Maharashtra State Board
Class IX Mathematics – Geometry
Board Paper 1
Solution

Time: 2 hours **Total Marks: 40**

1.

i. Let the measure of each interior opposite angle be x.

Since, Sum of two interior opposite angles = Measure of exterior angle

$\therefore x + x = 80°$

$\therefore 2x = 80°$

$\therefore x = 40°$

Hence, the measure of each interior opposite angle is 40°.

ii. $\angle ACB = 180° - 105°$

$\therefore \angle ACB = 75°$

$\angle ABC = \angle ACB = 75°$ ....(angles opposite to equal sides of a triangle)

Now, in ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$

$\therefore 75° + 75° + \angle BAC = 180°$

$\therefore 150° + \angle BAC = 180°$

$\therefore \angle BAC = 180° - 150° = 30°$

iii. Radius of the circle is 6.7 cm.

d(P, R) = 5.7 cm

$\therefore$ 5.7 cm < 6.7 cm

$\therefore$ The distance between P and R is less than the radius of the circle.

$\therefore$ Point R lies in the interior of the circle.

iv. Let $A \equiv (x_1, y_1)$ and $B \equiv (x_2, y_2)$.

Here, $x_1 = 2$, $y_1 = 5$ and $x_2 = -6$, $y_2 = 8$

By distance formula, we have

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-6-2)^2 + (8-5)^2} = \sqrt{(-8)^2 + (3)^2} = \sqrt{64+9}$$

$\therefore AB = \sqrt{73}$

Thus, the distance between the points A and B is $\sqrt{73}$ units.

v. $\dfrac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \times \operatorname{cosec} 31^\circ = \dfrac{\cos 80^\circ}{\cos(90^\circ - 10^\circ)} + \sin(90^\circ - 59^\circ) \times \dfrac{1}{\sin 31^\circ}$

$$= \frac{\cos 80^\circ}{\cos 80^\circ} + \sin 31^\circ \times \frac{1}{\sin 31^\circ}$$
$$= 1 + 1$$
$$= 2$$

vi. Area of the square-shaped field = $(\text{side})^2 = (300)^2 = 90000 \text{ m}^2$

Cost of leveling the field per square metre = Rs. 1.25

$\therefore$ Cost of leveling = Rate × Area

= Rs. 1.25×90000

= Rs. 1, 12, 500

Thus, the cost of leveling the field is Rs. 1, 12, 500.

2.

i. Seg QS is the angle bisector of $\angle PQR$ ....(Given)

$\therefore \angle PQS \cong \angle RQS$ ...(1)

Seg QS is the bisector of $\angle PSR$ ....(Given)

$\therefore \angle PSQ \cong \angle RSQ$ ...(2)

In ΔPQS and ΔRQS,

$\angle PQS \cong \angle RQS$ ...[From (1)]

seg QS ≅ seg QS ...(Common side)

$\angle PSQ \cong \angle RSQ$ .....[From (2)]

$\therefore \Delta PQS \cong \Delta RSQ$ ....(ASA test)

$\therefore \angle P \cong \angle R$ ....(c.a.c.t.)

ii. In ΔNYX, $m\angle NYX = 90^\circ$ ...(Given)

$\therefore m\angle N + m\angle X = 90^\circ$ ....(Acute angles of a right angled triangle)

$\therefore m\angle N + 45^\circ = 90^\circ$ ...(Given : $m\angle X = 45^\circ$)

$\therefore m\angle N = 45^\circ$

Now, in ΔNMZ,

$m\angle N + m\angle NMZ + m\angle Z = 180^\circ$ ...(Angle Sum property of a triangle)

$\therefore 45^\circ + 110^\circ + x = 180^\circ$

$\therefore 155^\circ + x = 180^\circ$

$\therefore x = 180^\circ - 155^\circ$

$\therefore x = 25^\circ$

iii. Given, angle = s

Then, its supplementary angle = $(180^\circ - s)$

According to given information, we have

$s = 4 \times (180 - s) + 20$

$\therefore s = 720 - 4s + 20$

$\therefore 5s = 740$

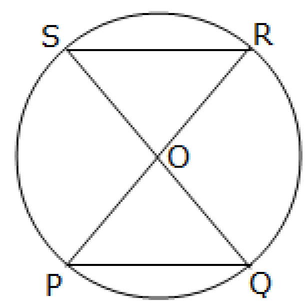
$\therefore s = 148$

Thus, the measure of angle 's' = 148°

iv. Given: In a circle with centre O, seg PQ and seg RS are two chords.

$\angle POQ \cong \angle ROS$

To prove: chord PQ ≅ chord RS



Proof:

In ΔPOQ and ΔROS,

seg OP ≅ seg OR ….(radii of same circle)

$\angle POQ \cong \angle ROS$ ….(given)

seg OQ ≅ seg OS ….(radii of same circle)

∴ ΔPOQ ≅ ΔROS ….(SAS test)

∴ seg PQ ≅ seg RS ….(c.s.c.t.)

∴ chord PQ ≅ chord RS

v. In □PQRS

$\angle P + \angle Q + \angle R + \angle S = 360^\circ$ ….(angle sum property of a quadrilateral)

$\therefore \angle P + \angle Q + 100^\circ + 30^\circ = 360^\circ$

$\therefore \angle P + \angle Q + 130^\circ = 360^\circ$

$\therefore \angle P + \angle Q = 230^\circ$

$\therefore \frac{1}{2}\angle P + \frac{1}{2}\angle Q = \frac{1}{2} \times 230^\circ$

i.e. $\angle APQ + \angle AQP = 115^\circ$ ….(1)

Now, in ΔAPQ,

$\angle APQ + \angle AQP + \angle PAQ = 180^\circ$

$\therefore 115^\circ + \angle PAQ = 180^\circ$

$\therefore \angle PAQ = 180^\circ - 115^\circ = 65^\circ$

$\therefore \angle PAQ = 65^\circ$

vi. $\text{L.H.S.} = \sin A = \sin 30^\circ = \frac{1}{2}$

$\text{R.H.S.} = \sqrt{\frac{1-\cos 2A}{2}} = \sqrt{\frac{1-\cos 60^\circ}{2}}$ ...(since $\angle A = 30^\circ$, $2\angle A = 60^\circ$)

$= \sqrt{\frac{1-\frac{1}{2}}{2}}$

$= \sqrt{\frac{\frac{1}{2}}{2}}$

$= \sqrt{\frac{1}{4}}$

$= \frac{1}{2}$

$\therefore$ L.H.S. = R.H.S. (Proved)

3.

i. l(PL) + l (LN) = l(PN) (P-L-N)

$\therefore$ l(PL) + 5 = 11

$\therefore$ l(PL) = 6 units

l(MN) + l(NR) = l(MR) (M-N-R)

$\therefore$ 7 + l(NR) = 13

$\therefore$ l(NR) = 6 units

l(LM) + l(MQ) = l(LQ) (L-M-Q)

$\therefore$ 6 + 2 = l(LQ)

$\therefore$ l(LQ) = 8 units

ii. In ΔOAB

OA + OB > AB ...(1)

In ΔOBC

OB + OC > BC ...(2)

In ΔOCD,

OC + OD > CD ...(3)

In ΔODA,

OD + OA > AD ...(4)

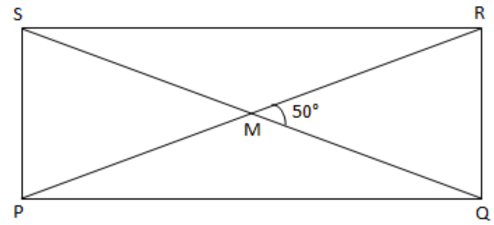
Adding (1), (2), (3) and (4)

2(OA + OB + OC + OD) > AB + BC + DC + DA

2[(OA + OC) + (OB + OD)] > AB + BC + CD + DA

2(AC + BD) > AB + BC + CD + DA

iii. ∠QMR = 50° …(given)



∠PMS = ∠QMR ….(vertically opposite angles)

∴ ∠PMS = 50°

The diagonals of a rectangle are congruent and bisect each other.

∴ MS = MP

∴ ∠MPS = ∠MSP ….(angles opposite to equal sides are equal)

Now, in ΔMSP,

∠PMS + ∠MPS + ∠MSP = 180°

∴ 50° + 2∠MPS = 180°

∴ 2∠MPS = 130°

∴ ∠MPS = 65°

iv. Let P ≡ (−2, 2) and Q ≡ (6, −6)

Segment PQ is divided into four equal parts by the points A, B and C.

Point B is the mid-point of segment PQ.

Then, by Mid point formula for B, we have

$$\left(\frac{-2+6}{2},\frac{2-6}{2}\right)=\left(\frac{4}{2},\frac{-4}{2}\right)=(2,-2)$$

$\therefore B \equiv (2,-2)$

Now, point A is the mid-point of segment PB.

Then, by Mid point formula, we have

$$\left(\frac{-2+2}{2},\frac{2-2}{2}\right)=\left(\frac{0}{2},\frac{0}{2}\right)=(0,0)$$

$\therefore A \equiv (0,0)$

Also, point C is the mid-point of segment BQ.

Then, by Mid point formula, we have

$$\left(\frac{6+2}{2},\frac{-6-2}{2}\right)=\left(\frac{8}{2},\frac{-8}{2}\right)=(4,-4)$$

$\therefore B \equiv (4,-4)$

Thus, the coordinates of the points A, B and C which divide the line segment into four equal parts are (0, 0), (2, −2) and (4, −4) respectively.

v. (a) $\frac{\cos 35^\circ}{\sin 55^\circ} + \frac{\sin 11^\circ}{\cos 79^\circ} - \cos 28^\circ \operatorname{cosec} 62^\circ$

$= \frac{\cos(90^\circ - 55^\circ)}{\sin 55^\circ} + \frac{\sin(90^\circ - 79^\circ)}{\cos 79^\circ} - \cos(90^\circ - 62^\circ)\operatorname{cosec} 62^\circ$

$= \frac{\sin 55^\circ}{\sin 55^\circ} + \frac{\cos 79^\circ}{\cos 79^\circ} - \sin 62^\circ \operatorname{cosec} 62^\circ$

$= 1 + 1 - 1$

$= 1$

(b) $\frac{\cos 81^\circ}{\sin 9^\circ} + \frac{\cos 14^\circ}{\sin 76^\circ} = \frac{\cos(90^\circ - 9)}{\sin 9^\circ} + \frac{\cos(90^\circ - 76^\circ)}{\sin 76^\circ}$

$= \frac{\sin 9^\circ}{\sin 9^\circ} + \frac{\sin 76^\circ}{\sin 76^\circ}$

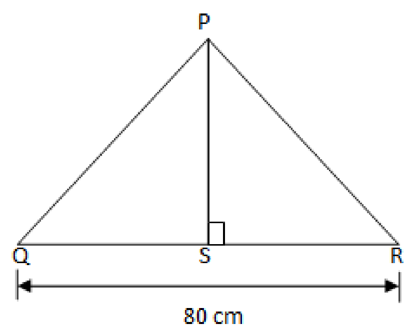
$= 1 + 1$

$= 2$

4.

i. Let ΔPQR be an isosceles triangle with base QR = 80 cm.

PQ = PR and PS is the height.



$A(\Delta PQR) = 360\ cm^2$ ....(given)

$A(\Delta PQR) = \frac{1}{2} \times \text{base} \times \text{height}$

$\therefore 360 = \frac{1}{2} \times 80 \times \text{height}$

$\therefore \text{height} = \frac{360 \times 2}{80} = 9\,\text{cm}$

$\therefore PS = 9$ cm ....(1)

In an isosceles triangle, the perpendicular to the base bisects the base.

$\therefore QS = \frac{1}{2} QR = \frac{1}{2} \times 80$ cm

$\therefore QS = 40$ cm ....(2)

In right angled ΔPQS, by Pythagoras Theorem,

$PQ^2 = QS^2 + PS^2$

$= (40)^2 + (9)^2$ ....[From (2) and (4)]

$= 1600 + 81$

$= 1681$

$\therefore PQ = 41$ cm

$\therefore PQ = PR = 41$ cm ....(3)

Perimeter of $\Delta PQR = PQ + QR + PR$

$= 41 + 80 + 41$ ....[From (3) and given]

$= 162$ cm

Thus, the perimeter of the given triangle is 162 cm.

ii.

(a) PQ ∥ SR

$\therefore \angle RFE = \angle EPQ$ ....(alternate angles)

$\angle REF = \angle PEQ$ ....(vertically opposite angles)

$\therefore \Delta FER \sim \Delta PEQ$

$\therefore \frac{ER}{EQ} = \frac{RF}{QP}$

$\therefore \frac{ER}{QR - ER} = \frac{9}{6}$

$\therefore \frac{ER}{10 - ER} = \frac{3}{2}$

$\therefore 2ER = 30 - 3ER$

$\therefore 5ER = 30$

$\therefore ER = 6$ cm

(b) In ΔOSF and ΔOQP,

$\angle QPO = \angle OFS$ ....(alternate angles)

$\angle POQ = \angle SOF$ ....(vertically opposite angels)

$\therefore \Delta OSF \sim \Delta OQP$

$\therefore \frac{OF}{OP} = \frac{SF}{QP}$

$\therefore \frac{OF}{4} = \frac{SR + RF}{6}$

$\therefore \frac{OF}{4} = \frac{12 + 9}{6}$

$\therefore \frac{OF}{4} = \frac{21}{6}$

$\therefore OF = \frac{4 \times 21}{6} = 14$ cm

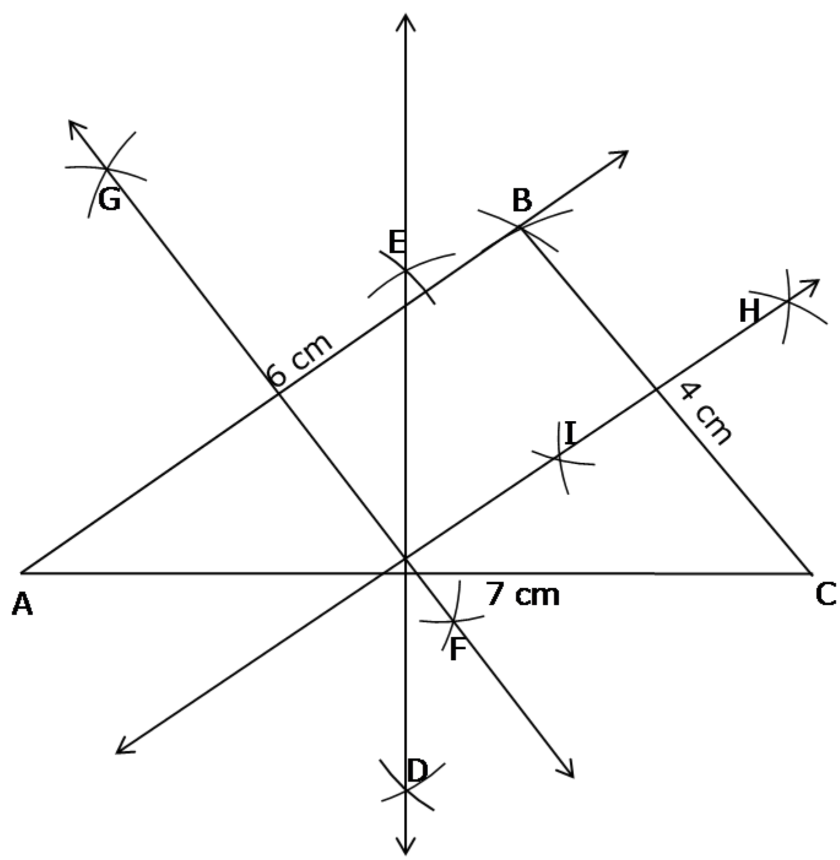
Now, PF = PO + OF

$\therefore PF = 4 + 14 = 18$ cm

iii. Steps of construction:

1. Draw seg AC = 7 cm.
2. Taking A as the centre and radius = 6 cm, draw arc of circle above the seg AC.
3. Taking C as the centre and radius = 4 cm, draw an arc of circle above seg AC intersecting the previous arc.
4. Mark the point of intersection as B.
5. Construct ΔABC by joining points A and B, B and C.
6. Taking A and C as the centres and radius greater than half of AC, draw arcs of circle above and below seg AC intersecting each other at points E and D. Join E and D and extend to get the perpendicular bisector of seg AC.
7. Draw perpendicular bisectors of seg BC and seg AB in the same manner.

   The perpendicular bisectors of seg AC, seg BC and seg AB intersect at one point.



5.

i. $\angle 2 = \angle 4$ ….(vertically opposite angles)

$\therefore 2x + 30 = x + 2y$

$\therefore 2x - x - 2y + 30 = 0$

$\therefore x - 2y + 30 = 0$ ….(1)

Also, $\angle 4 = \angle 6$ ….(alternate angles)

$x + 2y = 3y + 10$

$\therefore x + 2y - 3y - 10 = 0$

$\therefore x - y - 10 = 0$ ….(2)

Subtracting equation (2) from (1), we get

$-y + 40 = 0$

$\therefore -y = -40$

$\therefore y = 40$

Substituting y = 40 in equation (2), we get

$x - 40 - 10 = 0$

$\therefore x - 50 = 0$

$\therefore x = 50$

Now, $\angle 4 = (x + 2y)$

$\therefore \angle 4 = 50 + 2(40) = 50 + 80 = 130°$

But, $\angle 4 + \angle 5 = 180°$ …..(Interior angles)

$\therefore 130° + \angle 5 = 180°$

$\therefore \angle 5 = 180° - 130° = 50°$

ii.

(a) In ΔABC, side AB ≅ side BC and A-P-C. ...(Given)

$\therefore \angle A \cong \angle C$ ....(1)(Isosceles Δ Theorem)

$\angle BPC > A$ ....(2)(Exterior Angle Theorem)

From (1) and (2), $\angle BPC > \angle C$

$\therefore BC > BP$ ...(Side opposite to greater angle)

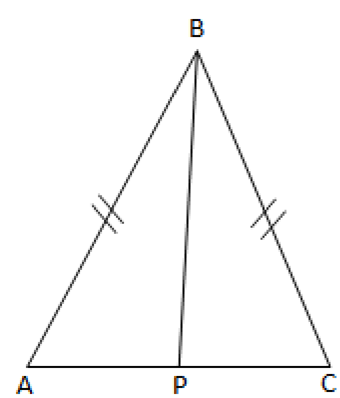
i.e. $BP < BC$ ...(3)

$AB \cong BC$ ....(4)(Given)

From (3) and (4),

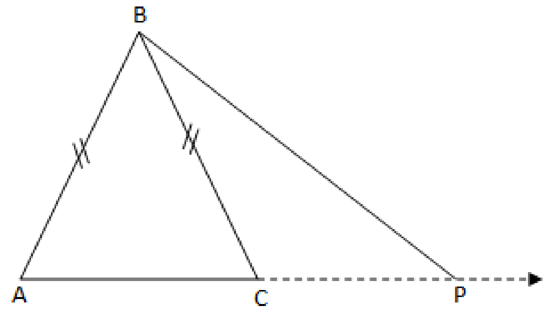
$BP < BC$ and $BP < AB$

$\therefore BP <$ congruent sides



(b) In ΔABC, side AB ≅ side BC and A-C-P. ....(Given)

∴ ∠A ≅ ∠BCA ....(1)(Isosceles Triangle Theorem)



∠BCA > ∠P ....(2)(Exterior Angle Theorem)

From (1) and (2), ∠A > ∠P

∴ BP > BA ....(3)(Side opposite to greater angle)

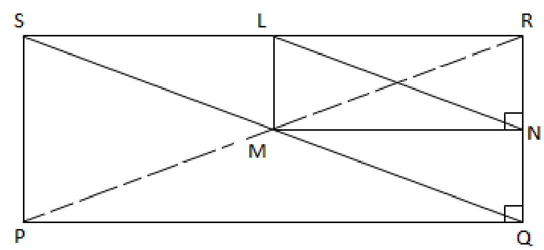
side AB ≅ side BC ....(4)(Given)

From (3) and (4),

BP > BA and BP > BC

∴ BP > congruent sides

iii. Construction: Join LN.



□LMNR is a rectangle.

∴ LM ∥ RN ....(opposite sides of a rectangle)

i.e. LM ∥ RQ

But RQ ∥ SP ....(opposite sides of a rectangle)

∴ LM ∥ RQ ∥ SP

In Δ PSR, M is the midpoint of PR and LM ∥ SP.

Then, by converse of midpoint theorem, point L is the midpoint of side SR.

∴ SL = LR (proved)

The diagonals of a rectangle are congruent.

∴ In rectangle LMNR

LN = MR ....(1)

In rectangle PQRS,

PR = SQ ...(2)

Also, $RM = \frac{1}{2}PR$ ....(since point M is the midpoint of PR)

$\therefore LN = \frac{1}{2}PR$ ....[From (1)]

$\therefore LN = \frac{1}{2}SQ$ ....[From (2)]