1. Let the measure of each interior opposite angle be $x$.

Since, Sum of two interior opposite angles = Measure of exterior angle
\[ x + x = 80^\circ \]
\[ 2x = 80^\circ \]
\[ x = 40^\circ \]
Hence, the measure of each interior opposite angle is $40^\circ$.

ii. $\angle ACB = 180^\circ - 105^\circ$
\[ \therefore \angle ACB = 75^\circ \]
$\angle ABC = \angle ACB = 75^\circ$  
...(angles opposite to equal sides of a triangle)

Now, in $\triangle ABC$,
\[ \angle ABC + \angle ACB + \angle BAC = 180^\circ \]
\[ 75^\circ + 75^\circ + \angle BAC = 180^\circ \]
\[ \angle BAC = 180^\circ - 150^\circ = 30^\circ \]

iii. Radius of the circle is $6.7$ cm.
\[ d(P, R) = 5.7 \text{ cm} \]
\[ 5.7 \text{ cm} < 6.7 \text{ cm} \]
\[ \therefore \text{The distance between P and R is less than the radius of the circle.} \]
\[ \therefore \text{Point R lies in the interior of the circle.} \]

iv. Let $A \equiv (x_1, y_1)$ and $B \equiv (x_2, y_2)$.

Here, $x_1 = 2, y_1 = 5$ and $x_2 = -6, y_2 = 8$

By distance formula, we have
\[ AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-6 - 2)^2 + (8 - 5)^2} = \sqrt{(-8)^2 + (3)^2} = \sqrt{64 + 9} \]
\[ \therefore AB = \sqrt{73} \]
Thus, the distance between the points A and B is $\sqrt{73}$ units.
v. \[
\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \times \csc 31^\circ = \frac{\cos 80^\circ}{\cos(90^\circ - 10^\circ)} + \sin(90^\circ - 59^\circ) \times \frac{1}{\sin 31^\circ}
\]
\[
= \frac{\cos 80^\circ}{\cos 80^\circ} + \sin 31^\circ \times \frac{1}{\sin 31^\circ}
\]
\[
= 1 + 1
\]
\[
= 2
\]

vi. Area of the square-shaped field = \((side)^2 = (300)^2 = 90000\) m\(^2\)

Cost of leveling the field per square metre = Rs. 1.25

\[\therefore\text{Cost of leveling} = \text{Rate} \times \text{Area}\]
\[= \text{Rs. 1.25} \times 90000\]
\[= \text{Rs. 1,12,500}\]

Thus, the cost of leveling the field is Rs. 1,12,500.

2.

i. Seg QS is the angle bisector of \(\angle PQR\) \(\ldots\)(Given)

\[\therefore \angle PQS \equiv \angle RQS \ldots(1)\]

Seg QS is the bisector of \(\angle PSR\) \(\ldots\)(Given)

\[\therefore \angle PSQ \equiv \angle RSQ \ldots(2)\]

In \(\triangle PQS\) and \(\triangle RQS\),

\[\angle PQS \equiv \angle RQS \ldots\text{[From (1)]}\]

seg QS \equiv seg QS \ldots\text{(Common side)}

\[\angle PSQ \equiv \angle RSQ \ldots\text{[From (2)]}\]

\[\therefore \triangle PQS \equiv \triangle RQS \ldots\text{(ASA test)}\]

\[\therefore \angle P \equiv \angle R \ldots\text{(c.a.c.t.)}\]

ii. In \(\triangle NYX\), \(m\angle NYX = 90^\circ\) \(\ldots\)(Given)

\[\therefore m\angle N + m\angle X = 90^\circ \ldots\text{(Acute angles of a right angled triangle)}\]

\[\therefore m\angle N + 45^\circ = 90^\circ \ldots\text{(Given : } m\angle X = 45^\circ)\]

\[\therefore m\angle N = 45^\circ\]

Now, in \(\triangle NMZ\),

\[m\angle N + m\angle NMZ + m\angle Z = 180^\circ \ldots\text{(Angle Sum property of a triangle)}\]

\[\therefore 45^\circ + 110^\circ + x = 180^\circ\]

\[\therefore 155^\circ + x = 180^\circ\]

\[\therefore x = 180^\circ - 155^\circ\]

\[\therefore x = 25^\circ\]
iii. Given, angle = s
Then, its supplementary angle = (180° – s)
According to given information, we have
\[ s = 4 \times (180° - s) + 20 \]
\[ \therefore s = 720 - 4s + 20 \]
\[ \therefore 5s = 740 \]
\[ \therefore s = 148 \]
Thus, the measure of angle 's' = 148°

iv. Given: In a circle with centre O, seg PQ and seg RS are two chords.
\[ \angle POQ \cong \angle ROS \]
To prove: chord PQ \cong chord RS
Proof:
In \( \triangle POQ \) and \( \triangle ROS \),
\[ \text{seg OP} \cong \text{seg OR} \quad \text{...(radii of same circle)} \]
\[ \angle POQ \cong \angle ROS \quad \text{...(given)} \]
\[ \text{seg OQ} \cong \text{seg OS} \quad \text{...(radii of same circle)} \]
\[ \therefore \triangle POQ \cong \triangle ROS \quad \text{...(SAS test)} \]
\[ \therefore \text{seg PQ} \cong \text{seg RS} \quad \text{...(c.s.c.t.)} \]
\[ \therefore \text{chord PQ} \cong \text{chord RS} \]

v. In \( \square PQRS \)
\[ \angle P + \angle Q + \angle R + \angle S = 360° \quad \text{...(angle sum property of a quadrilateral)} \]
\[ \therefore \angle P + \angle Q + 100° + 30° = 360° \]
\[ \therefore \angle P + \angle Q + 130° = 360° \]
\[ \therefore \angle P + \angle Q = 230° \]
\[ \therefore \frac{1}{2} \angle P + \frac{1}{2} \angle Q = \frac{1}{2} \times 230° \]
\[ \text{i.e. } \angle APQ + \angle AQP = 115° \quad \text{...(1)} \]
Now, in \( \triangle APQ \),
\[ \angle APQ + \angle AQP + \angle PAQ = 180° \]
\[ \therefore 115° + \angle PAQ = 180° \]
\[ \therefore \angle PAQ = 180° - 115° = 65° \]
\[ \therefore \angle PAQ = 65° \]
vi. \( \text{L.H.S.} = \sin A = \sin 30^\circ = \frac{1}{2} \)

\( \text{R.H.S.} = \sqrt{\frac{1 - \cos 2A}{2}} = \sqrt{\frac{1 - \cos 60^\circ}{2}} \) ...(since \( \angle A = 30^\circ, 2\angle A = 60^\circ \))

\( = \sqrt{\frac{1 - \frac{1}{2}}{2}} \)

\( = \sqrt{\frac{\frac{1}{2}}{2}} \)

\( = \sqrt{\frac{1}{4}} \)

\( = \frac{1}{2} \)

\( \therefore \text{L.H.S.} = \text{R.H.S.} \) (Proved)

3.

i. \( l(PL) + l(LN) = l(PN) \) (P-L-N)

\( \therefore l(PL) + 5 = 11 \)

\( \therefore l(PL) = 6 \text{ units} \)

\( l(MN) + l(NR) = l(MR) \) (M-N-R)

\( \therefore 7 + l(NR) = 13 \)

\( \therefore l(NR) = 6 \text{ units} \)

\( l(LM) + l(MQ) = l(LQ) \) (L-M-Q)

\( \therefore 6 + 2 = l(LQ) \)

\( \therefore l(LQ) = 8 \text{ units} \)

ii. In \( \triangle OAB \)

\( OA + OB > AB \) ...(1)

In \( \triangle OBC \)

\( OB + OC > BC \) ...(2)

In \( \triangle OCD, \)

\( OC + OD > CD \) ...(3)

In \( \triangle ODA, \)

\( OD + OA > AD \) ...(4)

Adding (1), (2), (3) and (4)

\( 2(OA + OB + OC + OD) > AB + BC + DC + DA \)

\( 2[(OA + OC) + (OB + OD)] > AB + BC + CD + DA \)

\( 2(AC + BD) > AB + BC + CD + DA \)
iii. \( \angle QMR = 50^\circ \) ...(given)
\[ \angle PMS = \angle QMR \] ...(vertically opposite angles)
\[ \therefore \angle PMS = 50^\circ \]
The diagonals of a rectangle are congruent and bisect each other.
\[ \therefore MS = MP \]
\[ \therefore \angle MPS = \angle MSP \] ....(angles opposite to equal sides are equal)
Now, in \( \Delta MSP \),
\[ \angle PMS + \angle MPS + \angle MSP = 180^\circ \]
\[ 50^\circ + 2\angle MPS = 180^\circ \]
\[ \therefore 2\angle MPS = 130^\circ \]
\[ \therefore \angle MPS = 65^\circ \]

iv. Let \( P \equiv (-2, 2) \) and \( Q \equiv (6, -6) \)
Segment \( PQ \) is divided into four equal parts by the points \( A, B \) and \( C \).
Point \( B \) is the mid-point of segment \( PQ \).
Then, by Mid point formula for \( B \), we have
\[ \left( \frac{-2 + 6}{2}, \frac{2 - 6}{2} \right) = \left( \frac{4}{2}, \frac{-4}{2} \right) = (2, -2) \]
\[ \therefore B \equiv (2, -2) \]
Now, point \( A \) is the mid-point of segment \( PB \).
Then, by Mid point formula, we have
\[ \left( \frac{-2 + 2}{2}, \frac{2 - 2}{2} \right) = \left( \frac{0}{2}, \frac{0}{2} \right) = (0, 0) \]
\[ \therefore A \equiv (0, 0) \]
Also, point \( C \) is the mid-point of segment \( BQ \).
Then, by Mid point formula, we have
\[ \left( \frac{6 + 2}{2}, \frac{-6 - 2}{2} \right) = \left( \frac{8}{2}, \frac{-8}{2} \right) = (4, -4) \]
\[ \therefore B \equiv (4, -4) \]
Thus, the coordinates of the points \( A, B \) and \( C \) which divide the line segment into four equal parts are \((0, 0), (2, -2)\) and \((4, -4)\) respectively.
v. (a)  \(\frac{\cos 35^\circ}{\sin 55^\circ} + \frac{\sin 11^\circ}{\cos 79^\circ} - \cos 28^\circ \csc 62^\circ\)

\[
= \frac{\cos(90^\circ - 55^\circ)}{\sin 55^\circ} + \frac{\sin(90^\circ - 79^\circ)}{\cos 79^\circ} - \cos(90^\circ - 62^\circ) \csc 62^\circ
\]

\[
= \frac{\sin 55^\circ}{\sin 55^\circ} + \frac{\cos 79^\circ}{\cos 79^\circ} - \sin 62^\circ \csc 62^\circ
\]

\[
= 1 + 1 - 1
\]

\[
= 1
\]

(b) \(\frac{\cos 81^\circ}{\sin 9^\circ} + \frac{\cos 14^\circ}{\sin 76^\circ} = \frac{\cos(90^\circ - 9^\circ)}{\sin 9^\circ} + \frac{\cos(90^\circ - 76^\circ)}{\sin 76^\circ}\)

\[
= \frac{\sin 9^\circ}{\sin 9^\circ} + \frac{\sin 76^\circ}{\sin 76^\circ}
\]

\[
= 1 + 1
\]

\[
= 2
\]

4.

i. Let \(\triangle PQR\) be an isosceles triangle with base \(QR = 80\) cm.

\(PQ = PR\) and \(PS\) is the height.

\(A(\triangle PQR) = 360\) cm\(^2\) ....(given)

\(A(\triangle PQR) = \frac{1}{2} \times \text{base} \times \text{height}\)

\[
\therefore 360 = \frac{1}{2} \times 80 \times \text{height}
\]

\[
\therefore \text{height} = \frac{360 \times 2}{80} = 9\text{ cm}
\]

\[
\therefore PS = 9\text{ cm} \quad \ldots(1)
\]

In an isosceles triangle, the perpendicular to the base bisects the base.

\[
\therefore QS = \frac{1}{2} QR = \frac{1}{2} \times 80\text{ cm}
\]

\[
\therefore QS = 40\text{ cm} \quad \ldots(2)
\]

In right angled \(\triangle PQS\), by Pythagoras Theorem,

\(PQ^2 = QS^2 + PS^2\)

\[
= (40)^2 + (9)^2 \quad \ldots[\text{From (2) and (4)}]
\]

\[
= 1600 + 81
\]

\[
= 1681
\]

\[
\therefore PQ = 41\text{ cm}
\]

\[
\therefore PQ = PR = 41\text{ cm} \quad \ldots(3)
\]

Perimeter of \(\triangle PQR = PQ + QR + PR\)

\[
= 41 + 80 + 41 \quad \ldots[\text{From (3) and given}]
\]

\[
= 162\text{ cm}
\]

Thus, the perimeter of the given triangle is 162 cm.
ii.

(a) PQ \parallel SR

\[ \angle RFE = \angle EPQ \quad \text{(alternate angles)} \]
\[ \angle REF = \angle PEQ \quad \text{(vertically opposite angles)} \]

\[ \therefore \triangle FER \sim \triangle PEQ \]

\[
\begin{align*}
\frac{ER}{RF} &= \frac{EQ}{QP} \\
\frac{ER}{QR} &= \frac{9}{6} \\
\frac{ER}{10} &= \frac{3}{2} \\
\therefore 2ER &= 30 - 3ER \\
5ER &= 30 \\
\therefore ER &= 6 \text{ cm}
\end{align*}
\]

(b) In \triangle OSF and \triangle OQP,

\[ \angle QPO = \angle OFS \quad \text{(alternate angles)} \]
\[ \angle POQ = \angle SOF \quad \text{(vertically opposite angles)} \]

\[ \therefore \triangle OSF \sim \triangle OQP \]

\[
\begin{align*}
\frac{OF}{SF} &= \frac{OP}{QP} \\
\frac{OF}{4} &= \frac{SR + RF}{6} \\
\frac{OF}{4} &= \frac{12 + 9}{6} \\
\therefore OF &= \frac{21}{6} \\
\therefore OF &= \frac{4 \times 21}{6} = 14 \text{ cm}
\end{align*}
\]

Now, PF = PO + OF

\[ \therefore PF = 4 + 14 = 18 \text{ cm} \]
iii. Steps of construction:

1. Draw seg AC = 7 cm.
2. Taking A as the centre and radius = 6 cm, draw arc of circle above the seg AC.
3. Taking C as the centre and radius = 4 cm, draw an arc of circle above seg AC intersecting the previous arc.
4. Mark the point of intersection as B.
5. Construct ΔABC by joining points A and B, B and C.
6. Taking A and C as the centres and radius greater than half of AC, draw arcs of circle above and below seg AC intersecting each other at points E and D. Join E and D and extend to get the perpendicular bisector of seg AC.
7. Draw perpendicular bisectors of seg BC and seg AB in the same manner.

The perpendicular bisectors of seg AC, seg BC and seg AB intersect at one point.
5.

i. \( \angle 2 = \angle 4 \quad \text{ ...(vertically opposite angles) } \)

\[ 2x + 30 = x + 2y \]
\[ 2x - x - 2y + 30 = 0 \]
\[ x - 2y + 30 = 0 \quad \text{ ....(1) } \]

Also, \( \angle 4 = \angle 6 \quad \text{ ...(alternate angles) } \)

\[ x + 2y = 3y + 10 \]
\[ x + 2y - 3y - 10 = 0 \]
\[ x - y - 10 = 0 \quad \text{ ....(2) } \]

Subtracting equation (2) from (1), we get

\[ -y + 40 = 0 \]
\[ -y = -40 \]
\[ y = 40 \]

Substituting \( y = 40 \) in equation (2), we get

\[ x - 40 - 10 = 0 \]
\[ x - 50 = 0 \]
\[ x = 50 \]

Now, \( \angle 4 = (x + 2y) \)

\[ \angle 4 = 50 + 2(40) = 50 + 80 = 130^\circ \]

But, \( \angle 4 + \angle 5 = 180^\circ \quad \text{ ....(Interior angles) } \)

\[ 130^\circ + \angle 5 = 180^\circ \]
\[ \angle 5 = 180^\circ - 130^\circ = 50^\circ \]

ii.

(a) In \( \triangle ABC \), side \( AB \equiv \) side \( BC \) and \( A-P-C \). \quad \text{ ... (Given) } \)

\[ \angle A \equiv \angle C \quad \text{ ....(1)(Isosceles \triangle \text{ Theorem}) } \]
\( \angle BPC > A \quad \text{ ....(2)(Exterior Angle Theorem) } \)

From (1) and (2), \( \angle BPC > \angle C \)

\[ \therefore \ BC > BP \quad \text{ ...(Side opposite to greater angle) } \]

i.e. \( BP < BC \quad \text{ ....(3) } \)

\( AB \equiv BC \quad \text{ ....(4)(Given) } \)

From (3) and (4),

\[ BP < BC \text{ and } BP < AB \]
\[ \therefore \ BP < \text{ congruent sides } \]
(b) In \( \triangle ABC \), side \( AB \equiv \) side \( BC \) and \( A \rightarrow C \rightarrow P \). ....(Given)

\[ \angle A \equiv \angle BCA \] ....(1)(Isosceles Triangle Theorem)

\[ \angle BCA > \angle P \] ....(2)(Exterior Angle Theorem)

From (1) and (2), \( \angle A > \angle P \)

\[ \therefore \text{ BP > BA } \] ....(3)(Side opposite to greater angle)

\[ \text{side } AB \equiv \text{side } BC \] ....(4)(Given)

From (3) and (4),

\[ \text{BP > BA and BP > BC} \]

\[ \therefore \text{ BP > congruent sides} \]

iii. Construction: Join LN.

\( \square \text{LMNR is a rectangle.} \)

\[ \therefore \text{ LM } \parallel \text{ RN } \] ....(opposite sides of a rectangle)

i.e. \( \text{ LM } \parallel \text{ RQ} \)

But \( \text{ RQ } \parallel \text{ SP } \) ....(opposite sides of a rectangle)

\[ \therefore \text{ LM } \parallel \text{ RQ } \parallel \text{ SP} \]

In \( \triangle PSR \), M is the midpoint of PR and LM \( \parallel \) SP.

Then, by converse of midpoint theorem, point L is the midpoint of side SR.

\[ \therefore \text{ SL = LR } \] (proved)

The diagonals of a rectangle are congruent.

\[ \therefore \text{ In rectangle LMNR} \]

\[ \text{LN = MR} \] ....(1)

In rectangle PQRS,

\[ \text{PR = SQ} \] ....(2)

Also, \( \text{RM} = \frac{1}{2}\text{PR} \) ....(since point M is the midpoint of PR)

\[ \therefore \text{ LN} = \frac{1}{2}\text{PR} \] ....[From (1)]

\[ \therefore \text{ LN} = \frac{1}{2}\text{SQ} \] ....[From (2)]