Maharashtra State Board Class IX Mathematics (Algebra) Board Paper 1 Solution

Time: 2 hours **Total Marks: 40**

1.

i. Given: $n(A \cup B) = 40$, $n(A) = 20$ and $n(A \cap B) = 12$

Now, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\therefore 40 = 20 + n(B) - 12$

$\therefore 40 = 8 + n(B)$

$\therefore n(B) = 40 - 8 = 32$

ii.
$$\begin{aligned}\sqrt{50} - \sqrt{98} + \sqrt{162} &= \sqrt{25 \times 2} - \sqrt{49 \times 2} + \sqrt{81 \times 2} \\ &= 5\sqrt{2} - 7\sqrt{2} + 9\sqrt{2} \\ &= \sqrt{2}(5 - 7 + 9) \\ &= 7\sqrt{2}\end{aligned}$$

iii.
$$\begin{aligned}8y^3 - \frac{125}{y^3} &= (2y)^3 - \left(\frac{5}{y}\right)^3 \\ &= \left(2y - \frac{5}{y}\right)\left(4y^2 + 2y \times \frac{5}{y} + \frac{25}{y^2}\right) \\ &= \left(2y - \frac{5}{y}\right)\left(4y^2 + 10 + \frac{25}{y^2}\right)\end{aligned}$$

iv. If the x-co-ordinate is negative and the y-co-ordinate is positive, then the point lies in the II^{nd} quadrant.

v. 2.5 kg = 2.5 × 1000 = 2500 gm

Ratio of 2500 gm to 8500 gm $= \frac{2500}{8500} = \frac{5}{17}$

$\therefore$ Ratio of 2.5 Kg to 8500 gm $= 5 : 17$

vi. Number of observations, n = 7

Mean$(\bar{x})$ of seven numbers $= 63$

$\therefore$ Sum of seven numbers = 63 × 7 = 441

Sum of six given numbers = 65 + 70 + 68 + 59 + 73 + 55 = 390

$\therefore$ Seventh number = Sum of seven numbers − Sum of six numbers

= 441 − 390

= 51

2.

i.

(a) B = {x|x is a capital of India}

$\therefore$ B = {Delhi}

i.e., Set B has only one element.

Hence, B is not an empty set, it is a singleton set.

(b) F = {y|y is a point of intersection of two parallel lines}

The two parallel lines do not intersect each other.

$\therefore$ There are 0 elements in set F.

Hence, F is an empty set.

ii. $\frac{1}{2\sqrt{3}+\sqrt{7}}$

$$= \frac{1}{2\sqrt{3}+\sqrt{7}} \times \frac{2\sqrt{3}-\sqrt{7}}{2\sqrt{3}-\sqrt{7}}$$

$$= \frac{2\sqrt{3}-\sqrt{7}}{\left(2\sqrt{3}\right)^2-\left(\sqrt{7}\right)^2}$$

$$= \frac{2\sqrt{3}-\sqrt{7}}{12-7}$$

$$= \frac{2\sqrt{3}-\sqrt{7}}{5}$$

iii. Let $p(x) = 2x^3 - 6x^2 + 5x + a$

Substituting x = 2 in p(x), we get

$p(2) = 2(2)^3 - 6(2)^2 + 5(2) + a$

$= 2(8) - 6(4) + 10 + a$

$= 16 - 24 + 10 + a$

$= 2 + a$

But p(2) must be 0, because (x − 2) is a factor of p(x).

$\therefore 2 + a = 0$

$\therefore a = -2$

iv. $\frac{27}{99} = 27 \div 99$

$$
\begin{array}{r|l}
 & 0.2727 \\
99 & 27.0000 \\
 & -198 \\
 & \quad 720 \\
 & \quad -693 \\
 & \qquad 270 \\
 & \qquad -198 \\
 & \qquad\quad 720 \\
 & \qquad\quad -693 \\
 & \qquad\qquad 27
\end{array}
$$

$\therefore \frac{27}{99} = 0.2727... = 0.\overline{27}$

v. $2x + y = 5$ ....(1)

$3x - y = 5$ ....(2)

Adding equations (1) and (2), we get

$$
\begin{array}{r}
2x + y = 5 \\
+\ 3x - y = 5 \\
\hline
5x \quad = 10
\end{array}
$$

$\therefore x = \frac{10}{5}$

$\therefore$ x = 2

Substituting x = 2 in equation (1), we get

2(2) + y = 5

$\therefore$ 4 + y = 5

$\therefore$ y = 5 − 4 = 1

$\therefore$ x = 2 and y = 1 is the solution of the given equations.

vi. $27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)[(3x)^2 + (y)^2 + (z)^2 - (3x)(y) - (y)(z) - (z)(3x)]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

3.

i. Mean weight of 25 students = 48 kg

$\therefore$ Total weight of 25 students = 48 × 25 = 1200 kg

Mean weight of first 13 students = 50 kg

$\therefore$ Total weight of first 13 students = 50 × 13 = 650 kg

Mean weight of last 13 students = 46 kg

$\therefore$ Total weight of last 13 students = 46 × 13 = 598 kg

Now, total weight of the first 13 students and last 13 students

= (650 + 598) kg

= 1248 kg

$\therefore$ Weight of 13^{th} student

= Total weight of the first 13 students and the last 13 students – Total weight of 25 students

= (1248 – 1200) kg

= 48 kg

Thus, the weight of 13^{th} students is 48 kg.

ii. $12x^3 - 11x^2 + 9x + 18 \div 4x + 3$

$$\begin{array}{r|l} & 3x^2 - 5x + 6 \\ \hline 4x+3 & 12x^3 - 11x^2 + 9x + 18 \\ & 12x^3 + 9x^2 \\ & - \quad\; - \\ \hline & -20x^2 + 9x + 18 \\ & -20x^2 - 15x \\ & + \quad\; + \\ \hline & 24x + 18 \\ & 24x + 18 \\ & - \quad - \\ \hline & 0 \end{array}$$

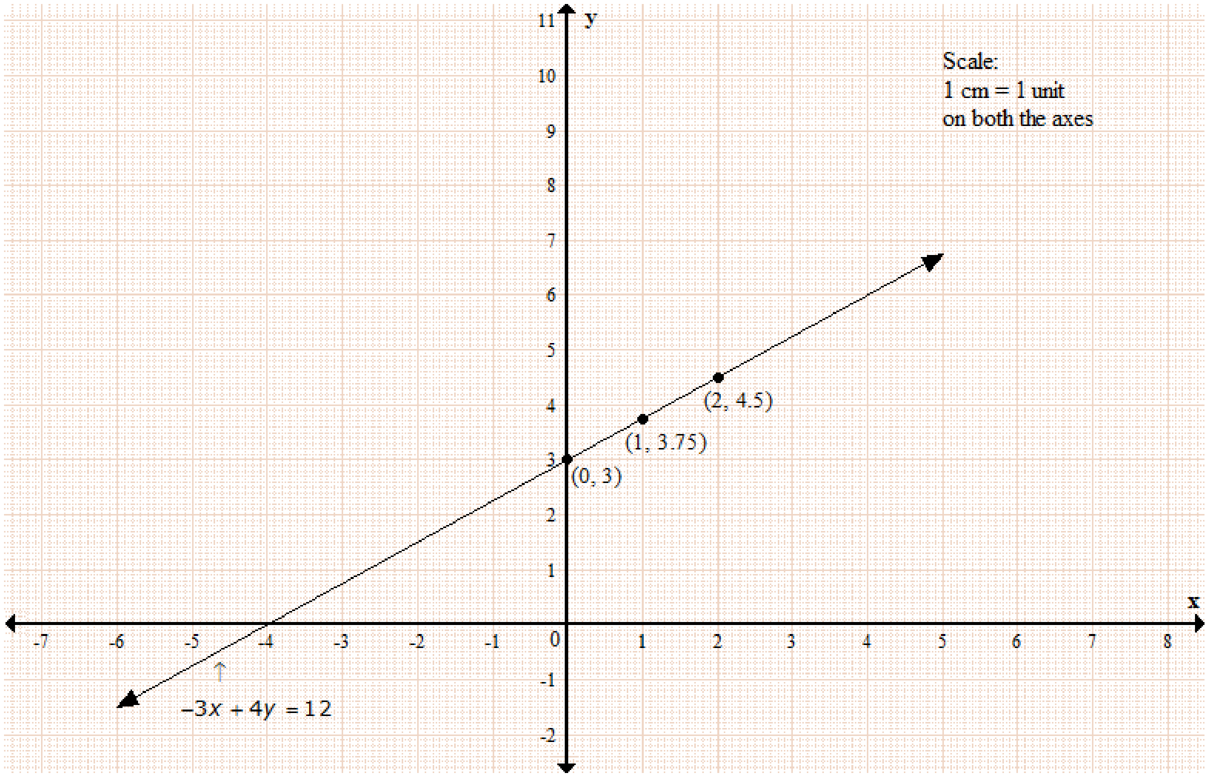
$\therefore 12x^3 - 11x^2 + 9x + 18 = (4x + 3)(3x^2 - 5x + 6) + 0$

iii. Given equation is $-3x + 4y = 12$.

Rewriting it, we get $4y = 3x + 12$

i.e. $y = 0.75x + 3$

x	0	1	2
y	3	3.75	4.5
(x, y)	(0, 3)	(1, 3.75)	(2, 4.5)



iv. $2x - y - 3 = 0$ ....(given)

$\therefore 2x - y - 3 = 0$

$\therefore y = 2x - 3$ ....(1)

$4x - y - 5 = 0$ ....(2)(given)

Substitute $y = 2x - 3$ in equation (2),

$4x - (2x - 3) - 5 = 0$

$\therefore 4x - 2x + 3 - 5 = 0$

$\therefore 2x - 2 = 0$

$\therefore 2x = 2$

$\therefore x = 1$

Substituting $x = 1$ in equation (1),

$y = 2(1) - 3$

$\therefore y = 2 - 3 \therefore y = -1$

$\therefore x = 1$ and $y = -1$

v. b is the geometric mean of a and c.

$\therefore b^2 = ac \quad ....(1)$

$$\text{L.H.S.} = a^2b^2c^2\left[\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right]$$

$$= a^2b^2c^2\left(\frac{b^3c^3 + c^3a^3 + a^3b^3}{a^3b^3c^3}\right)$$

$$= (ac)^2b^2\left(\frac{b^3c^3 + (ca)^3 + a^3b^3}{(ac)^3b^3}\right)$$

$$= \left(b^2\right)^2 b^2\left[\frac{b^3c^3 + \left(b^2\right)^3 + a^3b^3}{\left(b^2\right)^3 b^3}\right] \qquad ...[\text{from (1)}]$$

$$= b^4 b^2\left(\frac{b^3c^3 + b^6 + a^3b^3}{b^6 b^3}\right)$$

$$= b^6 b^3\left(\frac{c^3 + b^3 + a^3}{b^6b^3}\right)$$

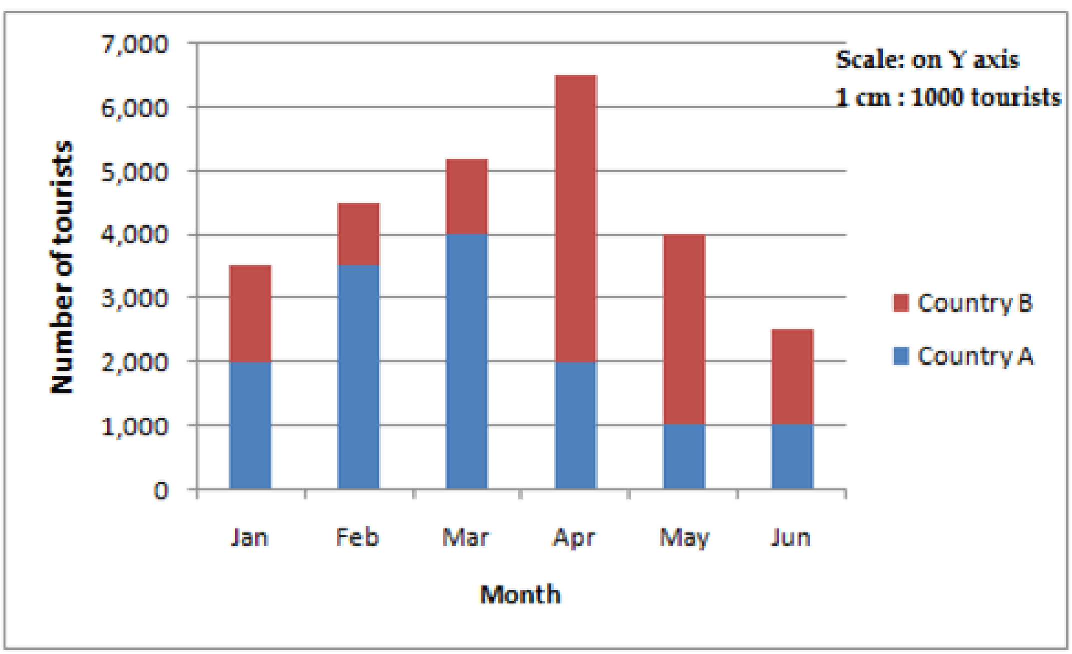
$$= a^3 + b^3 + c^3$$

$$= \text{R.H.S.}$$

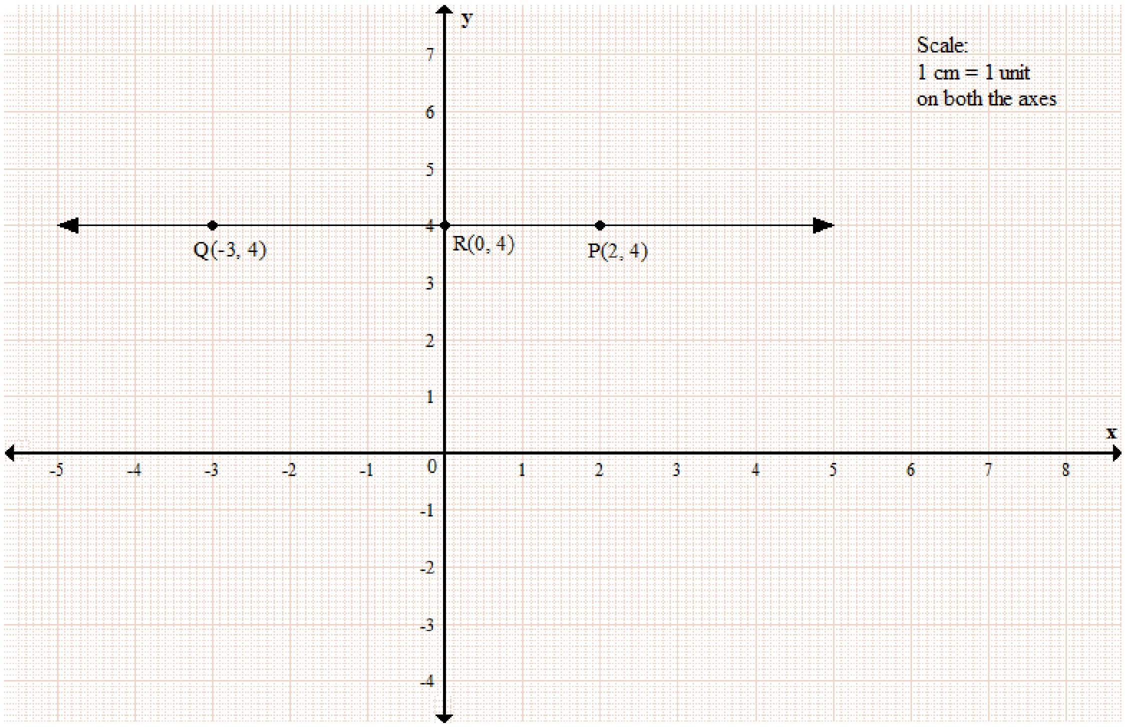
$$\therefore a^2b^2c^2\left[\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right] = a^3 + b^3 + c^3$$

4.

i. The subdivided bar diagram is as follows:



ii.



From the graph it can be clearly seen that the points P, R and Q are collinear.

Also the line passing through these points is parallel to the x-axis.

iii. A = {2, 4}

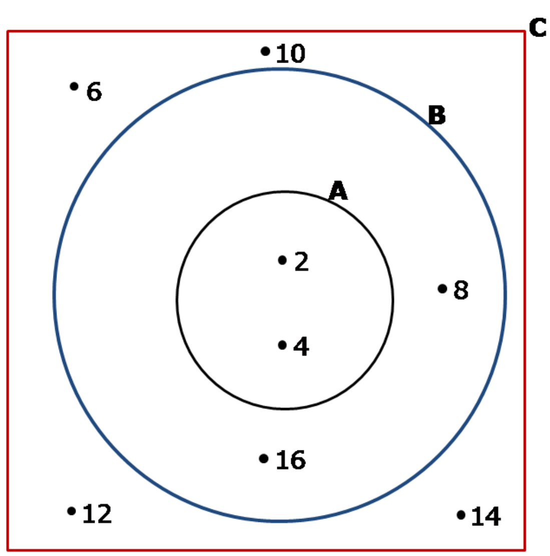
B = {x|x = 2^n, n < 5, n ∈ N}

∴ B = {2, 4, 8, 16}

C = {x|x is an even natural number ≤ 16}

∴ C = {2, 4, 6, 8, 10, 12, 14, 16}

The Venn diagram will be as given below:



5.

i. $\frac{\sqrt{7}-1}{\sqrt{7}+1}-\frac{\sqrt{7}+1}{\sqrt{7}-1}=a+b\sqrt{7}$

$\therefore \frac{\sqrt{7}-1}{\sqrt{7}+1}\times\frac{\sqrt{7}-1}{\sqrt{7}-1}-\frac{\sqrt{7}+1}{\sqrt{7}-1}\times\frac{\sqrt{7}+1}{\sqrt{7}+1}=a+b\sqrt{7}$

$\therefore \frac{\left(\sqrt{7}-1\right)^2}{\left(\sqrt{7}\right)^2-\left(1\right)^2}-\frac{\left(\sqrt{7}+1\right)^2}{\left(\sqrt{7}\right)^2-\left(1\right)^2}=a+b\sqrt{7}$

$\therefore \frac{7+1-2\sqrt{7}}{7-1}-\frac{7+1+2\sqrt{7}}{7-1}=a+b\sqrt{7}$

$\therefore \frac{8-2\sqrt{7}}{6}-\frac{8+2\sqrt{7}}{6}=a+b\sqrt{7}$

$\therefore \frac{8-2\sqrt{7}-8-2\sqrt{7}}{6}=a+b\sqrt{7}$

$\therefore \frac{-4\sqrt{7}}{6}=a+b\sqrt{7}$

$\therefore -\frac{2}{3}\sqrt{7}=a+b\sqrt{7}$

$\therefore 0+\left(-\frac{2}{3}\right)\sqrt{7}=a+b\sqrt{7}$

Equating the values of both the sides, we get $a=0$ and $b=-\frac{2}{3}$.

ii.

(a) $x^2 - 4x - 5 = x^2 - 5x + x - 5$

$= x(x - 5) + 1(x - 5)$

$= (x + 1)(x - 5)$

The polynomials have zeroes when p(x) = 0.

$\therefore$ x + 1 = 0 or x − 5 = 0

$\therefore$ x = −1 or x = 5

$\therefore$ The zeroes of the polynomial $x^2 - 4x - 5$ are −1 and 5.

Now, sum of zeroes $=-1+5=4=-\frac{(-4)}{1}=-\frac{b}{a}$

and product of zeroes $=(-1)\times(5)=-5=\frac{-5}{1}=\frac{c}{a}$

(b) Let the quadratic polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

We know $\alpha+\beta=-11=\frac{-b}{1}$ and $\alpha\beta=10=\frac{c}{a}$

If a = 1, then b = 11 and c = 10.

Thus, the required quadratic equation is $x^2 + 11x + 10$.

iii. a, b, c are in continued proportion.

Let $\frac{a}{b} = \frac{b}{c} = k$

$\therefore a = bk$ and $b = ck$

$\therefore a = ck.k = ck^2$

$$\text{L.H.S.} = \frac{(a + b)^2}{(b + c)^2}$$

$$= \frac{(ck^2 + ck)^2}{(ck + c)^2}$$

$$= \frac{[ck(k + 1)]^2}{[c(k + 1)]^2}$$

$$= \frac{c^2k^2}{c^2}$$

$$= k^2$$

$$\text{R.H.S.} = \frac{a^2 + b^2}{b^2 + c^2}$$

$$= \frac{(ck^2)^2 + (ck)^2}{(ck)^2 + c^2}$$

$$= \frac{c^2k^4 + c^2k^2}{c^2k^2 + c^2}$$

$$= \frac{c^2k^2(k^2 + 1)}{c^2(k^2 + 1)}$$

$$= k^2$$

$\therefore$ L.H.S. = R.H.S.

$$\therefore \frac{(a + b)^2}{(b + c)^2} = \frac{a^2 + b^2}{b^2 + c^2}$$