Maharashtra Board
Class IX Mathematics - Algebra
Sample Paper – 3
Solution

Time: 2 hours **Total Marks: 40**

1.

i. A = {January, February, March, May, July, August, October, December}

ii. Let $x = 1.\overline{3} = 1.33333....$

$\therefore 10x = 13.3333...$

Now, $10x - x = 12$

$\therefore 9x = 12$

$\therefore x = \frac{12}{9} = \frac{4}{3}$

iii. Let 'x' be the expression to be subtracted from 2a + 6b − 5 to get −3a + 2b + 3.

$\therefore$ 2a + 6b − 5 − x = −3a + 2b + 3

$\therefore$ 2a + 6b − 5 − x + 3a − 2b − 3 = 0

$\therefore$ 2a + 3a + 6b − 2b − 5 − 3 − x = 0

$\therefore$ 5a + 4b − 8 − x = 0

$\therefore$ x = 5a + 4b − 8

iv. Length of a rectangle is 4 cm more than its breadth, perimeter of the rectangle is 40 cm.

Let the length of the rectangle be x cm and the breadth be y cm.

According to the first condition,

x = y + 4

$\therefore$ x − y = 4 ....(1)

Now, perimeter of the rectangle = 2(length + breadth)

According to the second condition,

2(x + y) = 40

$\therefore$ x + y = 20 ....(2)

v. Given data: 7, 6, 10, 13, 1, 3, 4, 4

Number of observations, n = 8

$$\text{Mean, } \bar{x} = \frac{\text{Sum of observations}}{\text{Number of observations}} = \frac{7+6+10+13+1+3+4+4}{8} = \frac{48}{8} = 6$$

vi. $(a + b)(c + d) - a^2 + b^2$

$= (a + b)(c + d) - (a^2 - b^2)$

$= \underline{(a + b)}(c + d) - \underline{(a + b)}(a - b)$

$= (a + b)[(c + d) - (a - b)]$

$= (a + b)(c + d - a + b)$

$= (a + b)(-a + b + c + d)$

2.

i. The ungrouped frequency distribution table is as follows:

Number of children	Tally marks	Frequency (f)
1	~~IIII~~ ~~IIII~~ I	11
2	~~IIII~~ ~~IIII~~ ~~IIII~~ I	16
3	~~IIII~~ I	6
4	I	1
	Total	$N = \sum f_i = 34$

ii. E = {x|x ∈ N and x is a divisor of 12}

∴ E = {1, 2, 3, 4, 6, 12}

F = {y|y ∈ N and y is a divisor of 18}

∴ F = {1, 2, 3, 6, 9, 18}

∴ E ∪ F = {1, 2, 3, 4, 6, 9, 12, 18}

iii. $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} = a + b\sqrt{6}$

$\therefore \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = a + b\sqrt{6}$

$\therefore \frac{3+\sqrt{6}+\sqrt{6}+2}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} = a + b\sqrt{6}$

$\therefore \frac{5+2\sqrt{6}}{3-2} = a + b\sqrt{6}$

$\therefore 5 + 2\sqrt{6} = a + b\sqrt{6}$

Equating the values of both the sides, we get $a = 5$ and $b = 2$.

iv. Let x be the number to be added to each of the given numbers.

Then the numbers (1 + x), (7 + x), (9 + x) and (31 + x) are in proportion.

$\therefore \frac{(1+x)}{(7+x)} = \frac{(9+x)}{(31+x)}$

$\therefore (1+x)(31+x) = (7+x)(9+x)$

$\therefore 31 + x + 31x + x^2 = 63 + 7x + 9x + x^2$

$\therefore 31 + 32x + x^2 = 63 + 16x + x^2$

$\therefore 32x - 16x = 63 - 31$

$\therefore 16x = 32$

$\therefore x = \frac{32}{16}$

$\therefore x = 2$

Thus, 2 should be added to each of the given numbers to make them in proportion.

v. From the given figure, we have

(a) It is a subdivided bar diagram.

(b) There are 20 boys and 50 girls in division D.

(c) There are 30 girls in division B.

(d) Division C has equal number of boys and girls, i.e. 40 boys and 40 girls.

vi. Dividend: $x^3 - 4x^2 - 2x + 1$

Index form: $x^3 - 4x^2 - 2x + 1$

Co-efficient form: (1, −4, −2, 1)

Comparing the divisor (x − 3) with (x − a), we have a = 3

$$\begin{array}{c|cccc} 3 & 1 & -4 & -2 & 1 \\ & & 3 & -3 & -15 \\ \hline & 1 & -1 & -5 & \boxed{-14} \end{array}$$

$\therefore$ Quotient in co-efficient form: (1, −1, −5)

$\therefore$ Quotient = $x^2 - x - 5$ in the variable x

Remainder = −14

$\therefore (x^3 - 4x^2 - 2x + 1) = (x - 3)(x^2 - x - 5) + (-14)$

3.

i. 5x + 7y = 17 ....(1)

7x + 5y = 19 ....(2)

Adding equations (1) and (2), we have

$$\begin{array}{rrl} & 5x + 7y & = 17 \\ + & 7x + 5y & = 19 \\ \hline & 12x + 12y & = 36 \end{array}$$

$\therefore$ 12(x + y) = 36

$\therefore$ x + y = 3

Subtracting equation (1) from equation (2), we have

$$\begin{array}{rrl} & 7x + 5y & = 19 \\ & 5x + 7y & = 17 \\ - & - \quad - & \\ \hline & 2x - 2x & = 2 \end{array}$$

$\therefore$ 2(x − y) = 2

$\therefore$ x − y = 1

$\therefore$ x + y = 3 and x − y = 1

ii. $5\sqrt{3} + 2\sqrt{27} + 4\sqrt{\frac{1}{3}}$

$= 5\sqrt{3} + 2\sqrt{9 \times 3} + 4\sqrt{\frac{1}{3}} \times \sqrt{\frac{3}{3}}$

$= 5\sqrt{3} + 2 \times 3\sqrt{3} + 4\,\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 5\sqrt{3} + 6\sqrt{3} + \frac{4\sqrt{3}}{3}$

$= \sqrt{3}\left[5 + 6 + \frac{4}{3}\right]$

$= \sqrt{3}\left[11 + \frac{4}{3}\right]$

$= \sqrt{3}\left[\frac{33 + 4}{3}\right]$

$= \sqrt{3}\left[\frac{37}{3}\right]$

iii. (a) In Quadrant I, the coordinates of the required point will be (5, 4).

(b) In Quadrant II, the coordinates of the required point will be (−5, 4).

(c) In Quadrant III, the coordinates of the required point will be (−5, −4).

(d) In Quadrant IV, the coordinates of the required point will be (5, −4).

iv. $\frac{7a^2 + 2b^2}{7a^2 - 2b^2} = \frac{113}{13}$

$\therefore \frac{7a^2 + 2b^2 + 7a^2 - 2b^2}{7a^2 + 2b^2 - (7a^2 - 2b^2)} = \frac{113 + 13}{113 - 13}$ ...(Compodendo – Dividendo)

$\therefore \frac{14a^2}{7a^2 + 2b^2 - 7a^2 + 2b^2} = \frac{126}{100}$

$\therefore \frac{14a^2}{4b^2} = \frac{126}{100}$

$\therefore \frac{a^2}{b^2} = \frac{126}{100} \times \frac{4}{14}$

$\therefore \frac{a^2}{b^2} = \frac{9}{25}$

$\therefore \frac{a}{b} = \pm \frac{3}{5}$ ...(Taking square root)

$\therefore \frac{a}{b} = \frac{3}{5}$ or $\frac{a}{b} = -\frac{3}{5}$

v.

Weight (kg) x_i	Number of students f_i	c.f. (less than type)
35	6	6
36	5	6 + 5 = 11
38	8	11 + 8 = 19
40	9	19 + 9 = 28
42	2	28 + 2 = 30
44	7	30 + 7 = 37
45	4	37 + 4 = 41
	$n = \Sigma f_i = 41$	

Here, n = 41(odd number)

$\therefore$ Median $= \left(\frac{n+1}{2}\right)^{th}$ term $= \left(\frac{41+1}{2}\right)^{th}$ term $= \left(\frac{42}{2}\right)^{th}$ term $= 21^{st}$ term

Now, 21^{st} term in the cumulative frequency column corresponds to c.f. 28.

And, the weight corresponding to c.f. 28 is 40 kg.

$\therefore$ Median weight is 40 kg.

4.

i. U = {1, 3, 5, 7, 9, 11, 13, 15, 17}, A = {1, 3, 5, 7}, B = {1, 3, 9, 11}

Hence, A ∩ B = {1, 3} and

A ∪ B = {1, 3, 5, 7, 9, 11}

Now,

A' = U − A

= {1, 3, 5, 7, 9, 11, 13, 15, 17} − {1, 3, 5, 7}

= {9, 11, 13, 15, 17}

B' = U − B

= {1, 3, 5, 7, 9, 11, 13, 15, 17} − {1, 3, 9, 11}

= {5, 7, 13, 15, 17}

(A ∩ B)' = A' ∪ B'

= {9, 11, 13, 15, 17} ∪ {5, 7, 13, 15, 17}

= {5, 7, 9, 11, 13, 15, 17}

(A ∪ B)' = A' ∩ B'

= {9, 11, 13, 15, 17} ∩ {5, 7, 13, 15, 17}

= {13, 15, 17}

ii. $3x^5 - 4x^4 + 3x^3 + 2x \div x^2 - 3$

$$
\begin{array}{r|l}
 & 3x^3 - 4x^2 + 12x - 12 \\
\hline
x^2 - 3 & 3x^5 - 4x^4 + 3x^3 + 0x^2 + 2x + 0 \\
 & 3x^5 \qquad - 9x^3 \\
 & - \qquad\qquad + \\
\hline
 & -4x^4 + 12x^3 + 0x^2 + 2x + 0 \\
 & -4x^4 \qquad + 12x^2 \\
 & + \qquad\qquad - \\
\hline
 & 12x^3 - 12x^2 + 2x + 0 \\
 & 12x^3 \qquad - 36x \\
 & - \qquad\qquad + \\
\hline
 & -12x^2 + 38x + 0 \\
 & -12x^2 \qquad + 36 \\
 & + \qquad\qquad - \\
\hline
 & 38x - 36
\end{array}
$$

$\therefore 3x^5 - 4x^4 + 3x^3 + 2x = (x^2 - 3)(3x^3 - 4x^2 + 12x - 12) + (38x - 36)$

iii. Take point A on a number line such that A = −1.

∴ OA = 1

Construct AB ⊥ OA such that AB = 1 unit.

Since AB ⊥ OA, ΔOAB is a right triangle.

In ΔOAB, by Pythagoras theorem,

$(OB)^2 = (OA)^2 + (AB)^2$

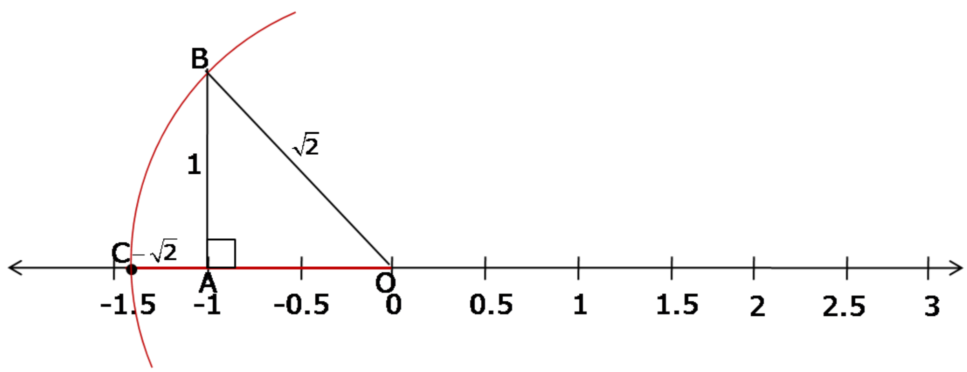
$\therefore (OB)^2 = 1^2 + 1^2 = 2$

$\therefore OB = \sqrt{2}$

Draw an arc of radius OB taking O as centre.

The arc intersects the number line, in the left direction at C.

$-\sqrt{2}$ is thus marked at point C on the number line.



5.

i. Since (x − 5) is a factor of p(x),

∴ p(5) = 0

$\therefore (5)^3 + a(5)^2 + b(5) + 30 = 0$

∴ 125 + 25a + 5b + 30 = 0

∴ 25a + 5b + 155 = 0

∴ 5a + b + 31 = 0 ….(1)

Also, p(−6) = −396

$\therefore (-6)^3 + a(-6)^2 + b(-6) + 30 = -396$

∴ −216 + 36a − 6b + 30 = −396

∴ 36a − 6b − 186 = −396

∴ 36a − 6b + 210 = 0

∴ 6a − b + 35 = 0 ….(2)

Adding equations (1) and (2), we get

11a + 66 = 0

∴ 11a = −66

∴ a = −6

Substituting a = −6 in equation (1), we get

$5(-6) + b + 31 = 0$

$\therefore -30 + b + 31 = 0$

$\therefore b + 1 = 0$

$\therefore b = -1$

Then, we have

$p(x) = x^3 - 6x^2 - x + 30$

$= x^3 - 5x^2 - x^2 + 5x - 6x + 30$

$= x^2(x - 5) - x(x - 5) - 6(x - 5)$

$= (x - 5)(x^2 - x - 6)$

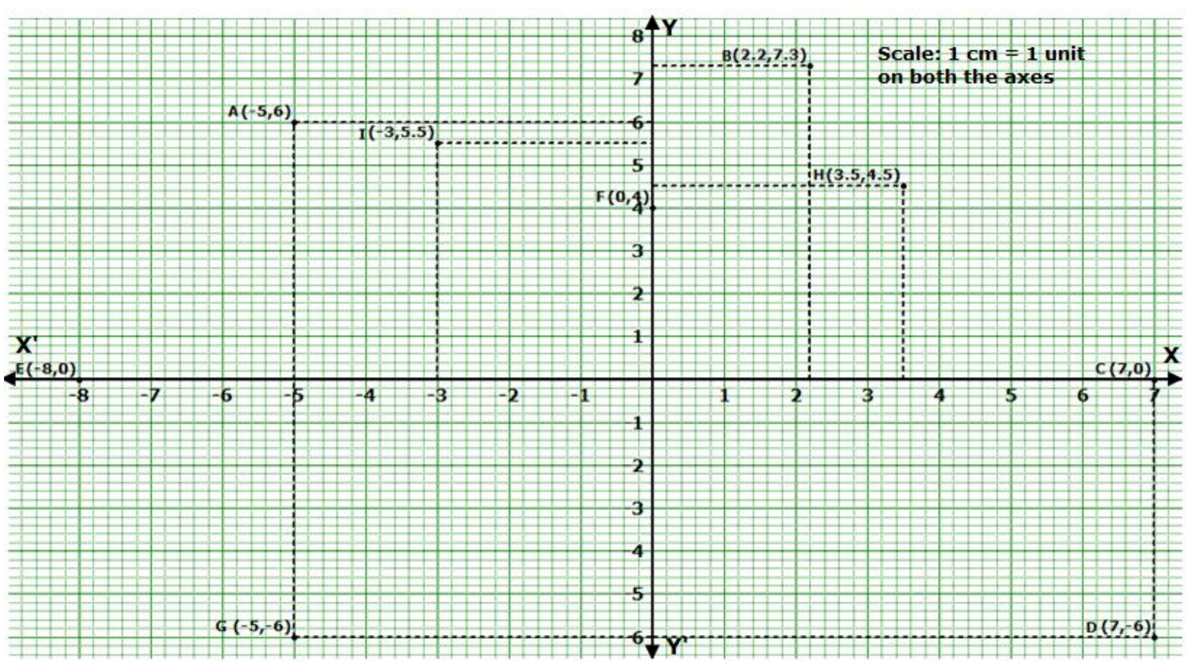
$= (x - 5)(x^2 - 3x + 2x - 6)$

$= (x - 5)[x(x - 3) + 2(x - 3)]$

$= (x - 5)(x - 3)(x + 2)$

ii. Following points are plotted on a graph paper:

1. A(-5, 6)
2. B(2.2, 7.3)
3. C(7, 0)
4. D(7, -6)
5. E(-8, 0)
6. F(0, 4)
7. G(-5, -6)
8. H(3.5, 4.5)
9. I(-3, 5.5)



iii.

Village	A	B	C	D
Female	150	240	90	140
Male	225	160	210	110
Total	375	400	300	250

Percentage of females in village A $= \frac{150 \times 100}{375} = 40\%$

$\therefore$ Percentage of males in village A $= (100 - 40)\% = 60\%$

Percentage of females in village B $= \frac{240 \times 100}{400} = 60\%$

$\therefore$ Percentage of males in village B $= (100 - 60)\% = 40\%$

Percentage of females in village C $= \frac{90 \times 100}{300} = 30\%$

$\therefore$ Percentage of males in village C $= (100 - 30)\% = 70\%$

Percentage of females in village D $= \frac{140 \times 100}{250} = 56\%$

$\therefore$ Percentage of males in village D $= (100 - 56)\% = 44\%$

The percentage bar diagram is as follows: