Differential Equation

What it is?

A differential equation is an equation that contains derivatives which are either partial derivatives or ordinary derivatives. The derivatives represent a rate of change, and the differential equation describes a relationship between the quantity that is continuously varying and the speed of change.

Real World Usage

To understand Differential equations, let us consider this simple example. Have you ever thought why a hot cup of coffee cools down when kept under normal conditions? According to Newton, cooling of a hot body is proportional to the temperature difference between its own temperature T and the temperature T_o of its surrounding. This statement in terms of mathematics can be written as:

$\frac{dT}{dt} \propto (T - T_o)$ ....(1)

Introducing a proportionality constant k, the above equation can be written as:

$\frac{dT}{dt} = k(T - T_o)$ ....(2)

Here, T is the temperature of the body,

t is the time,

T_o is temperature of the surrounding,

$\frac{dT}{dt}$ is the rate of cooling of the body

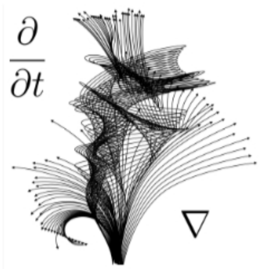
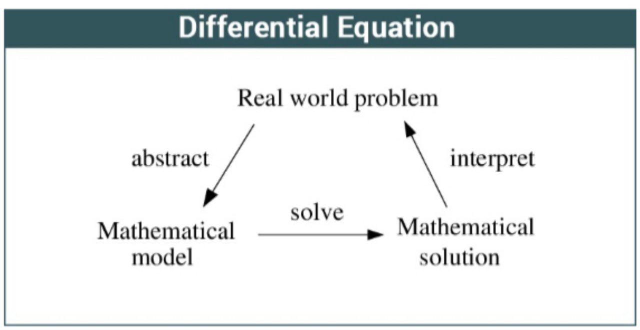


Fig: The path of the projectile follows a curve which can be derived from an ordinary differential equation.



Eg: dy/dx = 3x

Here, the differential equation contains a derivative that involves a variable (dependent variable,y) w.r.t another variable(independent variable,x). The types of differential equations are 1. An ordinary differential equation containd one independent variable and its derivatives. It is frequently called as ODE. The general definition of ordinary differential equation is of the form: Given an F, a function os x and y and derivative of y , we have

F(x, y, y' .....y^(n1)) = y (n) is an explicit ordinary differential equation of order n.

2. Partial differential equation that contains one or more independent variable.

There are **two ways to solve a differential equation ;**

1. Separation of variables
2. integrating factor

Separation of the variable is done when the differential equation can be written in the form of dy/dx= f(y)g(x) where f is the function of y only and g is the function of x only. Taking an initial condition We rewrite this problem as 1/f(y)dy= g(x)dx and then integrate them from both the sides.

Integrating factor technique is used when the differential equation is of the form dy/dx+p(x)y=q(x) where p and q are both the functions of x only.

First order differential equation is of the form y'+ P(x)y = Q(x). where p and q are both functions of x and hence called first order differential equation because it contains functions and the first derivative of y. Higher order differential equation is an equation that contains derivatives of an unknown function which can be either a partial or ordinary derivative.It can be represented in any order.

Application of differential equations

1) Differential equations describe various exponential growths and decays.

2) They are also used to describe the change in investment return over time.

3) They are used in the field of medicines for modeling cancer growth or spread of a disease in the body.

4) Movement of electricity can also be described with the help of differential equations.

5) They help economists in finding the optimum investment strategies.

6) Motion of waves or a pendulum can also be described using differential equations.

Illustration 1: Verify that the function $y = e^{-3x}$ is a solution to the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$.

Solution: The function given is $y = e^{-3x}$. We differentiate both the sides of the equation with respect to x,

$\frac{dy}{dx} = -3e^{-3x}$ ....(1)

Now we again differentiate the above equation with respect to x,

$\frac{d^2y}{dx^2} = 9e^{-3x}$ ....(2)

We substitute the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and y in the differential equation given in the question,

On left hand side we get, LHS = $9e^{-3x} + (-3e^{-3x}) - 6e^{-3x}$

= $9e^{-3x} - 9e^{-3x} = 0$ (which is equal to RHS)

Therefore the given function is a solution to the given differential equation.

Stochastic differential equation – that contains one or more terms that are stochastic and the solution it provides is also stochastic.

The various application of differential equations in engineering is : heat conduction analysis , in physics it can be used to understand the motion of waves, pendulums, in chemistry it is used for modeling the chemical reactions, in medical science for monitoring the cancer growth. The ordinary differential equation can be utilized as an application in engineering field like for finding the relationship between various parts of the bridge.

To gain better understanding about this topic, it would be ideal if students would be able to have hands on experience about the same by working on NCERT Solutions for Differential Equations.