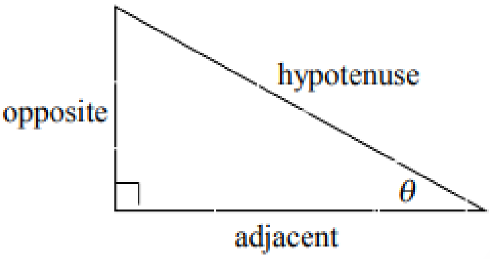
Trigonometric Formulas

Right-Triangle Definition:

For this definition we assume that

$0<\theta<\frac{\pi}{2}$ or $0°<\theta<90°$.



$$\sin\theta=\frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta=\frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta=\frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta=\frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta=\frac{\text{adjacent}}{\text{opposite}}$$

Reciprocal Identities:

$$\sin x=\frac{1}{\csc x} \qquad \csc x=\frac{1}{\sin x}$$

$$\cos x=\frac{1}{\sec x} \qquad \sec x=\frac{1}{\cos x}$$

$$\tan x=\frac{1}{\cot x} \qquad \cot x=\frac{1}{\tan x}$$

Ratio Identities:

$$\tan x=\frac{\sin x}{\cos x} \qquad \cot x=\frac{\cos x}{\sin x}$$

$$\sin x=\cos x\tan x \qquad \cos x=\sin x\cot x$$

Tangent and Cotangent Identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Reciprocal Identities:

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Half Angle Formulas:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum and Difference Formulas/Identities:

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$
$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$
$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Double-angle formulas:

Substitute $\alpha = \beta$ in the previous sum formulas, then we find the double-angle formulas: :

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \qquad (7)$$
$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$
$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

Two useful forms of (7) are derived by replacing $\cos^2 \alpha$ by $1 - \sin^2 \alpha$, resp. $\sin^2 \alpha$ by $1 - \cos^2 \alpha$:

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$
$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

And so:

$$\sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha)$$

$$\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha)$$

Inverse Trig Functions:

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x \qquad \cos^{-1}\left(\cos(\theta)\right) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}\left(\sin(\theta)\right) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x \qquad \tan^{-1}\left(\tan(\theta)\right) = \theta$

Alternate Notation

$\sin^{-1} x = \arcsin x$

$\cos^{-1} x = \arccos x$

$\tan^{-1} x = \arctan x$

Sum to Product Formulas:

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Degrees to Radians Formulas:

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Cofunction Formulas:

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta \qquad \sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$$

Law of Sines, Cosines and Tangents:

Law of Sines

$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2=b^2+c^2-2bc\cos\alpha$$

$$b^2=a^2+c^2-2ac\cos\beta$$

$$c^2=a^2+b^2-2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c}=\frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c}=\frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

How to Find Reference Angles:

Step 1: Determine which quadrant the angle is in

Step 2: Use the appropriate formula

Quad I	=	is the angle itself		
Quad II	=	$180 - \theta$	or	$\pi - \theta$
Quad III	=	$\theta - 180$	or	$\theta - \pi$
Quad IV	=	$360 - \theta$	or	$2\pi - \theta$

Odd-Even Identities:

Sin $(-x)$ = $-\sin x$ Csc $(-x)$ = $-\csc x$

Cos $(-x)$ = $\cos x$ Sec $(-x)$ = $\sec x$

Tan $(-x)$ = $-\tan x$ Cot $(-x)$ = $-\cot x$

Facts and Properties

Period:

The period of a function is the number, T, such that $f(\theta+T)=f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T=\frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T=\frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T=\frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T=\frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T=\frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T=\frac{\pi}{\omega}$$

Domain:

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq \left(n+\frac{1}{2}\right)\pi,\quad n=0,\pm1,\pm2,\ldots$

$\csc\theta$, $\theta \neq n\pi,\quad n=0,\pm1,\pm2,\ldots$

$\sec\theta$, $\theta \neq \left(n+\frac{1}{2}\right)\pi,\quad n=0,\pm1,\pm2,\ldots$

$\cot\theta$, $\theta \neq n\pi,\quad n=0,\pm1,\pm2,\ldots$

Range:

The range is all possible values to get out of the function.

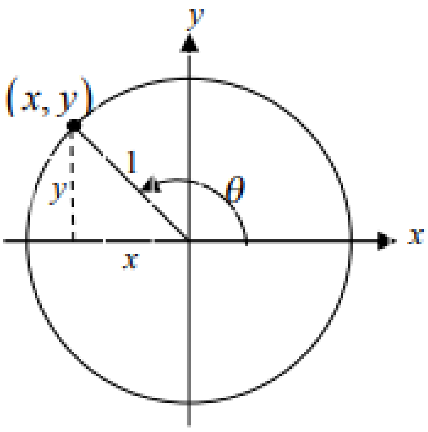
$-1 \leq \sin\theta \leq 1$ $\qquad$ $\csc\theta \geq 1$ and $\csc\theta \leq -1$

$-1 \leq \cos\theta \leq 1$ $\qquad$ $\sec\theta \geq 1$ and $\sec\theta \leq -1$

$-\infty < \tan\theta < \infty$ $\qquad$ $-\infty < \cot\theta < \infty$

Unit circle definition:

For this definition θ is any angle.

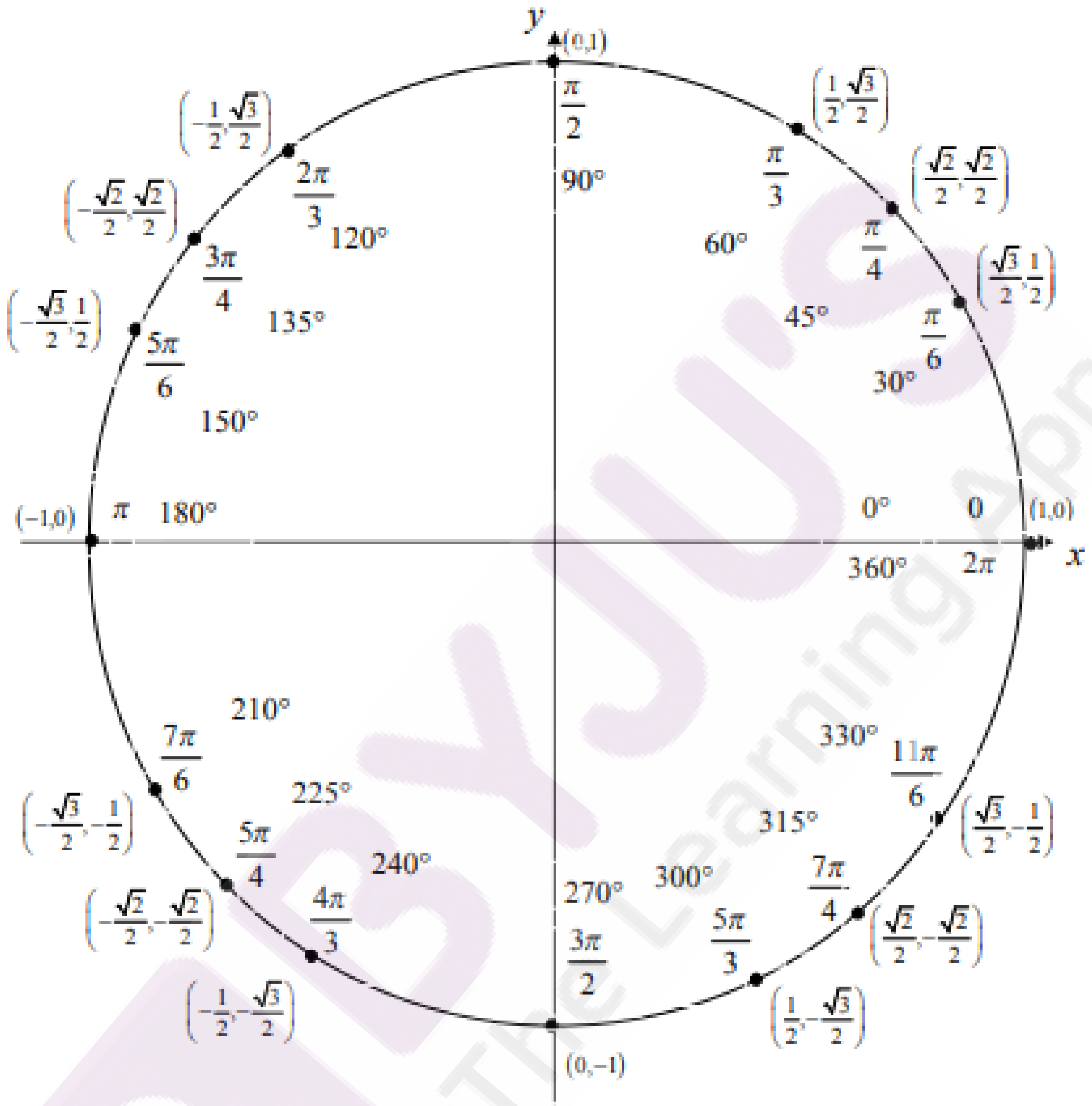


$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Unit Circle:


y
x
(0,1)
(1,0)
(−1,0)
(0,−1)
90°
60°
45°
30°
0°
360°
120°
135°
150°
180°
210°
225°
240°
270°
300°
315°
330°
$\frac{\pi}{2}$
$\frac{\pi}{3}$
$\frac{\pi}{4}$
$\frac{\pi}{6}$
0
2π
$\frac{2\pi}{3}$
$\frac{3\pi}{4}$
$\frac{5\pi}{6}$
π
$\frac{7\pi}{6}$
$\frac{5\pi}{4}$
$\frac{4\pi}{3}$
$\frac{3\pi}{2}$
$\frac{5\pi}{3}$
$\frac{7\pi}{4}$
$\frac{11\pi}{6}$
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$

Radian and Degree Measures of Angles:

$$1 \text{ rad} = \frac{180°}{\pi} \approx 57°17'45''$$

$$1° = \frac{\pi}{180} \text{ rad} \approx 0.017453 \text{ rad}$$

$$1' = \frac{\pi}{180 \cdot 60} \text{ rad} \approx 0.000291 \text{ rad}$$

$$1'' = \frac{\pi}{180 \cdot 3600} \text{ rad} \approx 0.000005 \text{ rad}$$

Angle (degrees)	0	30	45	60	90	180	270	360
Angle (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

Signs of Trigonometric Functions:

Quadrant	Sin α	Cos α	Tan α	Cot α	Sec α	Cosec α
I	+	+	+	+	+	+
II	+					+
III			+	+		
IV		+			+	

Trigonometric Functions of Common Angles:

$\alpha°$	α rad	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$	$\sec\alpha$	$\operatorname{cosec}\alpha$
0	0	0	1	0	∞	1	∞
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
90	$\frac{\pi}{2}$	1	0	∞	0	∞	1
120	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$
180	π	0	-1	0	∞	-1	∞
270	$\frac{3\pi}{2}$	-1	0	∞	0	∞	-1
360	2π	0	1	0	∞	1	∞

$\alpha°$	α rad	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$
15	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
18	$\frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$	$\sqrt{5+2\sqrt{5}}$
36	$\frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$
54	$\frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$
72	$\frac{2\pi}{5}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$
75	$\frac{5\pi}{12}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$

Most Important Formulas:

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\sec^2\alpha - \tan^2\alpha = 1$$

$$\csc^2\alpha - \cot^2\alpha = 1$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$$

$$\tan\alpha \cdot \cot\alpha = 1$$

$$\sec\alpha = \frac{1}{\cos\alpha}$$

$$\operatorname{cosec}\alpha = \frac{1}{\sin\alpha}$$