Maharashtra Board
Class IX Mathematics
(Geometry) Sample Paper – 2
Solution

Time: 2 hours **Total Marks: 40**

Note: (1) All questions are compulsory.
(2) Use of a calculator is not allowed.

1.

i. The triangles are congruent by the SAS criterion, i.e. two sides and the included angle.
The figure shows that side AC = side AD.
AC = AD,
$\angle$ CAB = $\angle$ DAB (AB bisects $\angle A$)
AB = AB (common side)
$\therefore$ Δ ABC $\cong$ Δ ABD

ii. In Δ ABC , $\angle B = 90°$,
We have
$$\frac{AB}{AC} = \sin 30° = \frac{1}{2}$$
$$\Rightarrow \frac{5}{AC} = \frac{1}{2}$$
$$\Rightarrow AC = 10cm$$
And
$$\frac{BC}{AC} = \cos 30° = \frac{\sqrt{3}}{2}$$
$$BC = 5\sqrt{3}$$

iii. For every line l and for every point P not lying on l, there exists a unique line m passing through P and parallel to l.

iv. Area of a regular polygon = $\frac{1}{2} \times n \times a \times r$
Given, n = 9, a = 6 cm, r = 8 cm
$\therefore$ Area = $\frac{1}{2} \times 9 \times 6 \times 8 = 216\ sqcm.$ = 216 sq cm

v. Perimeter of a n-sided regular polygon = $\frac{2 \times Area}{In\text{-}Radius} = \frac{2 \times 50}{10} = 10cm$

vi. PQ = PR

$PQ^2 = PR^2$

$(0-3)^2 + (2-s)^2 = (0-s)^2 + (2-5)^2$

$9 + 4 - 4s + s^2 = s^2 + 9$

$4 = 4s$

$s = 1$

vii. Circumference of a circle = $2\pi r$

Here, radius (r) = 14 cm, $\pi = \frac{22}{7}$

Thus, the circumference = $2 \times \frac{22}{7} \times 14 = 2 \times 22 \times 2 = 88$ cm

So, the circumference of the circle = 88 cm

2.

i. In $\triangle ABC$ and $\triangle ABD$,

AC = AD (given)

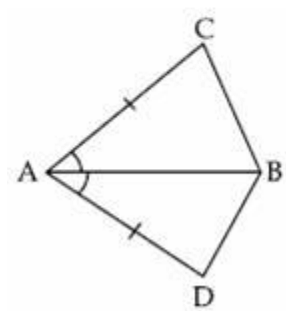
$\angle CAB = \angle DAB$ (AB is bisection of $\angle DAC$)

AB = AB (common)

So, by SAS congruency,

$\Delta ABC \cong \Delta ABD$

BC = BD (CPCT)



ii. Here, $\angle PRT = 64°$ (vertically opposite angles as PQ||TR)

Therefore, $\angle QRP = 180° - (64° + 67°)$ (linear pair)

Or $\angle QRP = 180° - 131° = 49°$

iii. Side = 10 cm

$S = \frac{10+10+10}{2} = 15cm$

Area of triangle $= \sqrt{15(15-10)(15-10)(15-10)}$

Area of triangle $= 25\sqrt{3}cm^2$

iv. Area of parallelogram = base × altitude

$\therefore AB \times BF = BC \times DE$

$\Rightarrow 20 \times 14 = BC \times 10$

$\Rightarrow BC = 28$ cm

AD = BC (opposite sides of a parallelogram)

$\therefore AD = 28$ cm

v. m∠BAC + m∠ABC = m∠ACD (exterior angle property)

∴x + 60° = 115°

∴x = 55°

vi. Side of the square = 14 cm

From the figure, we see that the radius of each semicircle is 7 cm.

Perimeter of the shaded region = AD + BC + length of arc DPC + length of arc APB

Length of arc DPC = Length of arc APB

= perimeter of a semicircle = $\pi r = \frac{22}{7} \times 7 = 22cm$

Therefore, the perimeter of the shaded region = 14 cm + 14 cm + 22 cm + 22 cm = 72 cm.

3.

i. ABCD is a parallelogram.

∴ two pairs of opposite sides are parallel and equal.

a) If AB = AD, then you can say that AB = BC = CD = AD.
A parallelogram having all sides equal is called a RHOMBUS.

b) ∠DAB = 90°. This means that all angles will be right angles, as the opposite angles of a parallelogram are equal, and the sum of the co-interior angles is 180°.
A parallelogram each of whose angles is 90° is called a RECTANGLE.

c) If AB = AD and ∠DAB = 90°, then this means that all the angles will be right angles as the opposite angles of a parallelogram are equal, the sum of the co-interior angles is 180° and all sides will be equal.
A parallelogram each of whose angles is 90° and all sides are equal is called a SQUARE.

ii. If (x,y), (1,2) and (7,0) are collinear then the area of the triangle formed by these points is zero.

Area of triangle= $\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

Here, $x_1 = x$, $y_1 = y$, $x_2 = 1$, $y_2 = 2$, $x_3 = 7$, $y_3 = 0$

Area of triangle = $\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

⇒ Area of triangle = $\frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]$

⇒ Area of triangle = $\frac{1}{2}\left[2x-y+7y-14\right]$

⇒ Area of triangle = $x+3y-7$

Now, area of ΔABC = 0

∴x + 3y −7 = 0

x + 3y = 7

iii. Let a, b, c represents the three sides of the triangle and a = 9cm.

Now, a + b + c = 22

⇒b + c = 22 − 9 = 13

⇒b + c = 13 … (1)

Also, b − c = 7 (given) … (2)

Adding (1) and (2), we get

2b = 20 or b = 10

From (1), c = 13 − 10 = 3

Hence, a = 9, b = 10, c = 3 and s = 11

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$Area = \sqrt{11(2)(1)(8)} = 4\sqrt{11} sq.cm$$

iv. Area of parallelogram ABCD = base × Altitude

= CD × AM

= 15 cm × 8 cm

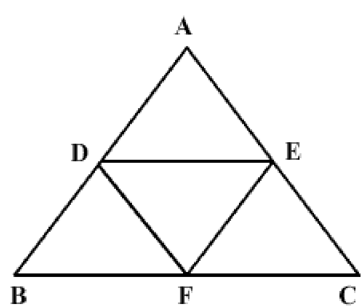
= 120 cm^2

Area of parallelogram ABCD = AD × CN

120 = AD × 10

AD = 12 cm

v. An isosceles triangle ABC



In ΔABC, AB = AC

D, E and F are the mid-points of sides AB, AC and BC, respectively.

By the mid-point theorem,

$$EF = \frac{1}{2}AB$$

$$DF = \frac{1}{2}AC$$

$$AB = AC$$

$$\therefore \frac{1}{2}AB = \frac{1}{2}AC$$

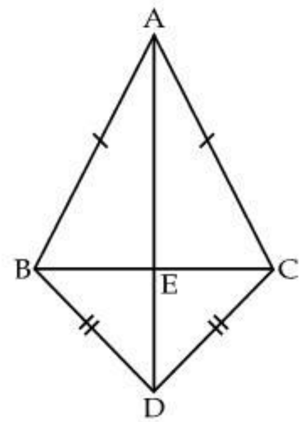
⇒EF = DF

In ΔDEF, EF = DF.

Hence, DEF is an isosceles triangle.

4.

i. In ΔABD and ΔACD,



AB = AC (given)
BD = DC (given)
AD = AD (common)
Therefore, $\Delta ABD \cong \Delta ACD$ (by the SSS rule)
So, $\angle BAD = \angle CAD$ (by cpct) ... (1)
In ΔABE and ΔACE,
AB = AC (given)
AE = AE (common)
$\angle BAD = \angle DAC$ [By (1)] ... (2)
Therefore, $\Delta ABE \cong \Delta ACE$ (by SAS)
$\therefore$ BE = CE (by cppt)
And $\angle BEA = \angle CEA$ (cpcpt)
But $\angle BEA + \angle CEA = 180°$
$\therefore \angle BEA = 90°$

ii. The perpendicular from the centre of the circle to a chord bisects the chord.
$\therefore$ P and Q are the mid-points of AB and CD.

$$AP = \frac{1}{2}AB = 3cm$$

$$CQ = \frac{1}{2}CD = 4cm$$

In right triangles OAP and OCQ, we have
$OA^2 = OP^2 + AP^2$ and $OC^2 = OQ^2 + CQ^2$
$5^2 = OP^2 + 3^2$ and $5^2 = OQ^2 + 4^2$
$OP^2 = 5^2 - 3^2$ and $\Rightarrow OQ^2 = 5^2 - 4^2$
$OP^2 = 16$ and $OQ^2 = 9$
$\therefore OP = 4$ and $OQ = 3$
$\therefore PQ = OP + OQ = 4 + 3 = 7$ cm

iii. Ratio of sides = 3:5:7

Let sides be 3x, 5x and 7x

So 3x + 5x + 7x = 300 m

15x = 300 m

x = 20 m

Sides are 60 m, 100 m and 140 m

s = $\frac{300}{2} = 150m$

$\Delta = \sqrt{S(s-a)(s-b)(s-c)}$

$\Delta = \sqrt{S(s-60)(s-100)(s-140)}$

$\Delta = \sqrt{150(90)(50)(10)}$

$\Delta = 1500\sqrt{3}m^2$

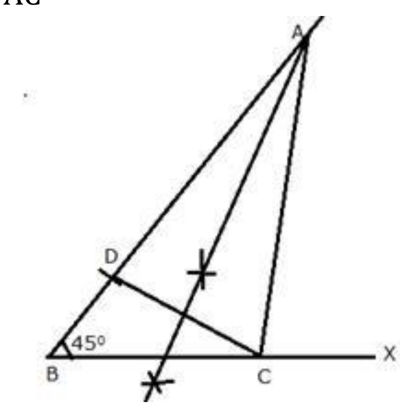
5.

i. Steps of construction:

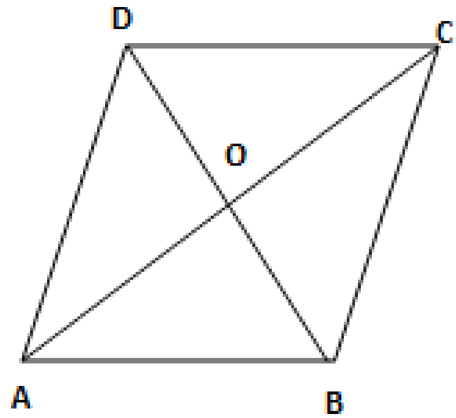
1. Draw a ray BX and cut off a line segment BC = 4.5 cm from it.
2. Construct $\angle$XBY = 45°.
3. Cut of a line segment BD = 2.5 cm from ray BY.
4. Join CD.
5. Draw a perpendicular bisector CD cutting BY at A.
6. Join AC.
7. ABC is the required triangle.
8. Justification:

Point A lies on the perpendicular bisector of DC. Therefore, AD = AC

So, BD = AB − AD = AB − AC



ii. Let ABCD be the rhombus, where AC = 10 cm and BD = 24 cm.



Let AC and BD intersect each other at O.
Now, the diagonals of the rhombus bisect each other at right angles.
Thus, we have

$$AO = \frac{1}{2} \times AC = \frac{1}{2} \times 10 = 5cm$$

$$BO = \frac{1}{2} \times BD = \frac{1}{2} \times 24 = 12cm$$

Because AOB is a right angled triangle, by Pythagoras theorem, we have

$$AB^2 = AO^2 + BO^2$$

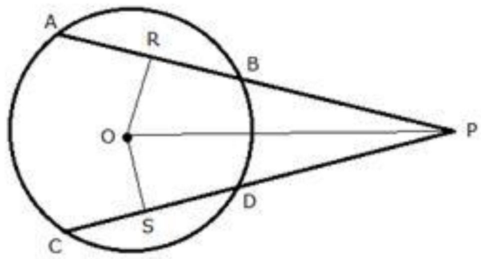
$$AB^2 = 5^2 + 12^2 = 25 + 144 = 169$$

Hence, AB = 13
Thus, the length of each side of the rhombus is 13 cm.

iii. Const: Join centre O to P. Draw $OR \perp AB$ and OS $\perp CD$

Proof: We have

AB = CD (given)



$\Rightarrow$ OR = OS (equal chords are equidistant from the centre)

In $\triangle$ ORP and $\triangle$ OSP,

OR = OS

$\angle ORP = \angle OSP$ (each 90°)

and OP = OP (common)

$\Rightarrow$ Δ ORP $\cong$ Δ OSP (RHS criterion)

$\Rightarrow$ RP = SP ... (i)

Now, AB = CD

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$

$\Rightarrow$ BR = DS ... (ii)

Subtracting (ii) from (i),

RP − BR = SP − DS

PB = PD