Maharashtra Board
Class IX Mathematics
(Geometry) Sample Paper – 3
Solution

Time: 2 hours **Total Marks: 40**

1.

i. Three sides of an equilateral triangle are congruent.

$\therefore$ Perimeter of an equilateral triangle = 3 × side

$\therefore$ 3 × side = 16.5

$\therefore \text{Side} = \dfrac{16.5}{3} = 5.5 \text{ cm}$

Thus, the length of each side of the equilateral triangle is 5.5 cm.

ii. AB = 5 cm, BC = 8 cm, AC = 10 cm.

$\therefore$ AC > BC > AB

$\therefore \angle B > \angle A > \angle C$ ....(Inequality property of a triangle)

The smallest angle is $\angle C$ and the greatest angle is $\angle B$.

iii. We know that equal chords of a circle are equidistant from the center.

Thus, OE = OF and hence, OF = 5 cm.

iv. Let P(x, y) be equidistant from the points A(2, –4) and B(–2, 6).

Hence, AP = BP

$\therefore AP^2 = BP^2$

$\therefore (x - 2)^2 + (y + 4)^2 = (x + 2)^2 + (y - 6)^2$

$\therefore x^2 - 4x + 4 + y^2 + 8y + 16 = x^2 + 4x + 4 + y^2 - 12y + 36$

$\therefore 8x - 20y + 20 = 0$

$\therefore 2x - 5y + 5 = 0$

Thus, the relation between x and y is 2x – 5y + 5 = 0.

v. $\tan^2 30 + \tan^2 45 + \tan^2 60 = \left(\dfrac{1}{\sqrt{3}}\right)^2 + (1)^2 + \left(\sqrt{3}\right)^2$

$$= \frac{1}{3} + 1 + 3$$

$$= \frac{1 + 3 + 9}{3}$$

$$= \frac{13}{3}$$

vi. Diagonal of a square = $5\sqrt{6}$ cm

$$\text{Area of a square} = \frac{(\text{Diagonal})^2}{2} = \frac{\left(5\sqrt{6}\right)^2}{2} = \frac{25 \times 6}{2} = 75 \text{ cm}^2$$

2.

i. In ΔLMP and ΔTSP,

$\angle$PLM = $\angle$PTS ....(alternate angles, since LM $\parallel$ ST)

$\angle$PML = $\angle$PST ....(alternate angles, since LM $\parallel$ ST)

$\angle$LPM = $\angle$TPS ....(vertically opposite angles)

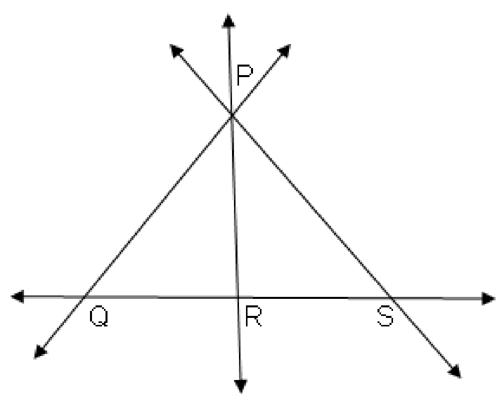
$\therefore$ ΔLMP ~ ΔTSP ....(by AAA test)

$$\therefore \frac{MP}{PS} = \frac{LP}{PT} = \frac{LM}{ST}$$

$$\therefore \frac{MP}{12} = \frac{3}{4} = \frac{9}{ST}$$

$$\therefore MP = \frac{12 \times 3}{4} = 9 \text{ cm and } ST = \frac{4 \times 9}{3} = 12 \text{ cm}$$

ii. Let Q, R and S be the three collinear points.



Four lines can be drawn through given four points such that three points are collinear.

The four lines are line PQ, line PS, line PR, and line QS.

iii. Since sum of two sides of a triangle is greater than the third side, we have

In ΔPQS, PQ + QS > PS ....(1)

Similarly, in ΔPRS, RP + RS > PS ....(2)

Adding equations (1) and (2), we get

PQ + QS + RP + RS > PS + PS

$\therefore$ PQ + (QS + RS) + RP > 2PS

$\therefore$ PQ + QR + RP > 2PS

iv. Join OQ and OR, where O is the center of the circle.

In ΔOPQ and ΔOPR,

seg OQ ≅ seg OR ....(Radii of the same circle)

seg PQ ≅ seg PR ....(Given)

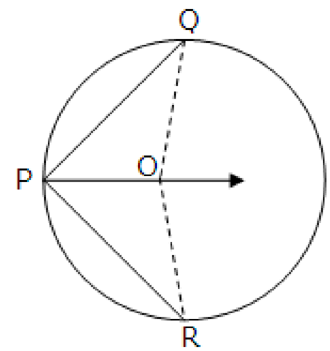
seg PO ≅ seg PO ....(Common side)

∴ ΔOPQ ≅ ΔOPR ....(SSS test)

∴ ∠OPQ ≅ ∠OPR ....(c.a.c.t.)

∴ seg PO is the bisector of ∠QPR.

i.e. the bisector of ∠RPQ passes through the centre of the circle.



v. Given ABCD is a parallelogram.

∴ ∠D = ∠B = 102° ...(opposite angles are equal)

For Δ ACD,

y = ∠DAC + ∠ADC ...(exterior angle property)

∴ y = 40° + 102° = 142°

Now, AD ∥ BC

∴ ∠DAB + ∠B = 180° ....(sum of co-interior angles is 180°)

∴ ∠DAB + 102° = 180°

∴ ∠DAB = 180° – 102° = 78°

Now, ∠DAB = ∠DAC + ∠BAC

∴ 78° = 40° + z

∴ z = 78° – 40° = 38°

vi. $\sin\theta = \frac{2\sqrt{2}}{3}$, $\cos\theta = \frac{1}{3}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\therefore \tan\theta = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}$$

$$\text{Now, } \tan\theta \times \cot\theta = 1$$

$$\therefore 2\sqrt{2} \times \cot\theta = 1$$

$$\therefore \cot\theta = \frac{1}{2\sqrt{2}}$$

3.

i. ΔUVW is a right-angled triangle.

Then, by Pythagoras theorem, we have

$UW^2 = UV^2 + VW^2$

$\therefore (8)^2 = (6)^2 + VW^2$

$\therefore VW^2 = (8)^2 - (6)^2 = 64 - 36 = 28$

$\therefore VW = 2\sqrt{7}$

$$\sin W = \frac{UV}{UW} = \frac{6}{8} = \frac{3}{4}$$

$$\cos W = \frac{VW}{UW} = \frac{2\sqrt{7}}{8} = \frac{\sqrt{7}}{4}$$

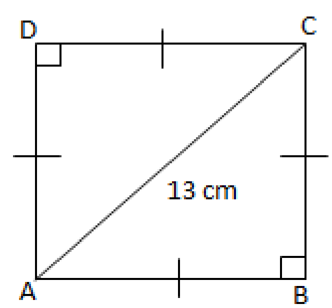
$$\tan W = \frac{UV}{VW} = \frac{6}{2\sqrt{7}} = \frac{3}{\sqrt{7}}$$

$$\cot W = \frac{VW}{UV} = \frac{2\sqrt{7}}{6} = \frac{\sqrt{7}}{3}$$

$$\sec W = \frac{UW}{VW} = \frac{8}{2\sqrt{7}} = \frac{4}{\sqrt{7}}$$

$$\operatorname{cosec} W = \frac{UW}{UV} = \frac{8}{6} = \frac{4}{3}$$

ii. Let □ABCD be a square with diagonal AC of length 13 cm.



$\therefore AB = BC = CD = AD$

In right-angled ΔABC,

$AC^2 = AB^2 + BC^2$ ….(by Pythagoras theorem)

$\therefore AC^2 = AB^2 + AB^2$

$\therefore AC^2 = 2AB^2$

$$\therefore AB^2 = \frac{AC^2}{2} = \frac{(13)^2}{2} = \frac{169}{2}$$

$$\therefore AB = \frac{13}{\sqrt{2}} = \frac{13}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{13\sqrt{2}}{2}$$

Thus, the length of each side of a square is $\frac{13\sqrt{2}}{2}$ cm.

iii. In ΔABC, $m\angle B = 30°$, $m\angle C = 25°$,

$\therefore \angle B > \angle C$

$\therefore$ side AC > side AB ....(1)

....(side opposite to the greater angle is greater)

$\angle ADB + \angle ADC = 180°$ ...(Angles in a linear pair)

$\therefore \angle ADB + 70° = 180°$

$\therefore \angle ADB = 110°$

In ΔABD, $\angle ADB = 110°$ and $\angle B = 30°$

$\therefore \angle ADB > \angle B$

$\therefore$ side AB > side AD ...(2)

From (1) and (2), we have

side AC > side AB > side AD.

iv. Let the angle be A.

Then its supplement is $(180° - A)$ and its complement is $(90° - A)$.

According to the given information, we have

$(180° - A) = 3 \times (90° - A) + 10$

$\therefore 180° - A = 270° - 3A + 10$

$\therefore 180° - A = 280° - 3A$

$\therefore 3A - A = 280° - 180°$

$\therefore 2A = 100°$

$\therefore A = \dfrac{100°}{2} = 50°$

Thus, the measure of an angle is 50°.

v. $L \equiv (1, 3)$, $M \equiv (-1, -1)$ and $N \equiv (-2, -3)$

By using distance formula, we have

$$LM = \sqrt{(-1-1)^2 + (-1-3)^2} = \sqrt{(-2)^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$\therefore LM = 2\sqrt{5}$ ....(1)

$$MN = \sqrt{(-2+1)^2 + (-3+1)^2} = \sqrt{(-1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$

$\therefore MN = \sqrt{5}$ ....(2)

$$LN = \sqrt{(-2-1)^2 + (-3-3)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

$\therefore LN = 3\sqrt{5}$ ....(3)

From (1), (2) and (3), we have LN = LM + MN

Thus, the points L, M and N are collinear.

4.

i. We have,

AR = 10 + 12 = 22 m

SQ = SR + RQ = 12 + 38 = 50 m

QD = QP + PD = 15 + 15 = 30 m

RP = RQ + QP = 38 + 15 = 53 m

AQ = AS + SR + RQ = 10 + 12 + 38 = 60

(1) Area of $\Delta DEP = \frac{1}{2} \times EP \times PD = \frac{1}{2} \times 40 \times 15 = 300\ m^2$

(2) Area of trapezium $EFRP = \frac{1}{2} \times RP \times (EP + FR) = \frac{1}{2} \times 53 \times 60 = 1590\ m^2$

(3) Area of $\Delta AFR = \frac{1}{2} \times FR \times AR = \frac{1}{2} \times 20 \times 22 = 220\ m^2$

(4) Area of $\Delta DQC = \frac{1}{2} \times DQ \times QC = \frac{1}{2} \times 30 \times 37 = 555\ m^2$

(5) Area of trapezium $BCQS = \frac{1}{2} \times QS \times (QC + SB) = \frac{1}{2} \times 50 \times 62 = 1550\ m^2$

(6) Area of $\Delta ABS = \frac{1}{2} \times AS \times BS = \frac{1}{2} \times 10 \times 25 = 125\ m^2$

$\therefore$ Area of the field = 300 + 1590 + 220 + 555 + 1550 + 125 = 4340 m^2

ii.

(a) Seg PN $\parallel$ seg LM and LN is the transversal ....(given)

$\therefore \angle MLN$ (i.e., $\angle MLA$) $\cong \angle LNP$ (i.e., $\angle ANP$) ....(alternate angles)

In ΔLAM and ΔNAP,

$\angle MLA \cong \angle ANP$ ....(proved)

$\angle LAM \cong \angle NAP$ ....(vertically opposite angles)

$\therefore \Delta LAM \sim \Delta NAP$ ....(AA test for similarity)

(b) $\Delta LAM \sim \Delta NAP$ ....(proved)

$\therefore \frac{LM}{NP} = \frac{LA}{NA}$ ....(Corresponding sides are proportional)

$\therefore \frac{8}{12} = \frac{5}{NA}$ ....(substituting the given values)

$\therefore NA = \frac{5 \times 12}{8}$

$\therefore NA = AN = 7.5$

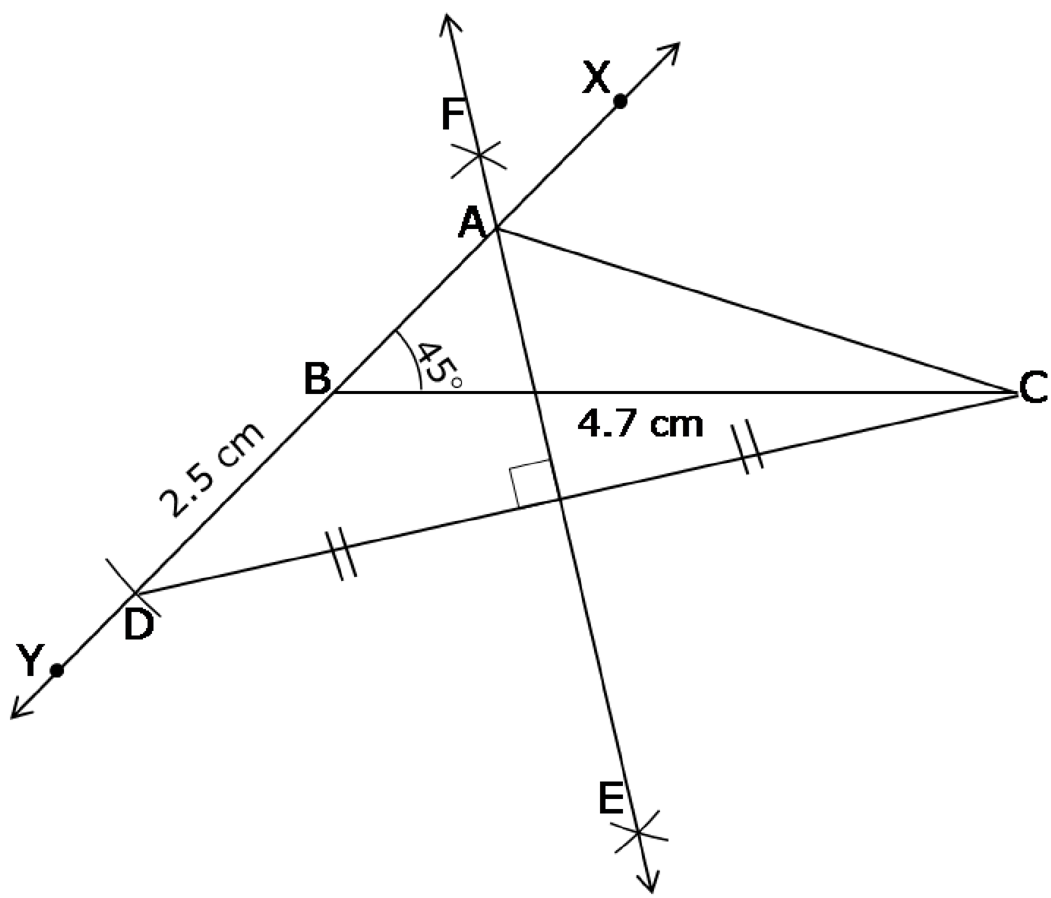
iii. Steps of construction:

1. Draw $\overline{BC}$ of length 4.7 cm.
2. Draw $\overrightarrow{BX}$, passing through point B, such that $\angle CBX = 45^\circ$
3. Draw a ray $\overrightarrow{BY}$ opposite to $\overrightarrow{BX}$
4. With centre B and radius 2.5 cm draw an arc which intersects $\overleftrightarrow{XY}$ at point D. Join D and C.
5. Keeping C and D as centres and radius greater than half of CD, draw arcs of circle above and below the line segment CD.
6. Mark the points of intersection of arcs as F and E.
7. Join $\overline{EF}$ which is the perpendicular bisector of $\overline{CD}$.
8. Mark the point of intersection of $\overline{EF}$ and $\overrightarrow{BX}$ as A. Join A and C.
9. Now by perpendicular bisector theorem, AD = AC.

   BD = 2.5 cm

   $\therefore AC - AB = AD - AB = 2.5 \text{ cm} \quad (\because AC = AD)$

Hence, $\triangle ABC$ is the required triangle.



5.

i. Given that $\angle PEB = 70°$

$\therefore \angle AEF = \angle PEB$ ....(vertically opposite angles)

$\therefore \angle AEF = 70°$

$\angle PEB + \angle BEF = 180°$

$\therefore 70 + \angle BEF = 180°$

$\therefore \angle BEF = 180° - 70°$

$\therefore \angle BEF = 110°$

$\angle PEA = \angle BEF$ (vertically opposite angles)

$\therefore \angle PEA = 110°$

$\angle BEF + \angle EFD = 180°$ ....(converse of interior angles theorem)

$\therefore 110° + \angle EFD = 180°$

$\therefore \angle EFD = 180° - 110°$

$\therefore \angle EFD = 70°$

$\angle EFD = \angle CFQ$ (vertically opposite angles)

$\therefore \angle CFQ = 70°$

$\angle EFD + \angle DFQ = 180°$ ....(angles in a linear pair)

$\therefore 70° + \angle DFQ = 180°$

$\therefore \angle DFQ = 180° - 70°$

$\therefore \angle DFQ = 110°$

$\angle DFQ = \angle CFE$ (vertically opposite angles)

$\therefore \angle CFE = 110°$

ii. Diagonal MP is the perpendicular bisector of diagonal NQ.

$\therefore$ seg NT $\cong$ seg QT ....(1)

$\angle NTM = \angle QTM = \angle PTN = \angle PTQ = 90^\circ$ .....(2)

Now, in ΔMNT and ΔMQT,

seg NT $\cong$ seg QT ...[From (1)]

$\angle MTN \cong \angle MTQ$ ...[From (2)]

seg MT $\cong$ seg MT ...(Common side)

$\therefore \Delta MNT \cong \Delta MQT$ ...(SAS test)

$\therefore$ seg MN $\cong$ seg MQ ...(c.s.c.t.)

In ΔPNT and ΔPQT,

seg NT ≅ seg QT ....[From (1)]

∠PTN ≅ ∠PTQ ....[From (2)]

seg PT ≅ seg PT ...(Common side)

∴ ΔPNT ≅ ΔPQT ...(SAS test)

∴ seg NP ≅ seg QP ...(c.s.c.t.)

In ΔNMP and ΔQMP,

seg MN ≅ seg MQ

seg NP ≅ seg QP

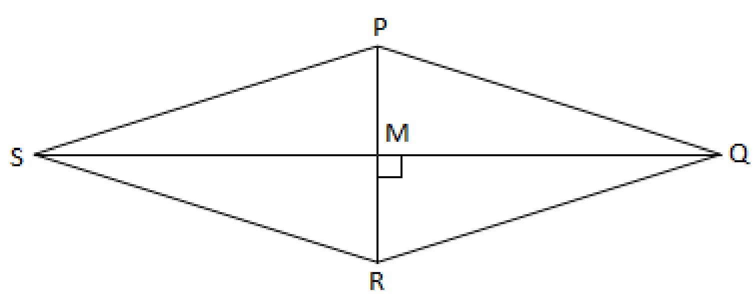
seg MP ≅ seg MP ...(Common side)

∴ ΔPNT ≅ΔPQT ...(SSS test)

iii. □PQRS is a rhombus having diagonals PR and QS of length 16 cm and 30 cm.

Let the diagonals PR and QS intersect at point M.

Now, diagonals of a rhombus are perpendicular bisectors of each other.



$\therefore$ seg PM $= $ seg RM $= \frac{1}{2}$PM $= \frac{1}{2} \times 16 = 8$ cm

seg SM $=$ seg QM $= \frac{1}{2}$SQ $= \frac{1}{2} \times 30 = 15$cm

Now, in ΔPMS,

$PS^2 = PM^2 + SM^2$ ....(by Pythagoras theorem)

$= (8)^2 + (15)^2$

$= 64 + 225$

$= 289$

∴ PS = 17

Since all the sides of a rhombus are congruent, the length of each side of a rhombus is 17 cm.