Maharashtra State Board Class IX Mathematics (Algebra) Board Paper 2 Solution

Time: 2 hours **Total Marks: 40**

1.

i. $C = \{p | p \in I, p^3 = -8\}$

$\therefore C = \{-2\}$

$\therefore C = \{p | p \in I, p^3 = -8\}$ is a singleton set.

ii. $27\sqrt[3]{18} \div 3\sqrt[3]{9}$

$= \frac{27\sqrt[3]{18}}{3\sqrt[3]{9}}$

$= \frac{9\sqrt[3]{9 \times 2}}{\sqrt[3]{9}}$

$= 9\sqrt[3]{\frac{9 \times 2}{9}}$

$= 9\sqrt[3]{2}$

iii. $p(x) = 2x^3 - 3x^2 + 4x - 5$

Divisor = $x - 1$

$\therefore$ Put $x = 1$ in $p(x)$

Then, by the Remainder Theorem

Remainder = $p(1) = 2(1)^3 - 3(1)^2 + 4(1) - 5$

$= 2(1) - 3(1) + 4 - 5$

$= 2 - 3 + 4 - 5$

$= -2$

iv. Arranging the given data in a tabular form, we have

Observations	14	17	18	22	23	25	28
Frequency	4	1	3	1	1	1	2

Here, the maximum frequency is 4 and corresponds to observation 14.

$\therefore$ The mode is 14.

v. $A = \{2, 3, 4\}$ and $B = \{3, 5\}$

$\therefore A \cap B = \{3\}$ and $A \cup B = \{2, 3, 4, 5\}$

vi. Given data: 5, 3, 11, 0, 7, 11, 4, 3, 8

Arranging the data in ascending order, we have

0, 3, 3, 4, 5, 7, 8, 11, 11

Here, n = 9 (odd)

$\therefore$ Median $= \left(\frac{n+1}{2}\right)^{th}$ term $= \left(\frac{9+1}{2}\right)^{th}$ term $= \left(\frac{10}{2}\right)^{th} = 5^{th}$ term

$\therefore$ Median $= 5$

2.

i. Square root of $8 + 2\sqrt{15}$

$= \sqrt{8 + 2\sqrt{15}}$

$= \sqrt{(5+3) + 2\sqrt{5 \times 3}}$

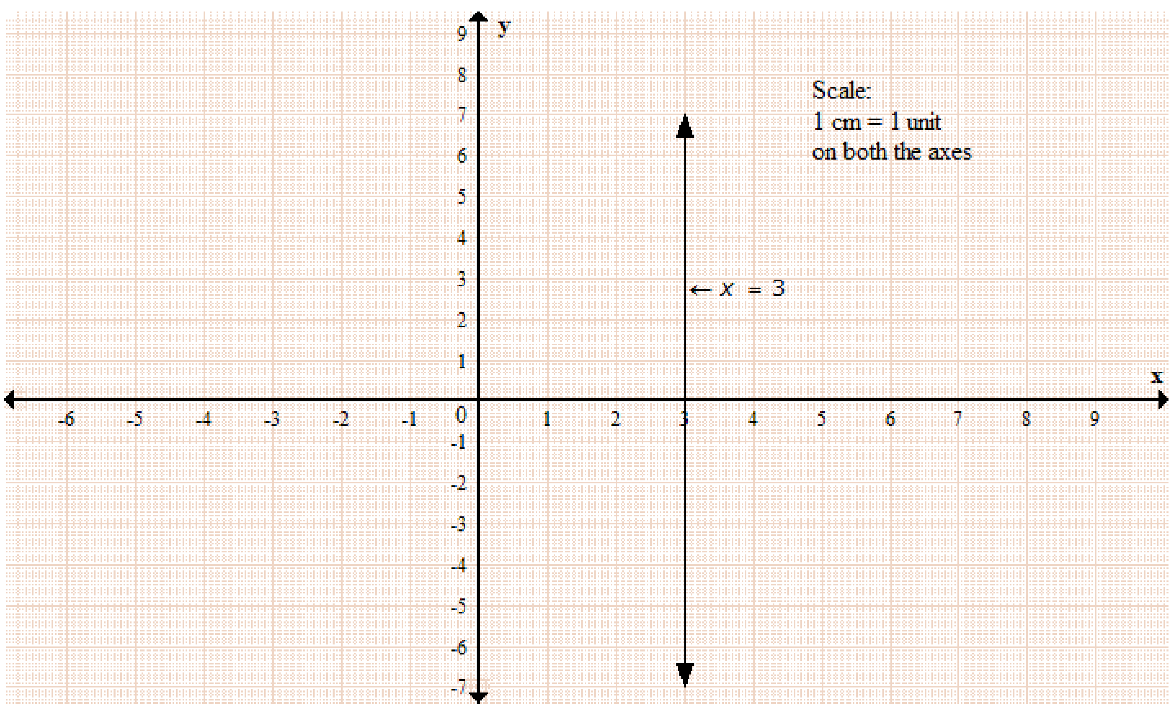
$= \sqrt{\left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2 + 2\sqrt{5} \times \sqrt{3}}$

$= \sqrt{\left(\sqrt{5} + \sqrt{3}\right)^2}$ ....$\left[\because a^2 + b^2 + 2ab = (a+b)^2\right]$

$= \sqrt{5} + \sqrt{3}$

$\therefore$ Square root of $8 + 2\sqrt{15}$ is $\sqrt{5} + \sqrt{3}$.

ii. Graph of x = 3



iii.

(a) (−3, 2)

(−, +) are the signs of the coordinates of points in the Quadrant II.

∴ The point (−3, 2) lies in the Quadrant II.

(b) (−x, −y)

(−, −) are the signs of the coordinates of points in the Quadrant III.

∴ The point (−x, −y) lies in the Quadrant III.

iv. Mode is the observation which appears most often.

∴ Mode = y − 1

Median is the middle most value.

∴ Median = 4^{th} observation = y + 4

Given, Mode + Median = 15

∴ y − 1 + y + 4 = 15

∴ 2y + 3 = 15

∴ 2y = 12

∴ y = 6

v. $\frac{a-1}{b-1} - \frac{a+1}{b+1}$

$= \frac{(a-1)(b+1) - (a+1)(b-1)}{(b-1)(b+1)}$

$= \frac{ab + a - b - 1 - ab + a - b + 1}{(b-1)(b+1)}$

$= \frac{2a - 2b}{(b-1)(b+1)}$

$= \frac{2(a-b)}{(b-1)(b+1)}$

Here we have $a > b$ and $b \neq \pm 1$

$\therefore \frac{2(a-b)}{(b-1)(b+1)}$ is positive

$\therefore \frac{a-1}{b-1} > \frac{a+1}{b+1}$

vi. The ungrouped frequency distribution table is as follows:

Number of saplings	Tally marks	Frequency (f)
2	II	2
3	~~IIII~~	5
4	~~IIII~~ III	8
5	~~IIII~~ II	7
6	IIII	4
7	III	3
8	I	1
	Total	$N = \sum f_i = 30$

3.

i. Given equation: $6a^4 + 11a^2b^2 - 10b^4$

Let $a^2 = m$ and $b^2 = n$

Then $a^4 = m^2$, $b^4 = n^2$ and $a^2b^2 = mn$

$\therefore 6a^4 + 11a^2b^2 - 10b^4$

$= 6m^2 + 11mn - 10n^2$

$= 6m^2 + 15mn - 4mn - 10n^2$

$= 3m(2m + 5n) - 2n(2m + 5n)$

$= (2m + 5n)(3m - 2n)$

Re-substituting the values of m and n, we get

$6a^4 + 11a^2b^2 - 10b^4 = (2a^2 + 5b^2)(3a^2 - 2b^2)$

ii. $\sqrt{294} - 3\sqrt{\frac{1}{6}} - 5\sqrt{6} + \sqrt{252}$

$$= \sqrt{6 \times 49} - 3\frac{1}{\sqrt{6}} - 5\sqrt{6} + \sqrt{7 \times 36}$$

$$= 7\sqrt{6} - \frac{3}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} - 5\sqrt{6} + 6\sqrt{7}$$

$$= 7\sqrt{6} - \frac{3\sqrt{6}}{6} - 5\sqrt{6} + 6\sqrt{7}$$

$$= 7\sqrt{6} - \frac{\sqrt{6}}{2} - 5\sqrt{6} + 6\sqrt{7}$$

$$= \sqrt{6}\left(7 - \frac{1}{2} - 5\right) + 6\sqrt{7}$$

$$= \sqrt{6}\left(\frac{14 - 1 - 10}{2}\right) + 6\sqrt{7}$$

$$= \frac{3}{2}\sqrt{6} + 6\sqrt{7}$$

iii.

(a) The y-co-ordinate of point B is zero.

(b) F ≡ (−1, −5) and A ≡ (3, 2).

(c) The line AH is parallel to the Y-axis.

(d) The x-co-ordinate of point P and Q is same.

(e) The y-co-ordinate of point E is 3.

(f) The x-co-ordinate of point M on line AH is 3.

iv. $y = \dfrac{(p+1)^{\frac{1}{3}} + (p-1)^{\frac{1}{3}}}{(p+1)^{\frac{1}{3}} - (p-1)^{\frac{1}{3}}}$

By componendo-dividendo,

$$\frac{y+1}{y-1} = \frac{2(p+1)^{\frac{1}{3}}}{2(p-1)^{\frac{1}{3}}}$$

Cubing both the sides,

$$\frac{(y+1)^3}{(y-1)^3} = \frac{p+1}{p-1}$$

$$\therefore \frac{y^3+3y^2+3y+1}{y^3-3y^2+3y-1} = \frac{p+1}{p-1}$$

By componendo-dividendo

$$\frac{2y^3+6y}{6y^2+2} = \frac{2p}{2}$$

$$\therefore \frac{2(y^3+3y)}{2(3y^2+1)} = p$$

$$\therefore y^3+3y = p(3y^2+1)$$

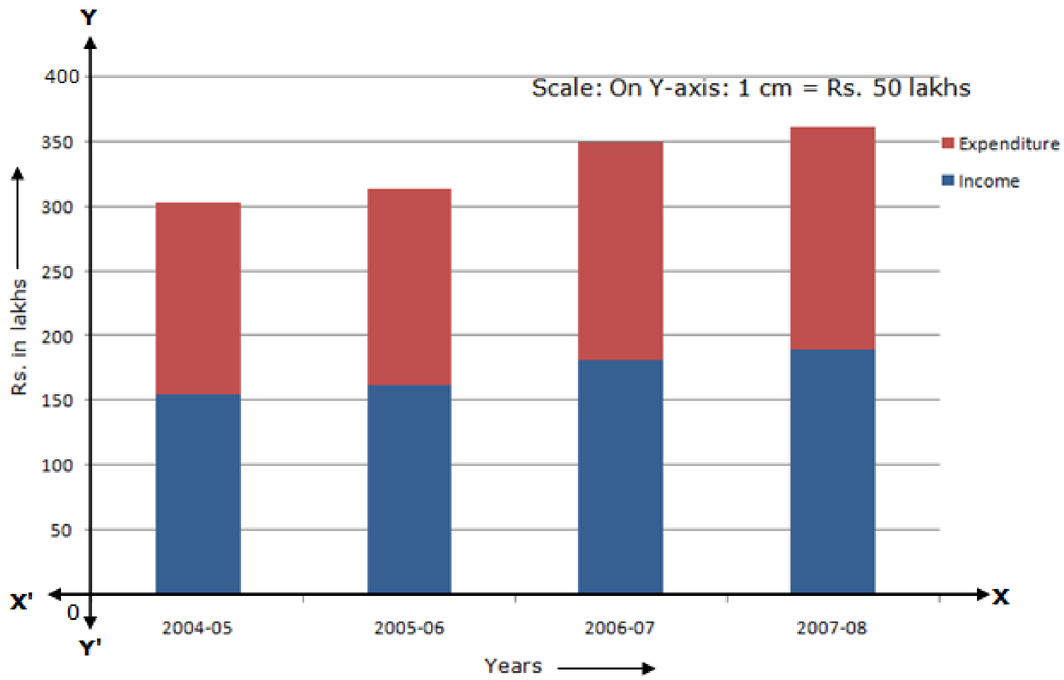
$$\therefore y^3+3y = 3py^2+p$$

$$\therefore y^3-3py^2+3y-p = 0$$

v.

Year	Income	Expenditure	Total
2004-05	155	148	303
2005-06	162	152	314
2006-07	181	169	350
2007-08	190	172	362

The subdivided bar diagram is as follows:



4.

i. Let a, b, c, d and e be the five numbers in continued proportion.

Second term = b = 6 and Fourth term = d = 54

Then, a, 6, c, 54 and e are in continued proportion.

$\therefore c^2 = 6 \times 54$

$\therefore c^2 = 6 \times 6 \times 3 \times 3$

$\therefore c^2 = 6^2 \times 3^2$

$\therefore c = 6 \times 3$

$\therefore c = 18$

Further, a, 6, c are in continued proportion.

$\therefore 6^2 = a \times c$

$\therefore 36 = a \times 18$

$\therefore a = \frac{36}{18} = 2$

Now, c, 54 and e are in continued proportion.

$\therefore 54^2 = c \times e$

$\therefore 54^2 = 18 \times e$

$\therefore e = \dfrac{54 \times 54}{18} = 3 \times 54 = 162$

Hence, 2, 6, 18, 54 and 162 are the required numbers.

ii. $\dfrac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}} = \dfrac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}} \times \dfrac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{\sqrt{6}+\sqrt{5}+\sqrt{11}}$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{\left(\sqrt{6}+\sqrt{5}\right)^2 - \left(\sqrt{11}\right)^2}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{6+5+2\sqrt{6}\sqrt{5}-11}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{2\sqrt{30}}$$

$$= \frac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{2\sqrt{30}} \times \frac{\sqrt{30}}{\sqrt{30}}$$

$$= \frac{\sqrt{30}\left(\sqrt{6}+\sqrt{5}+\sqrt{11}\right)}{2\left(\sqrt{30}\right)^2}$$

$$= \frac{\sqrt{30}\left(\sqrt{6}+\sqrt{5}+\sqrt{11}\right)}{60}$$

iii. Sum of zeroes $= -\dfrac{b}{a} = \dfrac{4+\sqrt{2}}{2}$......(1)

Product of zeroes $= \dfrac{c}{a} = \dfrac{4-\sqrt{2}}{2}$.........(2)

From (1) and (2) we can say that,

$a = 2,\ b = -\left(4+\sqrt{2}\right), c = 4-\sqrt{2}$

$\therefore$ Required quadratic polynomial is,

$ax^2 + bx + c = 0$

$2x^2 - \left(4+\sqrt{2}\right)x + \left(4-\sqrt{2}\right) = 0$

5.

i. Let the first number be x and the second number be y.

Their ratio is 5 : 6. ....(given)

$\therefore \dfrac{x}{y} = \dfrac{5}{6}$

$\therefore 6x = 5y$ ....(1)

If 8 is subtracted from each number, then the ratio becomes 4 : 5.

$\therefore \dfrac{x-8}{y-8} = \dfrac{4}{5}$

$\therefore 5(x-8) = 4(y-8)$

$\therefore 5x - 40 = 4y - 32$

$\therefore 5x - 4y = -32 + 40$

$\therefore 5x - 4y = 8$ ....(2)

$\therefore 4y = 5x - 8$

$\therefore y = \dfrac{5x-8}{4}$ ....(3)

Substituting this value of y in equation (1), we get,

$6x = 5\left(\dfrac{5x-8}{4}\right)$

$\therefore 6x \times 4 = 25x - 40$

$\therefore 24x = 25x - 40$

$\therefore 25x - 24x = 40$

$\therefore x = 40$

Substituting this value of x in equation (3), we get

$y = \dfrac{5(40)-8}{4}$

$= \dfrac{200-8}{4}$

$= \dfrac{192}{4}$

$= 48$

$\therefore$ The first number is 40 and the second number is 48.

ii.

(a) p(x) = (x − 2)(x − 9)

(x − 2)(x − 9) = 0

$\therefore$ x − 2 = 0 or x − 9 = 0

$\therefore$ x = 2 or x = 9

Hence, 2 and 9 are the zeroes of the given polynomial.

(b) $p(x) = x^3 + ax^2 + 4x - 5$

Divisor is $x + 1$

Put $x = -1$ in $p(x)$

Then, by Remainder Theorem, we have

Remainder $= p(-1) = (-1)^3 + a(-1)^2 + 4(-1) - 5$

$= -1 + a - 4 - 5$

$= a - 10$

But, remainder = 14

$\therefore$ $a - 10 = 14$

$\therefore$ $a = 14 + 10 = 24$

iii. Let B denote the set of students who play Basket ball and V denote the set of students who play Volley ball.

$\therefore$ $n(B) = 400$ and $n(V) = 170$

Then, $n(B \cap V)$ denote the set of students who play both the games.

$\therefore$ $n(B \cap V) = 100$

(a) Now, number of students who play Basket ball only

= Number of students who play Basket ball – Number of students who play both the games

$= n(B) - n(B \cap V)$

$= 400 - 100$

$= 300$

(b) Number of students who play Volley ball only

= Number of students who play Volley ball – Number of students who play both the games

$= n(V) - n(B \cap V)$

$= 170 - 100$

$= 70$

(c) Total number of students in the school

= Number of students who play Basket ball + Number of students who play Volley ball – Number of students who play both the games

$= n(B) + n(V) - n(B \cap V)$

$= 400 + 170 - 100$

$= 470$