
* Although the most suitable answer is (3). However, both option (3) & (1) can be considered as correct answers.

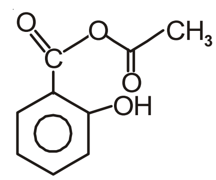
e.g. hydrolysis of $SiCl_4$

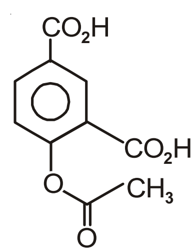


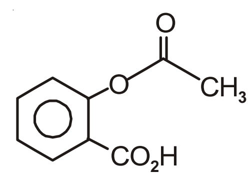
Hence option (1), $AlCl_3$ and $SiCl_4$ is also correct.

4. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2O$ in the presence of catalytic amount of H_2SO_4 produces

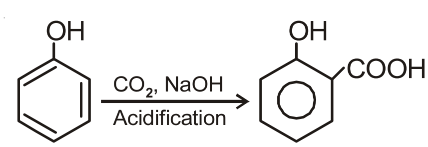
(1) 

(2) 

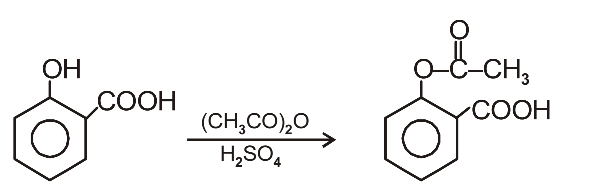
(3) 

(4) 

Answer (4)

Sol. 

(Major)



Acetyl salicylic acid
(Aspirin)

5. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

	Base	Acid	End point
(1)	Strong	Strong	Pinkish red to yellow
(2)	Weak	Strong	Yellow to pinkish red
(3)	Strong	Strong	Pink to colourless
(4)	Weak	Strong	Colourless to pink

Answer (2)

Sol. The pH range of methyl orange is

$$\xleftarrow[\text{Pinkish red}]{} 3.9 — 4.5 \xrightarrow[\text{Yellow}]{}$$

Weak base is having pH greater than 7. When methyl orange is added to weak base solution, the solution becomes yellow. This solution is titrated by strong acid and at the end point pH will be less than 3.1. Therefore solution becomes pinkish red.

6. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constant for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is

(1) 3×10^{-20}
(2) 6×10^{-21}
(3) 5×10^{-19}
(4) 5×10^{-8}

Answer (1)

Sol. In presence of external H^+,

$$H_2S \rightleftharpoons 2H^+ + S^{2-}\ ,\ K_{a_1} \cdot K_{a_2} = K_{eq}$$

$$\therefore\ \frac{[H^+]^2[S^{2-}]}{[H_2S]} = 1\times10^{-7}\times1.2\times10^{-13}$$

$$\frac{[0.2]^2[S^{2-}]}{[0.1]} = 1.2\times10^{-20}$$

$[S^{2-}] = 3 \times 10^{-20}$

7. The combustion of benzene (l) gives CO_2(g) and H_2O(l). Given that heat of combustion of benzene at constant volume is –3263.9 kJ mol^{-1} at 25° C; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be

($R = 8.314$ JK^{-1} mol^{-1})

(1) –452.46
(2) 3260
(3) –3267.6
(4) 4152.6

Answer (3)

Sol. $C_6H_6(l) + \frac{15}{2}O_2(g) \longrightarrow 6CO_2(g) + 3H_2O(l)$

$$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}$$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -3263.9 + \left(-\frac{3}{2}\right)\times 8.314\times298\times10^{-3}$$

$$= -3263.9 + (-3.71)$$

$$= -3267.6 \text{ kJ mol}^{-1}$$

8. The compound that does not produce nitrogen gas by the thermal decomposition is

(1) $(NH_4)_2Cr_2O_7$
(2) NH_4NO_2
(3) $(NH_4)_2SO_4$
(4) $Ba(N_3)_2$

Answer (3)

Sol. $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + 4H_2O + Cr_2O_3$

$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

$(NH_4)_2SO_4 \xrightarrow{\Delta} 2NH_3 + H_2SO_4$

$Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

Among all the given compounds, only $(NH_4)_2SO_4$ do not form dinitrogen on heating, it produces ammonia gas.

9. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u)

(1) 0.8 hours
(2) 3.2 hours
(3) 1.6 hours
(4) 6.4 hours

Answer (2)

Sol. $B_2H_6 + 3O_2 \longrightarrow B_2O_3 + 3H_2O$

27.66 of B_2H_6 = 1 mole of B_2H_6 which requires three moles of oxygen (O_2) for complete burning

$6H_2O \longrightarrow 6H_2 + 3O_2$ (On electrolysis)

Number of faradays = 12 = Amount of charge

$12 \times 96500 = i \times t$

$12 \times 96500 = 100 \times t$

$$t = \frac{12\times96500}{100} \text{ second}$$

$$t = \frac{12\times96500}{100\times3600} \text{ hour}$$

t = 3.2 hours

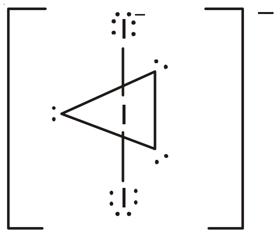
10. Total number of lone pair of electrons in I_3 ion is

(1) 6 (2) 9

(3) 12 (4) 3

Answer (2)

Sol. Structure of I_3^-



Number of lone pairs in $I_3^{\ominus}$ is 9.

11. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is

(1) Ca (2) Al

(3) Fe (4) Zn

Answer (2)

Sol. $Al^{3+} \xrightarrow{NaOH} \underset{\text{White gelatinous ppt.}}{Al(OH)_3 \downarrow} \xrightarrow[NaOH]{Excess} \underset{\text{Sodium meta aluminate (soluble)}}{NaAlO_2}$

$$2Al(OH)_3 \xrightarrow{\text{Strong heating}} Al_2O_3 + 3H_2O$$

Al_2O_3 is used in column chromatography.

12. According to molecular orbital theory, which of the following will not be a viable molecule?

(1) He_2^+ (2) H_2^-

(3) H_2^{2-} (4) He_2^{2+}

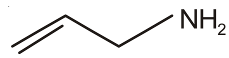
Answer (3)

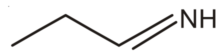
Sol.

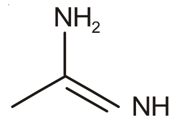
	Electronic configuration	Bond order
He_2^+	$\sigma_{1s^2}\sigma^*_{1s^1}$	$\frac{2-1}{2} = 0.5$
H_2^-	$\sigma_{1s^2}\sigma^*_{1s^1}$	$\frac{2-1}{2} = 0.5$
H_2^{2-}	$\sigma_{1s^2}\sigma^*_{1s^2}$	$\frac{2-2}{2} = 0$
He_2^{2+}	σ_{1s^2}	$\frac{2-0}{2} = 1$

Molecule having zero bond order will not be a viable molecule.

compound is

(a) 

(b) 

(c) 

(d) 

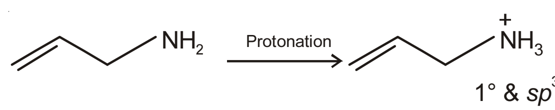
(1) (b) < (a) < (c) < (d)

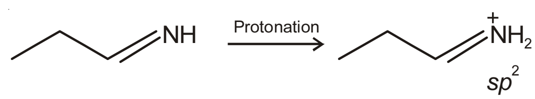
(2) (b) < (a) < (d) < (c)

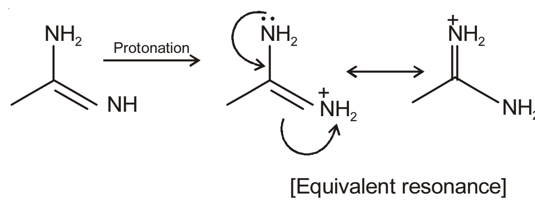
(3) (d) < (b) < (a) < (c)

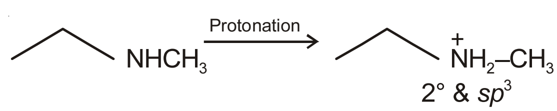
(4) (a) < (b) < (c) < (d)

Answer (2)

Sol. (a) 

(b) 

(c) 

(d) 

∴ Correct order of basicity : b < a < d < c.

14. Which type of 'defect' has the presence of cations in the interstitial sites?

(1) Vacancy defect

(2) Frenkel defect

(3) Metal deficiency defect

(4) Schottky defect

Answer (2)

Sol. In Frenkel defect, cation is dislocated from its normal lattice site to an interstitial site.

15. Which of the following compounds contain(s) no covalent bond(s)?

KCl, PH_3, O_2, B_2H_6, H_2SO_4

(1) KCl, H_2SO_4 (2) KCl

(3) KCl, B_2H_6 (4) KCl, B_2H_6, PH_3

Answer (2)

Sol. KCl – Ionic bond between K^+ and Cl^-

PH_3 – Covalent bond between P and H

O_2 – Covalent bond between O atoms

B_2H_6–Covalent bond between B and H atoms

H_2SO_4 – Covalent bond between S and O and also between O and H.

$\therefore$ Compound having no covalent bonds is KCl only.

16. The oxidation states of

Cr in $\left[Cr(H_2O)_6\right]Cl_3, \left[Cr(C_6H_6)_2\right]$, and

$K_2\left[Cr(CN)_2(O)_2(O_2)(NH_3)\right]$ respectively are

(1) +3, +2 and +4 (2) +3, 0 and +6

(3) +3, 0 and +4 (4) +3, +4 and +6

Answer (2)

Sol. $\left[Cr(H_2O)_6\right]Cl_3 \Rightarrow x + 0\times6 - 1\times3 = 0$

$\therefore x = +3$

$\left[Cr(C_6H_6)_2\right] \Rightarrow x + 2\times0 = 0$

$x = 0$

$K_2\left[Cr(CN)_2(O_2)(O_2)NH_3\right]$

$\Rightarrow 1\times2 + x - 1\times2 - 2\times2 - 2\times1 = 0$

$\Rightarrow x - 6 = 0$

$x = +6$

17. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$ to $[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively.

(1) $(H_2O + O_2)$ and $(H_2O + OH^-)$

(2) H_2O and $(H_2O + O_2)$

(3) H_2O and $(H_2O + OH^-)$

(4) $(H_2O + O_2)$ and H_2O

Answer (2)

Sol. $[Fe(CN)_6]^{4-} + \frac{1}{2}H_2O_2 + H^+ \longrightarrow [Fe(CN)_6]^{3-} + H_2O$

$[Fe(CN)_6]^{3-} + \frac{1}{2}H_2O_2 + OH^{\ominus} \longrightarrow$

$[Fe(CN)_6]^{4-} + H_2O + \frac{1}{2}O_2$

18. Glucose on prolonged heating with HI gives

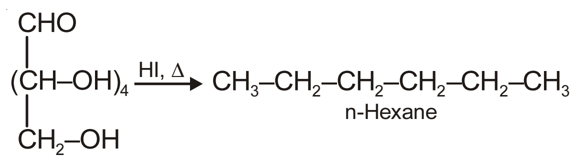
(1) 1-Hexene

(2) Hexanoic acid

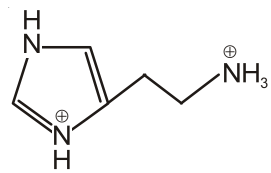
(3) 6-iodohexanal

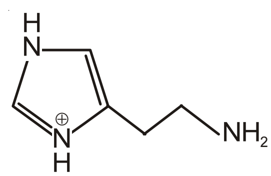
(4) *n*-Hexane

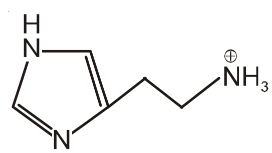
Answer (4)

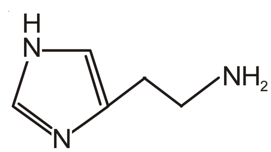
Sol. 

19. The predominant form of histamine present in human blood is (pK_a, Histidine = 6.0)

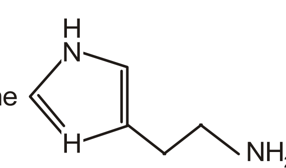
(1) 

(2) 

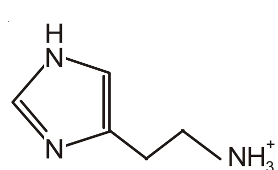
(3) 

(4) 

Answer (3)

Sol. Histamine 

At pH (7.4) major form of histamine is protonated at primary amine.



20. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2.Ca(OH)_2]$ to

(1) $[3(CaF_2).Ca(OH)_2]$ (2) $[3Ca_3(PO_4)_2.CaF_2]$

(3) $[3\{Ca(OH)_2\}.CaF_2]$ (4) $[CaF_2]$

Answer (2)

Sol. F^- ions make the teeth enamel harder by converting

$$\underset{\text{Hydroxyapatite}}{[3Ca_3(PO_4)_2.Ca(OH)_2]} \text{ to } \underset{\text{Fluorapatite}}{[3Ca_3(PO_4)_2.CaF_2]}$$

21. Consider the following reaction and statements

$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$

(I) Two isomers are produced if the reactant complex ion is a *cis*-isomer

(II) Two isomers are produced if the reactant complex ion is a *trans*-isomer.

(III) Only one isomer is produced if the reactant complex ion is a *trans*-isomer.

(IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.

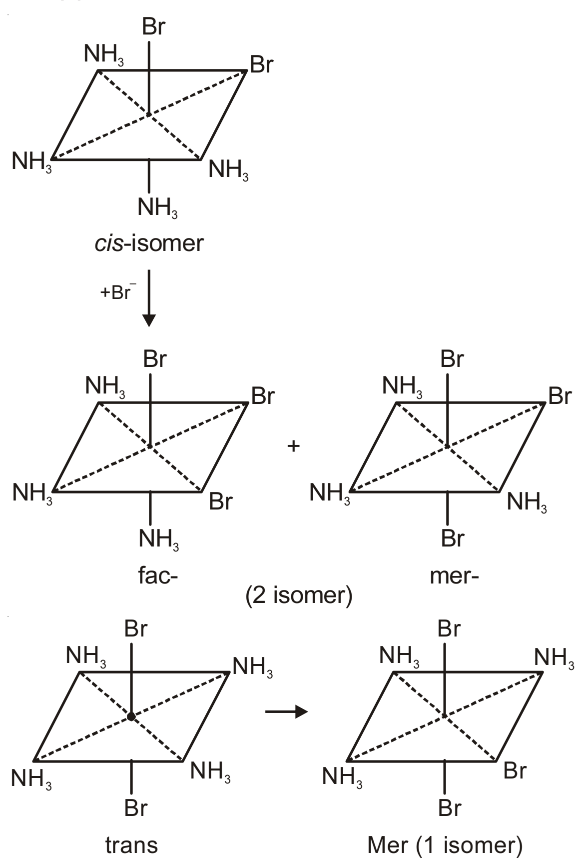
The correct statements are:

(1) (I) and (III) (2) (III) and (IV)

(3) (II) and (IV) (4) (I) and (II)

Answer (1)

Sol.



So option (1) is correct.

22. The *trans*-alkenes are formed by the reduction of alkynes with

(1) $NaBH_4$

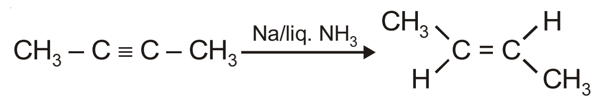
(2) Na/liq. NH_3

(3) Sn - HCl

(4) H_2 - Pd/C, $BaSO_4$

Answer (2)

Sol.



Trans alkene

So, option (2) is correct.

23. The ratio of mass percent of C and H of an organic compound $(C_XH_YO_Z)$ is 6 : 1. If one molecule of the above compound $(C_XH_YO_Z)$ contains half as much oxygen as required to burn one molecule of compound C_XH_Y completely to CO_2 and H_2O. The empirical formula of compound $C_XH_YO_Z$ is

(1) C_2H_4O (2) $C_3H_4O_2$

(3) $C_2H_4O_3$ (4) $C_3H_6O_3$

Answer (3)

Sol.

Element	Relative mass	Relative mole	Simplest whole number ratio
	6	$\frac{6}{12} = 0.5$	1
	1	$\frac{1}{1} = 1$	2

So, X = 1, Y = 2

Equation for combustion of C_XH_Y

$$C_XH_Y + \left(X + \frac{Y}{4}\right)O_2 \longrightarrow XCO_2 + \frac{Y}{2}H_2O$$

Oxygen atoms required = $2\left(X + \frac{Y}{4}\right)$

As per information,

$$2\left(X + \frac{Y}{4}\right) = 2Z$$

$\Rightarrow \left(1 + \frac{2}{4}\right) = Z$

$\Rightarrow$ Z = 1.5

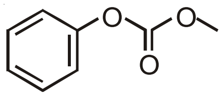
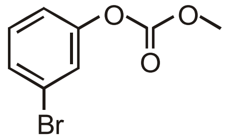
Molecule can be written

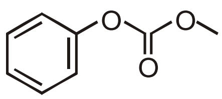
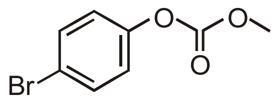
$C_XH_YO_Z$

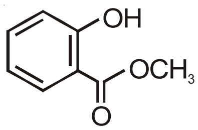
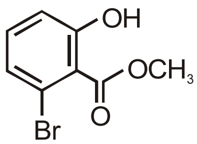
$C_1H_2O_{3/2}$

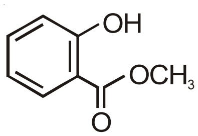
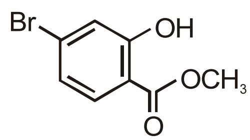
$\Rightarrow C_2H_4O_3$

24. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively

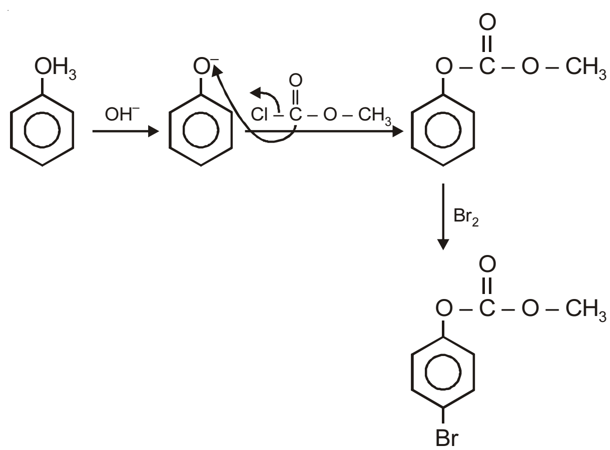
(1)  and 

(2)  and 

(3)  and 

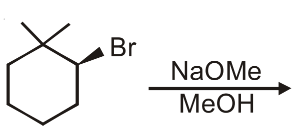
(4)  and 

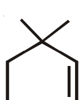
Answer (2)

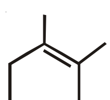
Sol. 

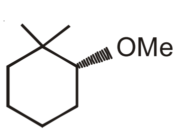
Hence, option (2) is correct.

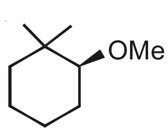
25. The major product of the following reaction is



(1) 

(2) 

(3) 

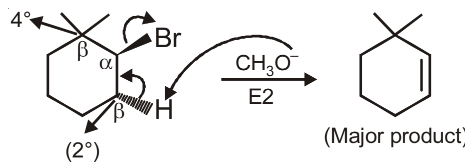
(4) 

Answer (1)

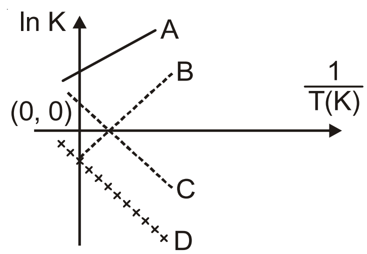
Sol. CH_3O^- is a strong base and strong nucleophile, so favourable condition is S_N2/E2.

Given alkyl halide is 2° and β C's are 4° and 2°, so sufficiently hindered, therefore, E2 dominates over S_N2.

Also, polarity of CH_3OH (solvent) is not as high as H_2O, so E1 is also dominated by E2.



26. Which of the following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction?



(1) B and C

(2) C and D

(3) A and D

(4) A and B

Answer (4)

Sol. Equilibrium constant $K = \left(\frac{A_f}{A_b}\right)e^{-\frac{\Delta H^\circ}{RT}}$

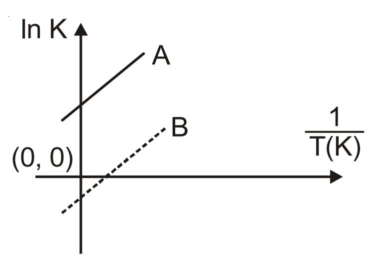
$$\ln K = \ln\left(\frac{A_f}{A_b}\right) - \frac{\Delta H^\circ}{R}\left(\frac{1}{T}\right)$$

$$y = C + m\ x$$

Comparing with equation of straight line,

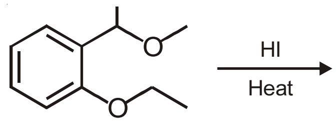
$$\text{Slope} = \frac{-\Delta H^\circ}{R}$$

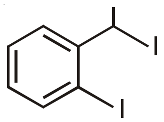
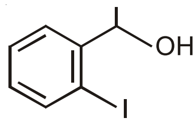
Since, reaction is exothermic, ΔH° = –ve, therefore, slope = +ve.

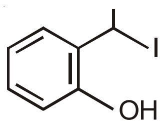
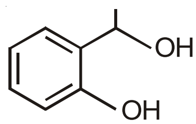


Hence, option (4) is correct.

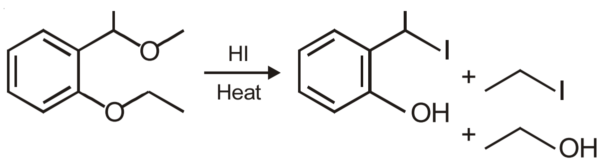
27. The major product formed in the following reaction is



(1)  (2) 

(3)  (4) 

Answer (3)

Sol. 

Hence, option (3) is correct.

28. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 mL of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 mL. The solubility product of $BaSO_4$ is 1×10^{-10}. What is original concentration of Ba^{2+}?

(1) 2×10^{-9} M

(2) 1.1×10^{-9} M

(3) 1.0×10^{-10} M

(4) 5×10^{-9} M

Answer (2)

Sol. Final concentration of $[SO_4^{--}] = \dfrac{[50 \times 1]}{[500]} = 0.1$ M

K_{sp} of $BaSO_4$,

$[Ba^{2+}][SO_4^{2-}] = 1 \times 10^{-10}$

$[Ba^{2+}][0.1] = \dfrac{10^{-10}}{0.1} = 10^{-9}$ M

Concentration of Ba^{2+} in final solution = 10^{-9} M

Concentration of Ba^{2+} in the original solution.

$M_1V_1 = M_2V_2$

M_1 (500 – 50) = 10^{-9} (500)

$M_1 = 1.11 \times 10^{-9}$ M

So, option (2) is correct.

29. At 518°C, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 torr, was 1.00 torr s^{-1} when 5% had reacted and 0.5 torr s^{-1} when 33% had reacted. The order of the reaction is

(1) 3

(2) 1

(3) 0

(4) 2

Answer (4)

Sol. Assume the order of reaction with respect to acetaldehyde is x.

Condition-1 :

Rate = $k[CH_3CHO]^x$

$1 = k[363 \times 0.95]^x$

$1 = k[344.85]^x$ ...(i)

Condition-2 :

$0.5 = k[363 \times 0.67]^x$

$0.5 = k[243.21]^x$ ...(ii)

Divide equation (i) by (ii),

$$\frac{1}{0.5} = \left(\frac{344.85}{243.21}\right)^x \Rightarrow 2 = (1.414)^x$$

$\Rightarrow$ x = 2

30. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

(1) $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$

(2) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O$

(3) $[Co(H_2O)_3Cl_3] \cdot 3H_2O$

(4) $[Co(H_2O)_6]Cl_3$

Answer (3)

Sol. The solution which shows maximum freezing point must have minimum number of solute particles.

(1) $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O \rightarrow [Co(H_2O)_5Cl]^{2+} + 2Cl^-$, i = 3

(2) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O \rightarrow [Co(H_2O)_4Cl_2]^+ + Cl^-$, i = 2

(3) $[Co(H_2O)_3Cl_3] \cdot 3H_2O \rightarrow [Co(H_2O)_3Cl_3]$, i = 1

(4) $[Co(H_2O)_6]Cl_3 \rightarrow [Co(H_2O)_6]^{3+} + 3Cl^-$, i = 4

So, solution of 1 molal $[Co(H_2O)_3Cl_3] \cdot 3H_2O$ will have minimum number of particles in aqueous state.

Hence, option (3) is correct.

PART–B : MATHEMATICS

31. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

is equal to

(1) $\frac{-1}{3(1+\tan^3 x)} + C$ (2) $\frac{1}{1+\cot^3 x} + C$

(3) $\frac{-1}{1+\cot^3 x} + C$ (4) $\frac{1}{3(1+\tan^3 x)} + C$

(where C is a constant of integration)

Answer (1)

Sol. $I = \int \frac{\sin^2 x.\cos^2 x\, dx}{\left\{(\sin^2 x + \cos^2 x)(\sin^3 x + \cos^3 x)\right\}^2}$

Dividing the numerator and denominator by $\cos^6 x$

$\Rightarrow \quad I = \int \frac{\tan^2 x \sec^2 x\, dx}{(1+\tan^3 x)^2}$

Let, $\tan^3 x = z$

$\Rightarrow \quad 3\tan^2 x.\sec^2 x dx = dz$

$I = \frac{1}{3}\int \frac{dz}{z^2} = \frac{-1}{3z} + C$

$= \frac{-1}{3(1+\tan^3 x)} + C$

32. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point $T(0, 3)$ then the area (in sq. units) of ΔPTQ is

(1) $54\sqrt{3}$ (2) $60\sqrt{3}$

(3) $36\sqrt{5}$ (4) $45\sqrt{5}$

Answer (4)

Sol. Clearly PQ is a chord of contact,

i.e., equation of PQ is $T \equiv 0$

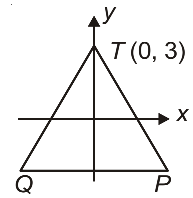
$\Rightarrow \quad y = -12$

Solving with the curve, $4x^2 - y^2 = 36$

$\Rightarrow \quad x = \pm 3\sqrt{5},\ y = -12$

i.e., $P(3\sqrt{5}, -12);\ Q(-3\sqrt{5}, -12);\ T(0,3)$

Area of ΔPQT is



$\Delta = \frac{1}{2} \times 6\sqrt{5} \times 15$

$= 45\sqrt{5}$

33. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is

(1) 2 (2) 3

(3) $\frac{4}{3}$ (4) $\frac{1}{2}$

Answer (1)

Sol. $y^2 = 16x$

Tangent at $P(16, 16)$ is $2y = x + 16$ ... (1)

Normal at $P(16, 16)$ is $y = -2x + 48$ ... (2)

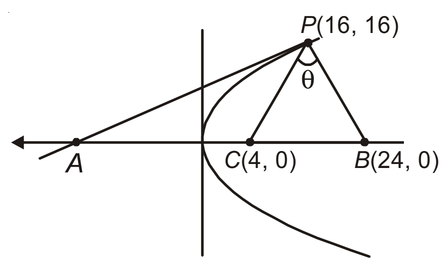
i.e., A is $(-16, 0)$; B is $(24, 0)$

Now, Centre of circle is $(4, 0)$

Now, $m_{PC} = \frac{4}{3}$

$m_{PB} = -2$

i.e., $\tan\theta = \left|\frac{\frac{4}{3}+2}{1-\frac{8}{3}}\right| = 2$



34. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}\cdot\vec{b} = 24$, then $|\vec{u}|^2$ is equal to

(1) 315 (2) 256

(3) 84 (4) 336

Answer (4)

Sol. Clearly, $\vec{u} = \lambda(\vec{a}\times(\vec{a}\times\vec{b}))$

$\Rightarrow \quad \vec{u} = \lambda((\vec{a}.\vec{b})\vec{a} - |\vec{a}|^2\,\vec{b})$

$\Rightarrow \quad \vec{u} = \lambda(2\vec{a} - 14\vec{b}) = 2\lambda\left\{(2\hat{i}+3\hat{j}-\hat{k}) - 7(\hat{j}+\hat{k})\right\}$

$\Rightarrow \quad \vec{u} = 2\lambda(2\hat{i} - 4\hat{j} - 8\hat{k})$

as, $\vec{u}\cdot\vec{b} = 24$

$\Rightarrow \quad 4\lambda(\hat{i} - 2\hat{j} - 4\hat{k})\cdot(\hat{j}+\hat{k}) = 24$

$\Rightarrow \quad \lambda = -1$

So, $\vec{u} = -4(\hat{i} - 2\hat{j} - 4\hat{k})$

$\Rightarrow \quad |\vec{u}|^2 = 336$

35. If α, $\beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

(1) 0 (2) 1

(3) 2 (4) –1

Answer (2)

Sol. $x^2 - x + 1 = 0$

Roots are $-\omega, -\omega^2$

Let $\alpha = -\omega$, $\beta = -\omega^2$

$\alpha^{101} + \beta^{107} = (-\omega)^{101} + (-\omega^2)^{107}$

$= -(\omega^{101} + \omega^{214})$

$= -(\omega^2 + \omega)$

$= 1$

36. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is

(1) $\frac{1}{2}(\sqrt{3}+1)$ (2) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

(3) $\frac{1}{2}(\sqrt{2}-1)$ (4) $\frac{1}{2}(\sqrt{3}-1)$

Answer (4)

Sol. $18x^2 - 9\pi x + \pi^2 = 0$

$(6x - \pi)(3x - \pi) = 0$

$\therefore \quad x = \frac{\pi}{6}, \frac{\pi}{3}$

$\alpha = \frac{\pi}{6}, \quad \beta = \frac{\pi}{3}$

$y = (gof)(x) = \cos x$

Area $= \int_{\pi/6}^{\pi/3} \cos x\,dx = (\sin x)_{\pi/6}^{\pi/3}$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}$

$= \frac{1}{2}(\sqrt{3}-1)$ sq. units

37. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is

(1) 0 (2) 1

(3) 2 (4) –1

Answer (3)

Sol. $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

$= 2\left[{}^5C_0\,x^5 + {}^5C_2\,x^3(x^3-1) + {}^5C_4\,x(x^3-1)^2\right]$

$= 2\left[x^5 + 10(x^6 - x^3) + 5x(x^6 - 2x^3 + 1)\right]$

$= 2\left[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x\right]$

$= 2\left[5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x\right]$

Sum of odd degree terms coefficients

$= 2(5 + 1 - 10 + 5)$

$= 2$

38. Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$.

If $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to

(1) 68 (2) 34

(3) 33 (4) 66

Answer (2)

Sol. Let $a_1 = a$ and common difference $= d$

Given, $a_1 + a_5 + a_9 + ... + a_{49} = 416$

$\Rightarrow \quad a + 24d = 32$ ...(i)

Also, $a_9 + a_{43} = 66 \Rightarrow a + 25d = 33$ ...(ii)

Solving (i) & (ii),

We get $d = 1$, $a = 8$

Now, $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$

$\Rightarrow \quad 8^2 + 9^2 + ... + 24^2 = 140m$

$\Rightarrow \quad \dfrac{24 \times 25 \times 49}{6} - \dfrac{7 \times 8 \times 15}{6} = 140m$

$\Rightarrow \quad \boxed{m = 34}$

39. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is

(1) 4 (2) 2

(3) 3 (4) 9

Answer (2)

Sol. Standard deviation of $x_i - 5$ is

$$\sigma = \sqrt{\frac{\sum_{i=1}^{9}(x_i - 5)^2}{9} - \left(\frac{\sum_{i=1}^{9}(x_i - 5)}{9}\right)^2}$$

$\Rightarrow \quad \sigma = \sqrt{5 - 1} = 2$

As, standard deviation remains constant if observations are added/subtracted by a fixed quantity.

So, σ of x_i is 2

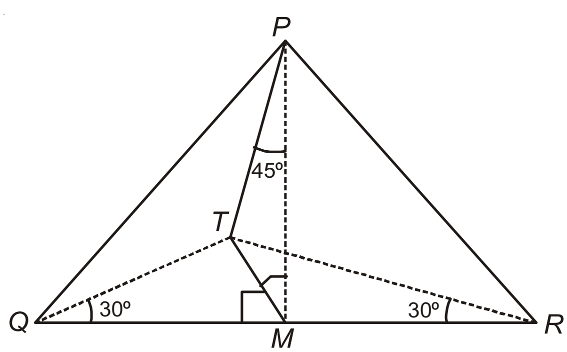
40. *PQR* is a triangular park with *PQ* = *PR* = 200 m. A T.V. tower stands at the mid-point of *QR*. If the angles of elevation of the top of the tower at *P*, *Q* and *R* are respectively 45º, 30º and 30º, then the height of the tower (in m) is

(1) 50 (2) $100\ 3$

(3) $50\sqrt{2}$ (4) 100

Answer (4)

Sol.



Let height of tower *TM* be *h*

$\therefore \quad PM = h$

In ΔTQM, $\tan 30º = \dfrac{h}{QM}$

$QM = \sqrt{3}\,h$

In ΔPMQ, $PM^2 + QM^2 = PQ^2$

$h^2 + (\sqrt{3}h)^2 = 200^2$

$\Rightarrow \quad 4h^2 = 200^2$

$\Rightarrow \quad h = 100$ m

41. Two sets *A* and *B* are as under :

$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$

$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \le 36\}$, then

(1) $A \subset B$

(2) $A \cap B = \phi$ (an empty set)

(3) Neither $A \subset B$ nor $B \subset A$

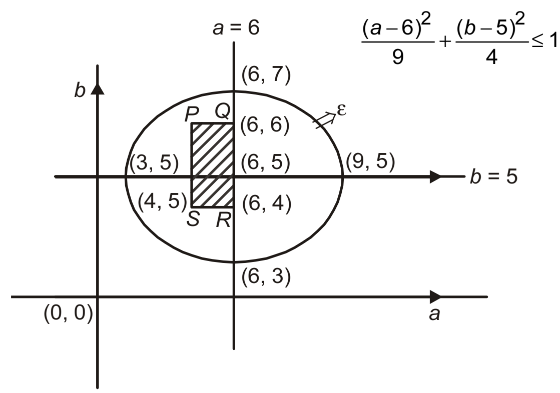
(4) $B \subset A$

Answer (1)

Sol. As, $|a - 5| < 1$ and $|b - 5| < 1$

$\Rightarrow \quad 4 < a, b < 6$ and $\dfrac{(a-6)^2}{9} + \dfrac{(b-5)^2}{4} \le 1$

Taking axes as *a*-axis and *b*-axis



The set *A* represents square *PQRS* inside set *B* representing ellipse and hence $A \subset B$.

42. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is

(1) Less then 500

(2) At least 500 but less than 750

(3) At least 750 but less than 1000

(4) At least 1000

Answer (4)

Sol. Number of ways of selecting 4 novels from 6 novels $= {}^6C_4$

Number of ways of selecting 1 dictionary from 3 dictionaries $= {}^3C_1$

Required arrangements $= {}^6C_4 \times {}^3C_1 \times 4! = 1080$

$\Rightarrow$ Atleast 1000

43. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$.

If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is:

(1) -3 (2) $-2\sqrt{2}$

(3) $2\sqrt{2}$ (4) 3

Answer (3)

Sol. $h(x) = \dfrac{x^2 + \frac{1}{x^2}}{x - 1/x}$

$= \left(x - 1/x\right) + \dfrac{2}{\left(x - 1/x\right)}$

$x - \frac{1}{x} > 0,\ \left(x - 1/x\right) + \dfrac{2}{\left(x - 1/x\right)} \in (2\sqrt{2},\ \infty]$

$x - \frac{1}{x} < 0,\ \left(x - 1/x\right) + \dfrac{2}{\left(x - 1/x\right)} \in (-\infty, -2\sqrt{2}]$

Local minimum is $2\sqrt{2}$

44. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then

$\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots\ldots + \left[\frac{15}{x}\right]\right)$

(1) Is equal to 15 (2) Is equal to 120

(3) Does not exist (in R) (4) Is equal to 0

Answer (2)

Sol. As $\frac{1}{x} - 1 < \left[\frac{1}{x}\right] \le \frac{1}{x}$

$\frac{2}{x} - 1 < \left[\frac{2}{x}\right] \le \frac{2}{x}$

$\sum_{r=1}^{15}\left(\frac{r}{x} - 1\right) < \sum_{r=1}^{15}\left(\frac{r}{x}\right) \le \sum_{r=1}^{15}\frac{r}{x}$

$120 < \lim_{x \to 0^+} x\left(\sum_{r=1}^{15}\left[\frac{r}{x}\right]\right) \le 120$

$\Rightarrow \lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots\ldots + \left[\frac{15}{x}\right]\right) = 120$

45. Then value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1 + 2^x} dx$ is :

(1) $\frac{\pi}{2}$

(2) 4π

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{8}$

Answer (3)

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x dx}{1 + 2^x}$ ... (i)

Also, $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2^x \sin^2 x dx}{1 + 2^x}$ ... (ii)

Adding (i) and (ii)

$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$

$2I = 2\int_0^{\frac{\pi}{2}} \sin^2 x dx \Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx$ ... (iii)

$I = \int_0^{\frac{\pi}{2}} \cos^2 x dx$ ... (iv)

Adding (iii) & (iv)

$2I = \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

46. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$

(3) $\frac{3}{4}$ (4) $\frac{3}{10}$

Answer (1)

Sol. E_1 : Event that first ball drawn is red.

E_2 : Event that first ball drawn is black.

E : Event that second ball drawn is red.

$$P(E) = P(E_1).P\left(\frac{E}{E_1}\right) + P(E_2).P\left(\frac{E}{E_2}\right)$$

$$= \frac{4}{10}\times\frac{6}{12}+\frac{6}{10}\times\frac{4}{12}=\frac{2}{5}$$

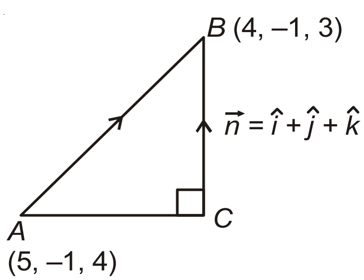
47. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, $x + y + z = 7$ is:

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$

(3) $\sqrt{\frac{2}{3}}$ (4) $\frac{2}{\sqrt{3}}$

Answer (3)

Sol.



Normal to the plane $x + y + z = 7$ is $\vec{n} = \hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{AB} = -\hat{i} - \hat{k} \Rightarrow |\overrightarrow{AB}| = AB = \sqrt{2}$

BC = Length of projection of $\overrightarrow{AB}$ on $\vec{n}$ $= |\overrightarrow{AB}\cdot\hat{n}|$

$$= \left|\left(-\hat{i}-\hat{k}\right)\cdot\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}}\right| = \frac{2}{\sqrt{3}}$$

Length of projection of the line segment on the plane is AC

$$AC^2 = AB^2 - BC^2 = 2 - \frac{4}{3} = \frac{2}{3}$$

$$AC^2 = \sqrt{\frac{2}{3}}$$

48. If sum of all the solutions of the equation $8\cos x\cdot\left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0, \pi]$ is $k\pi$, then k is equal to :

(1) $\frac{13}{9}$ (2) $\frac{8}{9}$

(3) $\frac{20}{9}$ (4) $\frac{2}{3}$

Answer (1)

Sol. $8\cos x\cdot\left(\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right)=1$

$\Rightarrow 8\cos x\left(\frac{3}{4}-\frac{1}{2}-1+\cos^2 x\right)=1$

$\Rightarrow 8\cos x\left(\frac{-3+4\cos^2 x}{4}\right)=1$

$\Rightarrow \cos 3x = 1$

$\Rightarrow \cos 3x = \frac{1}{2}$

$\Rightarrow 3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$\Rightarrow x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

$\Rightarrow$ Sum $= \frac{13\pi}{9}$

$\Rightarrow k = \frac{13}{9}$

49. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle $OPRQ$ is completed, then the locus of R is

(1) $2x + 3y = xy$ (2) $3x + 2y = xy$

(3) $3x + 2y = 6xy$ (4) $3x + 2y = 6$

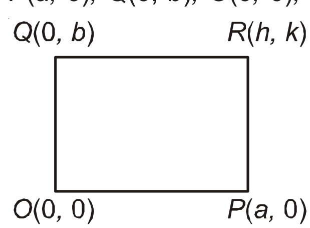
Answer (2)

Sol. Let the equation of line be $\frac{x}{a}+\frac{y}{b}=1$ ...(i)

(i) passes through the fixed point (2, 3)

$\Rightarrow \frac{2}{a}+\frac{3}{b}=1$ ...(ii)

$P(a, 0)$, $Q(0, b)$, $O(0, 0)$, Let $R(h, k)$,



Midpoint of OR is $\left(\frac{h}{2}, \frac{k}{2}\right)$

Midpoint of PQ is $\left(\frac{a}{2}, \frac{b}{2}\right) \Rightarrow h = a, \quad k = b$ ... (iii)

From (ii) & (iii),

$\frac{2}{h} + \frac{3}{k} = 1 \quad \Rightarrow$ locus of $R(h, k)$

$\frac{2}{x} + \frac{3}{y} = 1 \quad \Rightarrow 3x + 2y = xy$

50. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$

If $B - 2A = 100\lambda$, then λ is equal to

(1) 248 (2) 464

(3) 496 (4) 232

Answer (1)

Sol. $A = 1^2 + 2.2^2 + 3^2 + ... + 2.20^2$

$= (1^2 + 2^2 + 3^2 + ... + 20^2) + 4(1^2 + 2^2 + 3^2 + ... + 10^2)$

$= \frac{20 \times 21 \times 41}{6} + \frac{4 \times 10 \times 11 \times 21}{6}$

= 2870 + 1540 = 4410

$B = 1^2 + 2.2^2 + 3^2 + ... + 2.40^2$

$= (1^2 + 2^2 + 3^2 + ... + 40^2) + 4(1^2 + 2^2 + 3^2 + ... + 20^2)$

$= \frac{40 \times 41 \times 81}{6} + \frac{4 \times 20 \times 21 \times 41}{6}$

= 22140 + 11480 = 33620

$\Rightarrow$ $B - 2A$ = 33620 – 8820 = 24800

$\Rightarrow$ 100λ = 24800

λ = 248

51. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

(1) $\frac{7}{2}$ (2) 4

(3) $\frac{9}{2}$ (4) 6

Answer (3)

Sol. $y^2 = 6x$; slope of tangent at (x_1, y_1) is $m_1 = \frac{3}{y_1}$

also $9x^2 + by^2 = 16;$ slope of tangent at (x_1, y_1) is

$m_2 = \frac{-9x_1}{by_1}$

As $m_1 m_2 = -1$

$\Rightarrow \frac{-27x_1}{by_1^2} = -1$

$\Rightarrow b = \frac{9}{2}\left(\text{as } y_1^2 = 6x_1\right)$

52. Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is

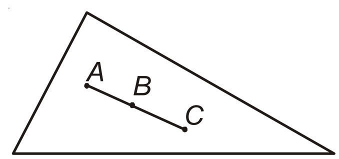
(1) $2\sqrt{10}$ (2) $3\sqrt{\frac{5}{2}}$

(3) $\frac{3\sqrt{5}}{2}$ (4) $\sqrt{10}$

Answer (2)

Sol. A (–3, 5)

B (3, 3)



So, $AB = 2\sqrt{10}$

Now, as, $AC = \frac{3}{2}AB$

So, radius = $\frac{3}{4}AB = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$

53. Let $S = \{t \in R : f(x) = |x - \pi| \cdot (e^{|x|} - 1)\sin|x|$ is not differentiable at $t\}$. Then the set S is equal to

(1) $\{0\}$ (2) $\{\pi\}$

(3) $\{0, \pi\}$ (4) ϕ (an empty set)

Answer (4)

Sol. $f(x) = |x - \pi|(e^{|x|} - 1)\sin|x|$

$x = \pi$, 0 are repeated roots and also continuous.

Hence, 'f' is differentiable at all x.

54. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to

(1) (–4, 3)

(2) (–4, 5)

(3) (4, 5)

(4) (–4, –5)

Answer (2)

Sol. $\Delta = \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$

$x = -4$ makes all three row identical

hence $(x + 4)^2$ will be factor

Also, $C_1 \to C_1 + C_2 + C_2$

$$\Delta = \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix}$$

$\Rightarrow$ $5x - 4$ is a factor

$\Delta = \lambda(5x-4)(x+4)^2$

$\therefore$ $B = 5$, $A = -4$

55. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to

(1) p

(2) q

(3) $\sim q$

(4) $\sim p$

Answer (4)

Sol. $\sim(p \vee q) \vee (\sim p \wedge q)$

By property, $(\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$= \sim p$

56. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to

(1) 10

(2) -30

(3) 30

(4) -10

Answer (1)

Sol. $\because$ System of equation has non-zero solution.

$$\therefore \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow$ $44 - 4k = 0$

$\therefore$ $k = 11$

Let $z = \lambda$

$\therefore$ $x + 11y = -3\lambda$

and $3x + 11y = 2\lambda$

$\therefore$ $x = \frac{5\lambda}{2}, y = -\frac{\lambda}{2}, z = \lambda$

$$\therefore \frac{xz}{y^2} = \frac{\frac{5\lambda}{2}\cdot\lambda}{\left(-\frac{\lambda}{2}\right)^2} = 10$$

57. Let $S = \{x \in R : x \geq 0$ and

$2\left|\sqrt{x}-3\right| + \sqrt{x}\left(\sqrt{x}-6\right) + 6 = 0\}$. Then S :

(1) Contains exactly one element

(2) Contains exactly two elements

(3) Contains exactly four elements

(4) Is an empty set

Answer (2)

Sol. $2|\sqrt{x}-3| + \sqrt{x}(\sqrt{x}-6) + 6 = 0$

$2|\ x-3| + (\ x-3+3)(\sqrt{x}-3-3) + 6 = 0$

$2|\ x-3| + (\ x-3)^2 - 3 = 0$

$(\ x-3)^2 + 2|\ x-3| - 3 = 0$

$(|\ x-3| + 3)(|\ x-3| - 1) = 0$

$\Rightarrow$ $|\sqrt{x}-3| = 1, |\sqrt{x}-3| + 3 \neq 0$

$\Rightarrow$ $\sqrt{x} - 3 = \pm 1$

$\Rightarrow$ $\sqrt{x} = 4, 2$

x = 16, 4

58. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is

(1) 185

(2) 85

(3) 95

(4) 195

Answer (3)

Sol. Equation of tangent at (1, 7) to curve $x^2 = y - 6$ is

$$x-1=\frac{1}{2}(y+7)-6$$

$2x - y + 5 = 0$ ...(i)

Centre of circle = (–8, –6)

Radius of circle $=\sqrt{64+36-c}=\sqrt{100-c}$

∵ Line (i) touches the circle

$$\therefore \quad \left|\frac{2(-8)-(-6)+5}{\sqrt{4+1}}\right|=\sqrt{100-c}$$

$$\sqrt{5}=\sqrt{100-c}$$

$\Rightarrow$ $c = 95$

59. let $y = y(x)$ be the solution of the differential equation

$\sin x\dfrac{dy}{dx}+y\cos x=4x$, $x \in (0, \pi)$. If $y=\left(\dfrac{\pi}{2}\right)=0$,

then $y\left(\dfrac{\pi}{6}\right)$ is equal to :

(1) $\dfrac{-8}{9\sqrt{3}}\pi^2$

(2) $-\dfrac{8}{9}\pi^2$

(3) $-\dfrac{4}{9}\pi^2$

(4) $\dfrac{4}{9\sqrt{3}}\pi^2$

Answer (2)

Sol. $\sin x\dfrac{dy}{dx}+y\cos x=4x$, $x \in (0, \pi)$

$$\frac{dy}{dx}+y\cot x=\frac{4x}{\sin x}$$

$\therefore$ I.F. $=e^{\int \cot x\, dx}=\sin x$

$\therefore$ Solution is given by

$$y\sin x=\int\frac{4x}{\sin x}\cdot\sin x\,dx$$

$y\cdot\sin x = 2x^2 + c$

when $x=\dfrac{\pi}{2}$, $y = 0$ $\Rightarrow$ $c=-\dfrac{\pi^2}{2}$

$\therefore$ Equation is : $y\sin x=2x^2-\dfrac{\pi^2}{2}$

when $x=\dfrac{\pi}{6}$ then $y\cdot\dfrac{1}{2}=2\cdot\dfrac{\pi^2}{36}-\dfrac{\pi^2}{2}$

$$\therefore \quad y=-\frac{8\pi^2}{9}$$

60. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane containing the lines L_1 and L_2, is

(1) $\dfrac{1}{3\sqrt{2}}$ (2) $\dfrac{1}{2\sqrt{2}}$

(3) $\dfrac{1}{\sqrt{2}}$ (4) $\dfrac{1}{4\sqrt{2}}$

Answer (1)

Sol. L_1 is parallel to $\begin{vmatrix} & \hat{j} & \hat{k} \\ 2 & -2 & 3 \\ 1 & -1 & 1\end{vmatrix}=\hat{i}+\hat{j}$

L_2 is parallel to $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2\end{vmatrix}=3\hat{i}-5\hat{j}-7\hat{k}$

Also, L_2 passes through $\left(\dfrac{5}{7},\dfrac{8}{7},0\right)$

So, required plane is $\begin{vmatrix} x-\frac{5}{7} & y-\frac{8}{7} & z \\ 1 & 1 & 0 \\ 3 & -5 & -7\end{vmatrix}=0$

$\Rightarrow$ $7x - 7y + 8z + 3 = 0$

Now, perpendicular distance $=\dfrac{3}{\sqrt{162}}=\dfrac{1}{3\sqrt{2}}$

PART–C : PHYSICS

61. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance?

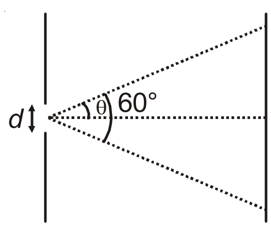
(*i.e.* distance between the centres of each slit.)

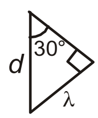
(1) 50 μm (2) 75 μm

(3) 100 μm (4) 25 μm

Answer (4)

Sol. $d\sin\theta = \lambda$





$\lambda = \frac{d}{2}$ $[d = 1 \times 10^{-6}\text{ m}]$

$\Rightarrow$ $\lambda = 5000$ Å

Fringe width, $B = \frac{\lambda D}{d'}$ (d' is slit separation)

$$10^{-2} = \frac{5000 \times 10^{-10} \times 0.5}{d'}$$

$\Rightarrow$ $d' = 25 \times 10^{-6}$ m = 25 μm

62. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n \approx A + B\lambda_n$ (2) $\Lambda_n^2 \approx A + B\lambda_n^2$

(3) $\Lambda_n^2 \approx \lambda$ (4) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

Answer (4)

Sol. $P_n = \frac{h}{\lambda_n}$, $P_g = \frac{h}{\lambda_g}$

$$k = \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}, \quad E = -k = -\frac{h^2}{2m\lambda^2}$$

$$E_n = -\frac{h^2}{2m\lambda_n^2}, \quad E_g = -\frac{h^2}{2m\lambda_g^2}$$

$$E_n - E_g = \frac{h^2}{2m}\left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right) = \frac{hc}{\Lambda_n}$$

$$\frac{h^2}{2m}\left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2\lambda_n^2}\right) = \frac{hc}{\Lambda_n}$$

$$\Lambda_n = \frac{2mc}{h}\left(\frac{\lambda_g^2\lambda_n^2}{\lambda_n^2 - \lambda_g^2}\right)$$

$$\Lambda_n = \frac{2mc\lambda_g^2}{h}\frac{\lambda_n^2}{\lambda_n^2\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)}$$

$$= \frac{2mc\lambda_g^2}{h}\left[1 - \frac{\lambda_g^2}{\lambda_n^2}\right]^{-1}$$

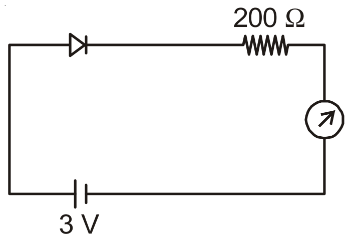
$$= \frac{2mc\lambda_g^2}{h}\left[1 + \frac{\lambda_g^2}{\lambda_n^2}\right]$$

$$= \frac{2mc\lambda_g^2}{h} + \left(\frac{2mc\lambda_g^4}{h}\right)\frac{1}{\lambda_n^2}$$

$$= A + \frac{B}{\lambda_n^2}$$

$$A = \frac{2mc\lambda_g^2}{h}, \quad B = \frac{2mc\lambda_g^4}{h}$$

63. The reading of the ammeter for a silicon diode in the given circuit is



(1) 15 mA (2) 11.5 mA

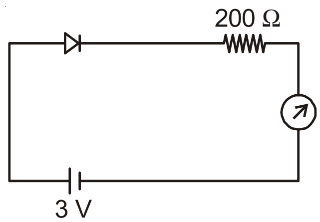
(3) 13.5 mA (4) 0

Answer (2)

Sol. $I = \dfrac{V - V_{\text{diode}}}{R}$

$= \left[\dfrac{3 - 0.7}{200} \times 1000\right]$ mA

= 11.5 mA



64. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(1) 3.5% (2) 4.5%

(3) 6% (4) 2.5%

Answer (2)

Sol. $\rho = \dfrac{m}{l^3}$

$\dfrac{d\rho}{\rho} = \dfrac{dm}{m} + 3\dfrac{dl}{l}$

= (1.5 + 3 × 1)

= 4.5%

65. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is

(1) $r_e < r_p = r_\alpha$ (2) $r_e < r_p < r_\alpha$

(3) $r_e < r_\alpha < r_p$ (4) $r_e > r_p = r_\alpha$

Answer (1)

Sol. $r = \dfrac{\sqrt{2mk}}{qB}$

$\dfrac{r_\alpha}{r_p} = \dfrac{\sqrt{2m_\alpha}}{q_\alpha} \times \dfrac{q_p}{\sqrt{2m_p}}$ $\left[\begin{aligned} m_\alpha &= 4m_p \\ q_\alpha &= 2q_p \end{aligned}\right]$

= 1

Mass of electron is least and charge $q_e = e$

So, $r_e < r_p = r_\alpha$

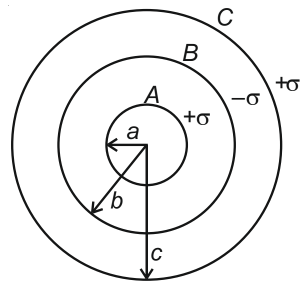
66. Three concentric metal shells A, B and C of respective radii a, b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is

(1) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{a^2 - b^2}{b} + c\right]$ (2) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{b^2 - c^2}{b} + a\right]$

(3) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{b^2 - c^2}{c} + a\right]$ (4) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{a^2 - b^2}{a} + c\right]$

Answer (1)

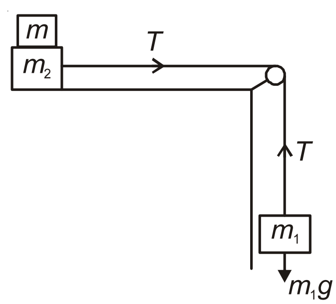
Sol.



$V_B = \left[\dfrac{\sigma 4\pi a^2}{4\pi\varepsilon_0 b} - \dfrac{\sigma 4\pi b^2}{4\pi\varepsilon_0 b} + \dfrac{\sigma 4\pi c^2}{4\pi\varepsilon_0 c}\right]$

$V_B = \dfrac{\sigma}{\varepsilon_0}\left(\dfrac{a^2 - b^2}{b} + c\right)$

67. Two masses m_1 = 5 kg and m_2 = 10 kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is



(1) 27.3 kg (2) 43.3 kg

(3) 10.3 kg (4) 18.3 kg

Answer (1)

Sol. To stop the moving block m_2, acceleration of m_2 should be opposite to velocity of m_2

$m_1 g < \mu(m + m_2)g$

$\Rightarrow$ 5 < 0.15(10 + m_2)

$\Rightarrow$ m_2 > 23.33 kg

$\therefore$ Minimum mass = 27.3 kg (according to given options)

68. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is

(1) $\frac{k}{2a^2}$ (2) Zero

(3) $-\frac{3}{2}\frac{k}{a^2}$ (4) $-\frac{k}{4a^2}$

Answer (2)

Sol. $F = \frac{-dU}{dr}$ $\left[U = -\frac{k}{2r^2}\right]$

$\frac{mv^2}{r} = \frac{k}{r^3}$ [This force provides necessary centripetal force]

$\Rightarrow mv^2 = \frac{k}{r^2}$

$\Rightarrow K.E = \frac{k}{2r^2}$

$\Rightarrow P.E = -\frac{k}{2r^2}$

Total energy = Zero

69. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be

(1) 0.3 nC (2) 2.4 nC

(3) 0.9 nC (4) 1.2 nC

Answer (4)

Sol. $C' = KC_0$

$Q = KC_0V$

$Q_{\text{induced}} = Q\left(1 - \frac{1}{K}\right)$

$= \frac{5}{3} \times 90 \times 10^{-12} \times 20\left(1 - \frac{3}{5}\right)$

= 1.2 nC

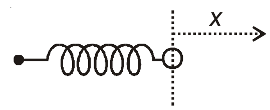
70. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/second. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = 6.02×10^{23} gm mole^{-1})

(1) 7.1 N/m (2) 2.2 N/m

(3) 5.5 N/m (4) 6.4 N/m

Answer (1)

Sol.



$Kx = ma \Rightarrow a = (K/m)x$

$T = 2\pi\sqrt{\frac{m}{K}}$

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = 10^{12}$

$= \frac{1}{4\pi^2} \times \frac{K}{m} = 10^{24}$

$K = 4\pi^2 m \times 10^{24} = \frac{4 \times 10 \times 108 \times 10^{-3}}{6.02 \times 10^{23}} \times 10^{24}$

= 7.1 N/m

71. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively

(1) (.28, .89) (2) (0, 0)

(3) (0, 1) (4) (.89, .28)

Answer (4)

Sol. $mu = mv_1 + 2m \times v_2$ ...(i)

$u = (v_2 - v_1)$ ...(ii)

$\Rightarrow v_1 = -\frac{u}{3}$

$\therefore \frac{\Delta E}{E} = p_d = \frac{\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{3}\right)^2}{\frac{1}{2}mu^2}$

$= \frac{8}{9} = 0.89$

And $mu = mv_1 + (12m) \times v_2$ ...(iii)

$u = (v_2 - v_1)$ ...(iv)

$$\Rightarrow v_1 = -\frac{11}{13}u$$

$$\therefore \frac{\Delta E}{E} = p_c = \frac{\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{11}{13}u\right)^2}{\frac{1}{2}mu^2} = \frac{48}{169} = 0.28$$

72. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is

(1) $\sqrt{3}$ (2) $\sqrt{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) 2

Answer (2)

Sol. $m = I(\pi R^2)$, $m' = 2m = I \times \left(\pi\sqrt{2}R\right)^2$

$$\therefore R' = \sqrt{2}R$$

$$B_1 = \frac{\mu_0 I}{2R}$$

$$B_2 = \frac{\mu_0 I}{2\times\left(\sqrt{2}R\right)}$$

$$\therefore \frac{B_1}{B_2} = \sqrt{2}$$

73. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 1.5 Ω

(2) 2 Ω

(3) 2.5 Ω

(4) 1 Ω

Answer (1)

Sol. $\because E \propto l_1$

and $E - ir \propto l_2$

$$\therefore \frac{E}{E - ir} = \frac{l_1}{l_2}$$

$$\Rightarrow \frac{E}{E - \left(\frac{E}{r+5}\right)\times r} = \frac{52}{40}$$

$$\Rightarrow \frac{r+5}{5} = \frac{13}{10}$$

$\Rightarrow r = 1.5\ \Omega$

74. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(1) 2×10^4

(2) 2×10^5

(3) 2×10^6

(4) 2×10^3

Answer (2)

Sol. Frequency of carrier = 10×10^9 Hz

Available bandwidth = 10% of 10×10^9 Hz

= 10^9 Hz

Bandwidth for each telephonic channel = 5 kHz

$\therefore$ Number of channels $= \frac{10^9}{5\times 10^3}$

$= 2 \times 10^5$

75. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is

(1) 30°

(2) 45°

(3) 60°

(4) 0°

Answer (2)

Sol. Polaroids A and B are oriented with parallel pass axis

Let polaroid C is at angle θ with A then it makes θ with B also.

$$\because \quad \frac{I}{8} = \left(\frac{I}{2} \times \cos^2\theta\right) \times \cos^2\theta$$

$$\Rightarrow \quad \cos^2\theta = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 45°$$

76. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances?

(1) 505 Ω (2) 550 Ω

(3) 910 Ω (4) 990 Ω

Answer (2)

Sol. $\dfrac{R_1}{R_2} = \dfrac{l}{(100 - l)}$

$$\frac{R_2}{R_1} = \frac{(l - 10)}{(110 - l)}$$

$(100 - l)(110 - l) = l(l - 10)$

$11000 + l^2 - 210l = l^2 - 10l$

$\Rightarrow \quad l = 55$ cm

$$R_1 = R_2\left(\frac{55}{45}\right)$$

$R_1 + R_2 = 1000\ \Omega$

$R_1 = 550\ \Omega$

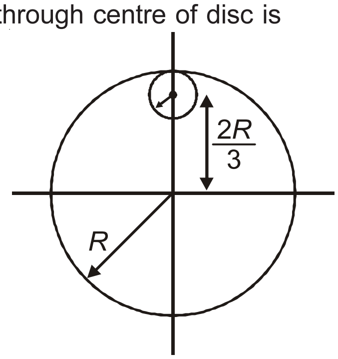
77. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\dfrac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

(1) $\dfrac{40}{9}MR^2$

(2) $10MR^2$

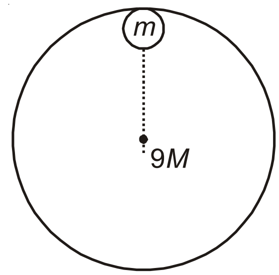
(3) $\dfrac{37}{9}MR^2$

(4) $4MR^2$



Answer (4)

Sol.



$$m = \frac{(9M)}{9} = M$$

$$I_1 = \frac{(9M) \times R^2}{2}$$

$$I_2 = \frac{M \times \left(\frac{R}{3}\right)^2}{2} + M \times \left(\frac{2R}{3}\right)^2 = \frac{MR^2}{2}$$

$\therefore \quad I_{req} = I_1 - I_2$

$$= \frac{9}{2}MR^2 - \frac{MR^2}{2}$$

$= 4MR^2$

78. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

(1) $\sqrt{2}v_0$ (2) $\dfrac{v_0}{2}$

(3) $\dfrac{v_0}{\sqrt{2}}$ (4) $\dfrac{v_0}{4}$

Answer (1)

Sol. It is a case of superelastic collision

$mv_0 = mv_1 + mv_2$ ...(i)

$\Rightarrow \quad v_1 + v_2 = v_0$

$$\frac{1}{2}m\left(v_1^2 + v_2^2\right) = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow \quad \left(v_1^2 + v_2^2\right) = \frac{3}{2}v_0^2 \quad \text{...(ii)}$$

$$\Rightarrow \quad (v_1 + v_2)^2 = v_1^2 + v_2^2 + 2v_1v_2$$

$$\Rightarrow \quad v_0^2 = \frac{3v_0^2}{2} + 2v_1v_2$$

$$\Rightarrow \quad 2v_1v_2 = -\frac{v_0^2}{2} \quad \text{...(iii)}$$

$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v_0^2 + v_0^2$

$$\Rightarrow \quad v_1 - v_2 = \sqrt{2}\,v_0$$

79. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z-ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic. If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$ (2) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

(3) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$ (4) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$

Answer (2)

Sol. $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right]$ air

$\vec{E}_2 = E_{02}\hat{x}\cos\left[k(2z-ct)\right]$ medium

During refraction, frequency remains unchanged, whereas wavelength gets changed.

$\therefore \; k' = 2k$ (From equations)

$\Rightarrow \frac{2\pi}{\lambda'} = 2\left(\frac{2\pi}{\lambda_0}\right)$

$\Rightarrow \lambda' = \frac{\lambda_0}{2}$

$\Rightarrow v = \frac{c}{2}$

$\Rightarrow \frac{1}{\sqrt{\mu_0\varepsilon_2}} = \frac{1}{2}\times\frac{1}{\sqrt{\mu_0\varepsilon_1}}$

$\Rightarrow \frac{\varepsilon_1}{\varepsilon_2} = \frac{1}{4}$

80. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exibits resonance. The quality factor, Q is given by

(1) $\frac{\omega_0 R}{L}$ (2) $\frac{R}{(\omega_0 C)}$

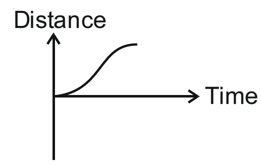
(3) $\frac{CR}{\omega_0}$ (4) $\frac{\omega_0 L}{R}$

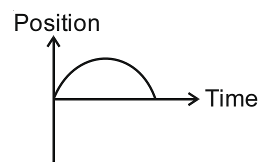
Answer (4)

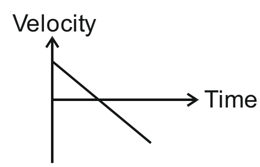
Sol. Quality factor, $Q = \frac{\omega_0}{(2\Delta\omega)}$

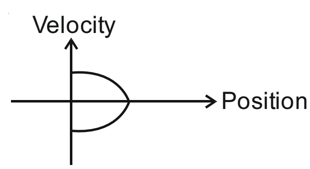
$Q = \frac{\omega_0 L}{R}$

81. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. Options (2), (3) and (4) correspond to uniformly accelerated motion in a straight line with positive initial velocity and constant negative acceleration, whereas option (1) does not correspond to this motion.

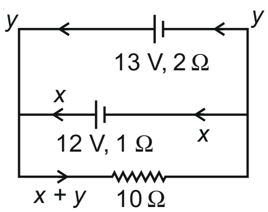
82. Two batteries with e.m.f 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between

(1) 11.5 V and 11.6 V (2) 11.4 V and 11.5 V

(3) 11.7 V and 11.8 V (4) 11.6 V and 11.7 V

Answer (1)

Sol.



Applying KVL in loops

$12 - x - 10(x + y) = 0$

$\Rightarrow \quad 12 = 11x + 10y \qquad \text{...(i)}$

$\quad\;\; 13 = 10x + 12y \qquad \text{...(ii)}$

Solving $x = \dfrac{7}{16}\text{A},\ y = \dfrac{23}{32}\text{A}$

$V = 10(x + y) = 11.56$ V

Aliter : $r_{eq} = \dfrac{2}{3}\Omega$, $R = 10\ \Omega$

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} \Rightarrow E_{eq} = \frac{37}{3}\text{ V}$$

$$V = \frac{E_{eq}}{R + r_{eq}}R = 11.56\text{ V}$$

83. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then

(1) $T \propto R^{\frac{n}{2}+1}$ (2) $T \propto R^{(n+1)/2}$

(3) $T \propto R^{n/2}$ (4) $T \propto R^{3/2}$ for any n

Answer (2)

Sol. $m\omega^2 R = k\,R^{-n} = \dfrac{k}{R^n}$

$$\Rightarrow \quad \frac{1}{T^2} \propto \frac{1}{R^{n+1}}$$

$$\Rightarrow \quad T \propto R^{\left(\frac{n+1}{2}\right)}$$

84. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is

(1) $16\ \nu_L$ (2) $\nu_L/16$

(3) $\nu_L/25$ (4) $25\ \nu_L$

Answer (3)

Sol. $h\nu_L = E\left[\dfrac{1}{1^2} - \dfrac{1}{\infty}\right] = E$

$$h\nu_P = E\left[\frac{1}{5^2} - \frac{1}{\infty}\right] = \frac{E}{25}$$

$$\Rightarrow \quad \nu_P = \frac{\nu_L}{25}$$

85. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$e = 100\ \sin 30t$

$$i = 20\sin\left(30t - \frac{\pi}{4}\right)$$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively

(1) $\dfrac{1000}{\sqrt{2}}, 10$ (2) $\dfrac{50}{\sqrt{2}}, 0$

(3) 50, 0 (4) 50, 10

Answer (1)

Sol. $P_{av} = E_{rms}\ I_{rms}\ \cos\phi$

$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$$

$$i_{wattless} = i_{rms}\ \sin\phi = \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 10$$

86. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2 V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 195 K (b) –2.7 kJ

(2) (a) 189 K (b) –2.7 kJ

(3) (a) 195 K (b) 2.7 kJ

(4) (a) 189 K (b) 2.7 kJ

Answer (2)

Sol. $TV^{\gamma - 1}$ = Constant

$$T_f = 300\left(\frac{V}{2V}\right)^{\frac{5}{3}-1} = 189\text{ K}$$

$$\Delta U = nC_v\Delta T = 2 \times \frac{3R}{2} \times [189 - 300]$$

$$= -2.7\text{ kJ}$$

87. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area of a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is

(1) $\frac{Ka}{3mg}$

(2) $\frac{mg}{3Ka}$

(3) $\frac{mg}{Ka}$

(4) $\frac{Ka}{mg}$

Answer (2)

Sol. $K = -V\frac{dP}{dV}$

$$\Rightarrow \frac{-dV}{V} = \frac{dP}{K} = \frac{mg}{Ka}$$

$$\Rightarrow \frac{-3dr}{r} = \frac{mg}{Ka}$$

$$\Rightarrow \frac{dr}{r} = -\frac{mg}{3Ka}$$

88. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations?

(1) 2.5 kHz (2) 10 kHz

(3) 7.5 kHz (4) 5 KHz

Answer (4)

Sol. $f_0 = \frac{V}{2L} = \frac{1}{2L}\sqrt{\frac{Y}{\rho}}$

$$= \frac{1}{2\times 0.6}\sqrt{\frac{9.27\times 10^{10}}{2.7\times 10^3}} = 4.88 \text{ kHz} \approx 5 \text{ kHz}$$

89. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly

(1) 4.70×10^3 N/m^2 (2) 2.35×10^2 N/m^2

(3) 4.70×10^2 N/m^2 (4) 2.35×10^3 N/m^2

Answer (4)

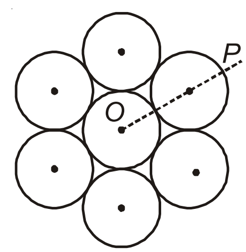
Sol. $F = nmv\cos\theta \times 2$

$$P = \frac{F}{A} = \frac{2.nmv\cos\theta}{A}$$

$$= \frac{2\times 10^{23}\times 3.32\times 10^{-27}\times 10^3}{\sqrt{2}\times 2\times 10^{-4}} \text{N/m}^2$$

$= 2.35 \times 10^3$ N/m^2

90. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is



(1) $\frac{55}{2}MR^2$

(2) $\frac{73}{2}MR^2$

(3) $\frac{181}{2}MR^2$

(4) $\frac{19}{2}MR^2$

Answer (3)

Sol. $I_0 = \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M(2R)^2\right)$

$I_P = I_0 + 7M(3R)^2$

$= \frac{181}{2}MR^2$