Maharashtra Board
Class IX Mathematics
(Geometry) Sample Paper – 1
Solution

Time: 2 hours **Total Marks: 40**

Note: (1) All questions are compulsory.
(2) Use of a calculator is not allowed.

1.

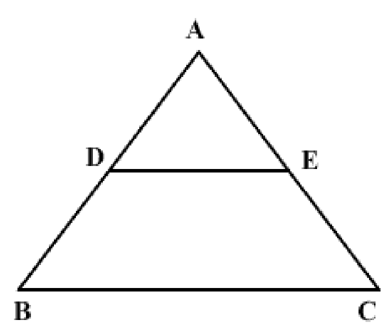
i. In the two triangles ABD and ACD,
AB = AC, BD = CD, AD = AD
∴ The two triangles ABD and ACD are congruent by the SSS criterion.

ii. In triangle PSQ and SRQ we have, $\angle PSQ = \angle RSQ$ and $\angle$PQS= $\angle$SQR and SQ= SQ
$\therefore \Delta PSQ \cong \Delta RSQ$ ... By ASA criteria
Hence, PS = SR = 5 cm

iii. In ΔABC, $\angle$B=90°, AB=12 and AC= 13
Using Pyrthagoras Theorem
$AC^2 = AB^2 + BC^2$
$\Rightarrow 13^2 = 12^2 + BC^2$
$\Rightarrow BC = 5$
$\therefore \tan A - \cot C = \frac{BC}{AB} - \frac{BC}{AB} = \frac{5}{12} - \frac{5}{12} = 0$

iv. Euclid's fifth postulate implies the existence of parallel lines.
It states: 'For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l'.

v. The mid-point theorem states that the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and half of it.
In the figure,



DE ∥ BC and DE = $\frac{1}{2}$BC (D and E are the mid-points of sides AB and AC)

vi. Let the length of the fourth side be x cm.

Perimeter = 130 cm

30 + 40 + 25 + x = 130 cm

95 + x = 130 cm

x = 130 − 95

x = 35 cm

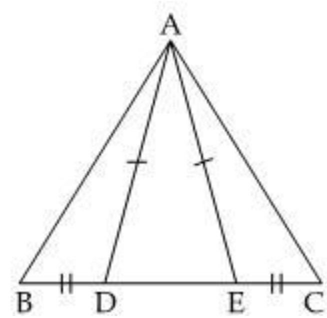
Therefore, the length of the fourth side is 35 cm.

vii. a) ABCDE is a pentagon.

b) The 5 pairs of adjacent sides are (i) AB, BC; (ii) BC, CD; (iii) CD, ED; (iv) ED, EA; (v) EA, AB.

2.

i. In $\triangle$ACD and $\triangle$ABE,



AD = AE (given)

BD = EC (given)

$\Rightarrow$ BD + DE = DE + EC

$\Rightarrow$ BE = CD

$\angle$ADC = $\angle$AEB ($\because$ AD = AE)

$\therefore \Delta ABE \cong \Delta ACD$ ... by SAS

ii. Parallel lines: Railway lines, opposite edges of a cuboid, opposite sides of a football field

Intersecting lines: Adjacent sides of a cuboid, adjacent sides of a tennis court, adjacent sides of a kite

iii. Let the angles be x, 2x and 3x.

Thus, by the angle sum property,

x + 2x + 3x = 180°

Or 6x = 180°

Or x = 30°

Hence, the angles are 30°, 60° and 90°.

iv. Here, P = 6.5 + 7 + 7.5 = 21 cm

So, s = $\frac{21}{2}$ = 10.5 cm

s – 6.5 = 4, s −7 = 3.5, s – 7.5 = 3

Area of Δ=$\sqrt{10.5\times4\times3.5\times3}$=$\sqrt{441}=21cm^2$

v. Here, Diagonal AC = 8 cm

Perpendicular DP = 4.5 cm

And Perpendicular BQ = 3.5 cm

Therefore, the sum of the perpendiculars = DP + BQ = 4.5 + 3.5 = 8 cm

Now, Area of quadrilateral = $\frac{1}{2}\times AC\times(DP+BQ)=\frac{1}{2}\times8\times8=32cm^2$

vi. Let the coordinates of end point B be (x, y).

∴By the section formula, coordinates of C are

$\left(\frac{3x+8}{3+4},\frac{3y+20}{3+4}\right)=(-1,2)$

$\frac{3x+8}{7}$=-1 and $\frac{3y+20}{7}=2$

3x = −7 − 8 = −15 and 3y = 14 − 20 = −6

⇒ x = −5 and y = −2

Hence, point B is (−5,−2).

3.

i. The mid-point theorem states that the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and half of it.

$BC = \frac{1}{2} \times QR = \frac{1}{2} \times 8 = 4\,\text{cm}$

$AB = \frac{1}{2} \times PQ = \frac{1}{2} \times 7 = 3.5\,\text{cm}$

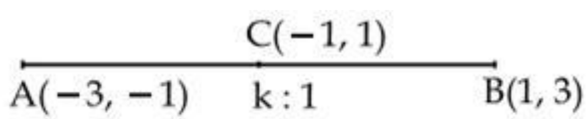
$AC = \frac{1}{2} \times PR = \frac{1}{2} \times 9 = 4.5\,\text{cm}$

The perimeter of ΔABC = AB+BC+CA

=3.5 + 4 + 4.5

= 12 cm

ii. Let C(−1,1) divide AB in the ratio of k:1.



Using the section formula, we have

$(k - 3)/(k + 1) = -1$ ... (1)

$(3k - 1)/(k + 1) = 1$ ... (2)

From (1),

$k - 3 = -k - 1$

$2k = 2$

$k = 1$

Thus, C divides AB in the ratio 1:1, i.e. C is the mid-point of AB.

$\therefore$ A, B and C are collinear.

iii. The diagonals are equal in the rectangle.

So, HF = EG

$4x + 2 = 5x - 1$

$2 + 1 = 5x - 4x$

$3 = x$

So, HF = $4 \times 3 + 2 = 14$ units

And EG = $5 \times 3 - 1 = 14$ units

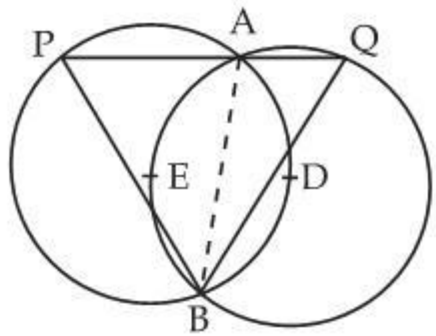
Now, it is also known that the diagonals of a rectangle bisect each other.

Therefore,

$OH = \frac{1}{2} \times HF = \frac{1}{2} \times 14$ units = 7 units

$\therefore$ $OE = \frac{1}{2} \times EG = \frac{1}{2} \times 14$ units = 7 units

iv. Join AB.



AB is the common chord.
We know that equal chords subtend equal angles, and here, two circles are congruent.
$\angle APB = \angle AQB$
In ΔBPQ,
PB = QB (lines opposite equal angles are equal)

v. In the figure, by the exterior angle property, we have
$\angle ACD = \angle A + \angle B$
$\angle CBF = \angle A + \angle C$
$\angle BAE = \angle B + \angle C$
Adding the above equations, we get
$\angle ACD + \angle CBF + \angle BAE = \angle A + \angle B + \angle A + \angle C + \angle B + \angle C$
$= 2 (\angle A + \angle B + \angle C)$
$= 2 \times 180°$ (angle sum of a triangle)
$\therefore \angle ACD + \angle CBF + \angle BAE = 360°$
The sum of the exterior angles = four right angles ($360° = 4 \times 90°$)

4.

i. Steps of construction

The following steps will be followed to construct the required triangle.

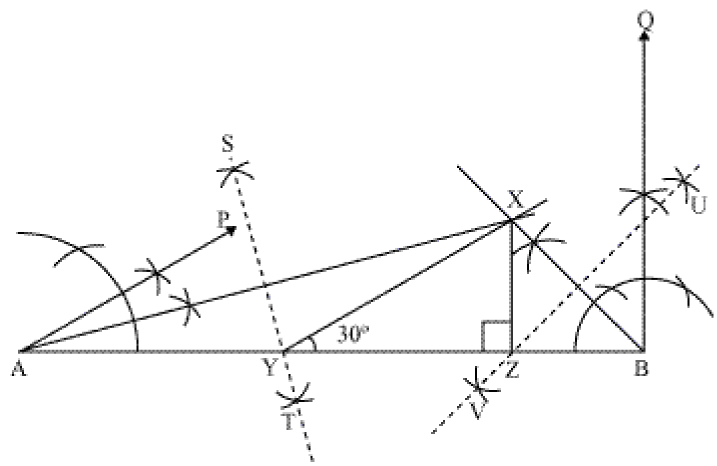
Step I: Draw a line segment AB of 11 cm (as XY + YZ + ZX = 11 cm)

Step II: Construct an angle PAB of 30° at point A and an angle QBA of 90° at point B.

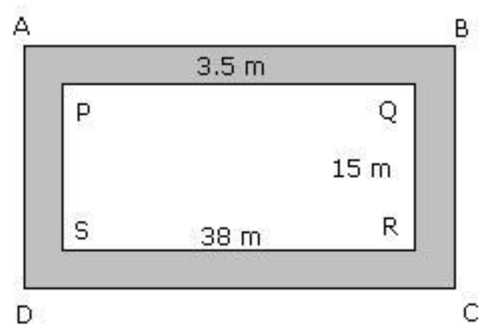
Step III: Bisect PAB and QBA. Let these bisectors intersect each other at point X.

Step IV: Draw perpendicular bisector ST of AX and UV of BX.

Step V: Let ST intersect AB at Y and UV intersect AB at Z. Join XY, XZ. DXYZ is the required triangle.



ii. The above data can be shown in a figure as follows:



Let PQRS represent the rectangular park and the shaded region represent the path 3.5 m wide.

Thus, to find the length AB and breadth BC, we have to add 3.5 m to both sides of the rectangular park whose dimensions are 38 × 15.

So, the length and breadth of the path are

Length AB = (38 + 3.5 + 3.5) m = 45 m

Breadth BC = (15 + 3.5 + 3.5) m = 22 m

So, the outer perimeter of the path = 2 × (l + b)

= 2 × (45 + 22)

= 2 × 67 = 134 m

Thus, the outer perimeter of the path is 134 m.

iii. Let us mark the four unshaded areas as I, II, III and IV.

Area of I + Area III = Area of square − Area of two semicircles

Area of I + Area III = $\left(10\times10-2\times\frac{1}{2}\times\pi\times5^2\right)cm^2$

Area of I + Area III= $(100 - 3.14 \times 25)\ cm^2$

Area of I + Area III= $(100 - 78.5)\ cm^2 = 21.5\ cm^2$

Similarly, Area of II + Area of IV = 21.5 cm^2

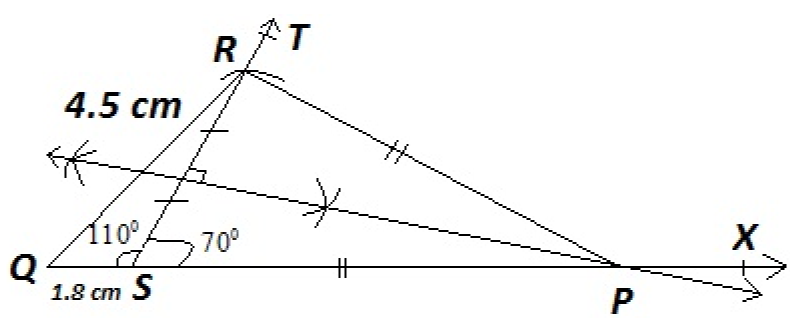
So, the area of the shaded region = Area of ABCD − Area (I + II + III + IV)

Area of the shaded region = $(100 - 2 \times 21.5)\ cm^2 = 100 - 43 = 57\ cm^2$

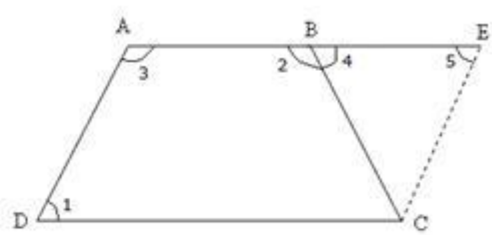
5.

i. Steps of construction:

1. Draw a ray QX. Mark a point S on ray QX such that QS = 1.8 cm.
2. Draw a ray ST such that $\angle$QST = 110°.
3. With Q as the centre and radius 4.5 cm, draw an arc intersecting ray ST at R.
4. Join points Q and R. Draw a perpendicular bisector of seg RS.
5. Let it intersect ray QX at P.
6. Join P and R. Hence, $\triangle$PQR is the required triangle.
7. Measure $\angle$P and verify it is 40°.

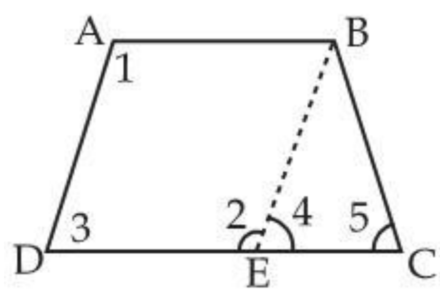


ii. Const: Draw EC || AD



Proof:
AD = CE (opposite sides of parallelogram AECD)
However, AD = BC (given)
Therefore, BC = CE
$\angle$CEB = $\angle$CBE (angle opposite to equal sides are also equal)
Consider parallel lines AD and CE. AE is the transversal line for them.
$\angle$A + $\angle$CEB = 180° (angles on the same side of transversal)
$\angle$A + $\angle$CBE = 180° (using the relation $\angle$CEB = $\angle$CBE) ... (1)
However, $\angle$ABC + $\angle$CBE = 180° (linear pair angles) ... (2)
From equations (1) and (2), we obtain
$\angle$DAB = $\angle$ABC
i.e. $\angle$A = $\angle$B

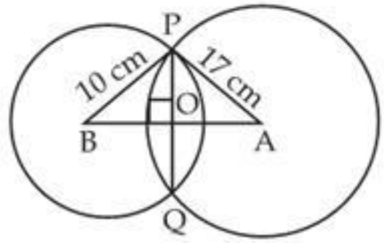
(b)



Construction: Draw BE||AD
Proof: ABED is a parallelogram.
$\angle$1 = $\angle$2 (opposite angle of a parallelogram)
$\angle$4 = $\angle$5 (angle opposite to equal sides)
$\angle$ 1 + $\angle$3 = 180° (adjacent angles of a parallelogram)
$\Rightarrow \angle 2 + \angle 3 = 180°$

Also, $\angle$2 = 180° − $\angle$4 (linear pair)
$\Rightarrow \angle 3 = 180° - \angle 2 = 180° - (180° - \angle 4)$
$\Rightarrow \angle 3 = \angle 4$
But $\angle$4 = $\angle$5
$\Rightarrow \angle 3 = \angle 5$
$\Rightarrow \angle C = \angle D$

iii. OP = OQ and $\angle$BOQ = 90°



Let OB = x cm, OA = 21 − x cm

In $\triangle$AOP,

$OA^2 + OP^2 = AP^2$

$(21 - x)^2 + OP^2 = 17^2$

$OP^2 = 289 - (21 - x)^2$

In $\triangle$BOP,

$OB^2 + OP^2 = BP^2$

$x^2 + OP^2 = 10^2$

$OP^2 = 100 - x^2$

$\therefore 289 - (21 - x)^2 = 100 - x^2$

$\Rightarrow 289 - (441 + x^2 - 42x) = 100 - x^2$

$\Rightarrow 289 - 441 - x^2 + 42x = 100 - x^2$

$\Rightarrow 42x = 252$

$\Rightarrow x = 6$

$\therefore OP^2 = 100 - 6^2 = 64$

OP = 8

PQ = 2 × OP = 2 × 8 = 16 cm