Maharashtra State Board
Class IX Mathematics – Geometry
Board Paper 2
Solution

Time: 2 hours **Total Marks: 40**

1.

i. The ratio of the angles of the triangle is 2 : 2 : 5.

Let the measures of the angles be $2x°$, $2x°$ and $5x°$.

Then $2x° + 2x° + 5x° = 180°$

$\therefore 9x° = 180°$

$\therefore x° = 20°$

$\therefore 2x° = 2 \times 20° = 40°$ and $5x° = 5 \times 20 = 100°$

Since two angles of the triangle are equal, it is an isosceles triangle.

ii. AB + BC > CA, BC + AC > AB, AC + AB > BC

Yes, they form a triangle as sum of two sides is greater than the third side

iii. $AM = \frac{AB}{2} = \frac{24}{2} = 12$ cm and $AO = \frac{AD}{2} = \frac{30}{2} = 15$ cm

In ΔAOM,

$AO^2 = AM^2 + OM^2$ ….(Pythagoras theorem)

$\therefore (15)^2 = (12)^2 + OM^2$

$\therefore 225 = 144 + OM^2$

$\therefore OM^2 = 225 - 144 = 81$

$\therefore OM = 9$ cm

Thus, the distance of AB from the centre of the circle is 9 cm.

iv. Let $J \equiv (x_1, y_1)$ and $L \equiv (x_2, y_2)$

Here, $x_1 = -8$, $y_1 = -4$, $x_2 = 1$, $y_2 = 2$, $m = 1$, $n = 2$

Let $P \equiv (x, y)$ divides JL externally in the given ratio.

Then, by section formula for external division, we have

$$x = \frac{mx_2 - nx_1}{m - n} = \frac{1(1) - 2(-8)}{1 - 2} = \frac{1 + 16}{-1} = -17$$

$$y = \frac{my_2 - ny_1}{m - n} = \frac{1(2) - 2(-4)}{1 - 2} = \frac{2 + 8}{-1} = -10$$

Thus, the co-ordinates of the point $P \equiv (-17, -10)$.

v. cot45° = 1, sec60° = 2, cosec30° = 2 and cot90° = 0

$\therefore$ $4\cot^2 45° - \sec^2 60° + \mathrm{cosec}^2 30° + \cot 90°$

$= 4(1)^2 - (2)^2 + (2)^2 + 0$

= 4 − 4 + 4

= 4

vi. Radius of the circle = r = 28 cm

$\therefore$ Diameter of the circle = d = 2r = 2 × 28 = 56 cm

Perimeter of the semicircle = $\pi r + d$

$$= \frac{22}{7} \times 28 + 56$$
$$= 22 \times 4 + 56$$
$$= 88 + 56$$
$$= 144 \text{ cm}$$

2.

i. Measure of the given angle $= \frac{3}{5} \times$ right angle

$$= \frac{3}{5} \times 90°$$
$$= 3 \times 18°$$
$$= 54°$$

Thus, the measure of its supplementary angle = 180° − 54° = 126°

ii. $\Delta CAB \sim \Delta FDE$

$\therefore \frac{CA}{FD} = \frac{AB}{DE} = \frac{CB}{FE}$ ....(Corresponding sides are proportional)

$\therefore \frac{13}{n} = \frac{12}{m} = \frac{5}{2}$

$\therefore \frac{12}{m} = \frac{5}{2}$

$\therefore m = \frac{12 \times 2}{5} = \frac{24}{5}$

$\therefore m = 4.8$

Also, $\frac{13}{n} = \frac{5}{2}$

$\therefore n = \frac{13 \times 2}{5} = \frac{26}{5}$

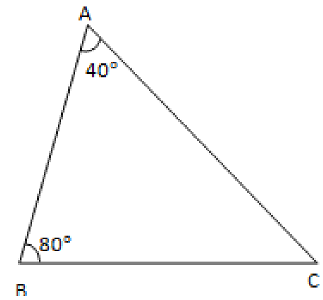
$\therefore n = 5.2$

iii. In ΔABC, $\angle A + \angle B + \angle C = 180°$

$\therefore 40° + 80° + \angle C = 180°$

$\therefore \angle C = 180° - 120° = 60°$

The descending order of the measures of the angles is $\angle B > \angle C > \angle A$.



$\therefore$ side AC > side AB > side BC ....(Sides opposite to the angles)

Thus, the shortest side is side BC and the longest side is side AC.

iv. BD = 3DC ....(given)

Let DC = x

Then, BD = 3x ....(1)

Now, BC = BD + DC = 3x + x = 4x

$\therefore BC^2 = 16x^2$ ....(2)

In right-angled ΔADB, by Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ ....(3)

Similarly, in right-angled ΔADC,

$AC^2 = AD^2 + DC^2$

$\therefore AD^2 = AC^2 - DC^2$ ....(4)

From (3) and (4), we have

$AB^2 = AC^2 - DC^2 + BD^2$

$= AC^2 - x^2 + (3x)^2$

$= AC^2 - x^2 + 9x^2$

$= AC^2 + 8x^2$

$= AC^2 + \frac{1}{2} \times 16x^2$ ...(5)

$\therefore AB^2 = AC^2 + \frac{1}{2}BC^2$ ... [From (5) and (2)]

v. Let the length of one side of the parallelogram be x cm.

Then, the length of other side is (25 + x) cm.

Perimeter of the parallelogram is 150 cm.

$\therefore x + (25 + x) + x + (25 + x) = 150$

$\therefore 4x + 50 = 150$

$\therefore 4x = 100$

$\therefore x = 25$ cm

$\therefore 25 + x = 25 + 25 = 50$ cm

Thus, the lengths of the sides of the parallelogram are 25 cm, 50 cm, 25 cm and 50 cm.

vi. $\dfrac{\tan^2 60^\circ + 4\cos^2 45^\circ + \sec^2 30^\circ + 5\cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$

$= \dfrac{\left(\sqrt{3}\right)^2 + 4\left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{2}{\sqrt{3}}\right)^2 + 5 \times 0}{2 + 2 - \left(\sqrt{3}\right)^2}$

$= \dfrac{3 + 4 \times \dfrac{1}{2} + \dfrac{4}{3} + 0}{4 - 3}$

$= \dfrac{3 + 2 + \dfrac{4}{3}}{1}$

$= \dfrac{9 + 6 + 4}{3}$

$= \dfrac{19}{3}$

3.

i. PQ is a straight line.

$\therefore$ a + b = 180° ....(Linear pair of angles)(1)

Now, a − b = 80° ....(given)(2)

Adding equation (1) and (2), we get

a + b + a − b = 180° + 80°

$\therefore$ 2a = 260°

$\therefore$ a = 130°

Substituting a = 130° in equation (1), we get

130° + b = 180°

$\therefore$ b = 180° − 130°

$\therefore$ b = 50°

$\therefore$ ∠POR = 130° and ∠ROQ = 50°

ii.

(a) In ΔABC and ΔCDE,

m∠B = m∠D = 90°

hypotenuse AC ≅ hypotenuse CE ....(Given)

seg BC ≅ seg ED ....(Given)

$\therefore$ ΔABC ≅ ΔCDE ....(Hypotenuse-side theorem)

(b) As ΔABC ≅ ΔCDE

$\therefore$∠BAC ≅ ∠DCE ....(1)(c.a.c.t.)

(c) In ΔABC,

$\angle BAC + \angle ACB = 90°$ ...(2)(Acute angles of a right-angled triangle)

From (1) and (2),

$\angle DCE + \angle ACB = 90°$ ....(3)

Now, $\angle ACB + \angle ACE + \angle DCE = 180°$

$\therefore \underline{\angle ACB + \angle DCE} + \angle ACE = 180°$

$\therefore 90° + \angle ACE = 180°$ ....[From (3)]

$\therefore \angle ACE = 90°$

iii. ΔABC is an isosceles triangle. ….(given)

$\therefore AB = AC$

Now, D, E and F are the mid-points of sides AB, AC and BC respectively.

Then, by mid-point theorem, we have

$EF = \frac{1}{2}AB$ and $DF = \frac{1}{2}AC$

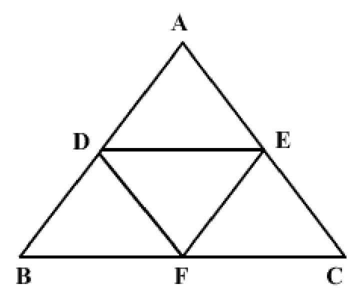
Since, $AB = AC$,

$\frac{1}{2}AB = \frac{1}{2}AC$

$\therefore EF = DF$

Now, in ΔDEF, $EF = DF$

Hence, DEF is an isosceles triangle.



iv. $A \equiv (-3, 0)$ and $B \equiv (3, 0)$

Let $C \equiv (x, y)$

Since ΔABC is an equilateral triangle, we have

$AB = BC = AC$

$\therefore AB^2 = BC^2 = AC^2$

Consider, $AB^2 = BC^2$

$\therefore [3 - (-3)]^2 + (0)^2 = (x - 3)^2 + (y - 0)^2$

$\therefore 36 = x^2 - 6x + 9 + y^2$

$\therefore x^2 - 6x + y^2 = 27$ ….(1)

Consider, $BC^2 = AC^2$

$\therefore (x - 3)^2 + (y - 0)^2 = [x - (-3)]^2 + (y - 0)^2$

$\therefore x^2 - 6x + 9 + y^2 = (x + 3)^2 + y^2$

$\therefore x^2 - 6x + 9 + y^2 = x^2 + 6x + 9 + y^2$

$\therefore 12x = 0$

$\therefore x = 0$

Substituting x = 0 in equation (1), we get

$(0)^2 - 6(0) + y^2 = 27$

$\therefore y^2 = 27$

$\therefore y = \pm 3\sqrt{3}$

Thus, the coordinates of C are $(0, 3\sqrt{3})$ or $(0, -3\sqrt{3})$.

v. In ΔABC, ∠B = 90°, ∠ ACB = x

$\therefore$ ∠BAC = 90° − x

In ΔADC, ∠D = 90°, ∠ ACD = y

$\therefore$ ∠DAC = 90° − y

$\tan x = \frac{AB}{BC}$

$\cot(90^\circ - y) = \cot \angle DAC = \frac{AD}{DC}$

$\sec y = \frac{AC}{CD}$

$\sin(90^\circ - x) = \sin \angle BAC = \frac{BC}{AC}$

$\operatorname{cosec}(90^\circ - y) = \operatorname{cosec} \angle DAC = \frac{AC}{DC}$

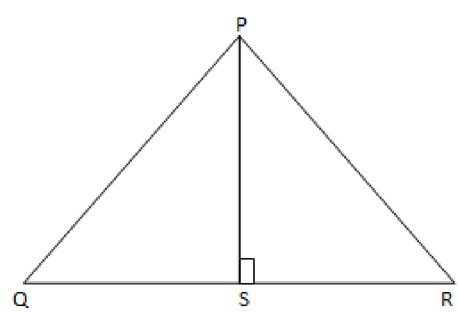
$\cos(90^\circ - x) = \cos \angle BAC = \frac{AB}{AC}$

4.

i. Let ΔPQR be an isosceles triangle.

(a) Let PQ = PR = x cm ....(1)

QR = 1.5x ....(2)



Now, perimeter of ΔPQR = 42 cm ....(given)

$\therefore$ PQ + PR + QR = 42

$\therefore$ x + x + 1.5x = 42 [From (1) and (2)]

$\therefore$ 3.5x = 42

$\therefore$ x = 12

$\therefore$ PQ = PR = 12 cm

QR = 1.5x = 1.5 × 12 = 18 cm

Thus, the length of congruent sides of a triangle, PQ = PR = 12 cm.

(b) In an isosceles triangle, the perpendicular to the base bisects the base.

Let PS $\perp$ QR.

Le PS = h

$\therefore QS = \frac{1}{2} \times QR = \frac{1}{2} \times 18 = 9\,cm$

In ΔPQS, using Pythagoras theorem,

$PQ^2 = QS^2 + PS^2$

$\therefore (12)^2 = (9)^2 + h^2$

$\therefore 144 = 81 + h^2$

$\therefore h^2 = 144 - 81 = 63$

$\therefore h = 3\sqrt{7}\,cm$

Thus, the height of a triangle, $PS = 3\sqrt{7}\,cm$.

(c) Area of $\Delta PQR = \frac{1}{2} \times base \times height$

$\therefore A(\Delta PQR) = \frac{1}{2} \times QR \times PS = \frac{1}{2} \times 18 \times 3\sqrt{7} = 27\sqrt{7}\ cm^2$

Thus, the area of a triangle is $27\sqrt{7}\ cm^2$.

ii. Let the measure of $\angle D$ be x°.

Then, $4\angle S = 3\angle D = 3x°$

$\therefore \angle S = \frac{3x°}{4}$

Also, $6\angle R = 3\angle D = 3x°$

$\therefore \angle R = \frac{3x°}{6} = \frac{x°}{2}$

Now, in ΔDSR,

$\angle D + \angle S + \angle R = 180°$ ....(Angle sum property of a triangle)

$\therefore x° + \frac{3x°}{4} + \frac{x°}{2} = 180°$

$\therefore 4x° + 3x° + 2x° = 180° \times 4$

$\therefore 9x° = 180° \times 4$

$\therefore x° = \frac{180° \times 4}{9}$

$\therefore x° = 80°$

$\therefore \angle S = \frac{3x°}{4} = \frac{3 \times 80°}{4} = 3 \times 20° = 60°$

$\angle R = \frac{x°}{2} = \frac{80°}{2} = 40°$

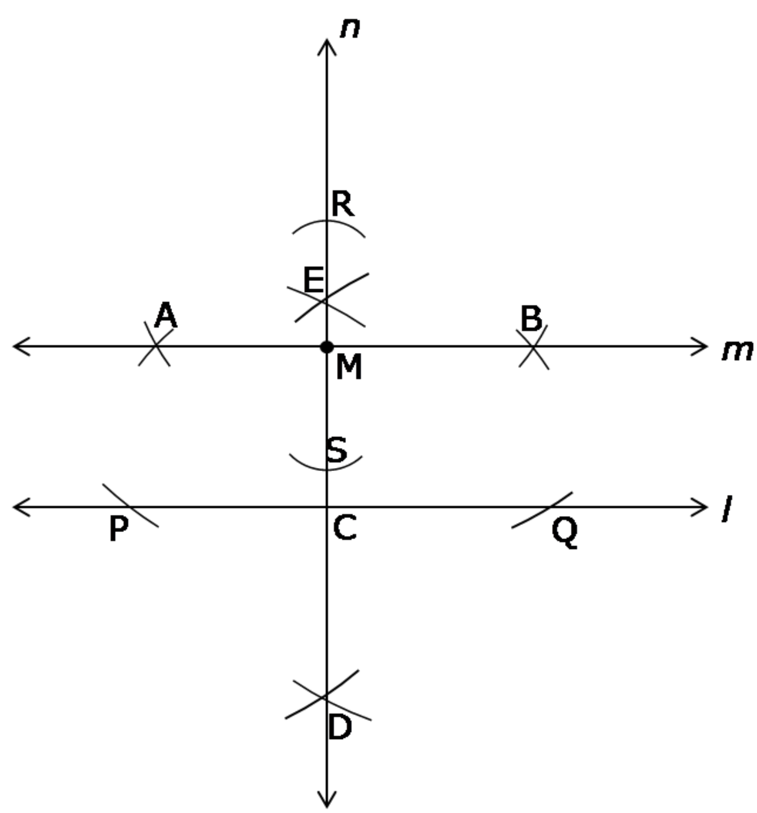
$\therefore \angle D = 80°$, $\angle S = 60°$ and $\angle R = 40°$

iii. Steps of construction:

1. Draw a line l, take any point M outside it.
2. Taking M as the centre and any radius, draw two arcs of circle on line l.
3. Name the points of intersection of the arcs and line l as P and Q respectively.
4. Taking P and Q as the centres, and radius more than half of PQ, draw arcs of circle above and below line l.
5. Name the points of intersection as E and D.
6. Draw line n ⊥ line l by joining the points E and D.
7. Taking M as the centre and any radius, draw arcs of circle above and below point M on line n. Name the points of intersection as R and S.
8. Taking R and S as the centres and radius more than half of RS, draw arcs of circle on both sides of line n. Name the points of intersection as A and B.
9. Draw line m ⊥ line n by joining the points A and B.

   Now, line l ⊥ line n and line m ⊥ line n.

   Hence, line m and line l are parallel to each other.



5.

i. l(AB) + l(BC) = l(AC)   (A-B-C)

$\therefore$ l(AB) + 5 = 8

$\therefore$ l(AB) = 3 units   ….(1)

seg AC $\cong$ seg BD   ….(given)

$\therefore$ l(BD) = 8

l(BC) + l(CD) = l(BD)   (B-C-D)

$\therefore$ 5 + l(CD) = 8

$\therefore$ l(CD) = 3 units   ….(2)

seg BD $\cong$ seg CE   ….(given)

$\therefore$ l(CE) = 8

l(CD) + l(DE) = l(CE)   (C-D-E)

$\therefore$ 3 + l(DE) = 8

$\therefore$ l(DE) = 5 units   ….(3)

(a) l(BC) = l(DE) = 5 units   ….[From (3) and given that l(BC) = 5]

$\therefore$ seg BC $\cong$ seg DE

(b) l(AB) = l(CD) = 3 units   ….[From (1) and (2)]

$\therefore$ seg AB $\cong$ seg CD

ii. In ΔABM and ΔPQN,

$\angle$AMB = $\angle$PNQ ….(each right angle)

AB = PQ   ....(given)

AM = PN   ….(given)

$\therefore$ ΔABM $\cong$ ΔPQN   …(hypotenuse-side congruence of right triangles)

$\therefore$ BM = QN   ….(1)(c.p.c.t.)

In ΔAMC and ΔPNR,

$\angle$AMC = $\angle$PNR   ….(each right angle)

AC = PR   ….(given)

AM = PN   ….(given)

$\therefore$ ΔAMC = ΔPNR   ….(hypotenuse-side congruence of right triangles)

$\therefore$ MC = NR ….(2)(c.p.c.t.)

Adding (1) and (2), we get

BM + MC = QN + NR

$\therefore$ BC = QR

Now, in ΔABC and ΔPQR,

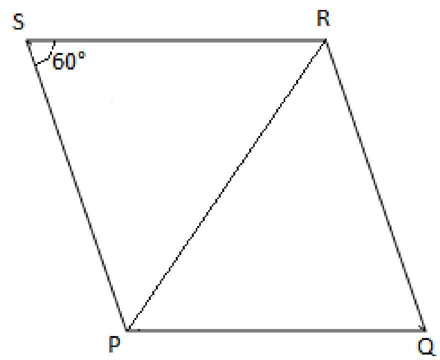
AB = PQ ….(given)

AC = PR ….(given)

BC = QR ….(proved above)

∴ ΔABC ≅ ΔPQR

iii. Let □PQRS be a rhombus with ∠PSR = 60°.



In ΔPSR,

PS = SR ….(sides of a rhombus)

∴ ∠SPR = ∠SRP ….(1)(angles opposite to equal sides are equal)

Now, in ΔPSR,

∠PSR + ∠SPR + ∠SRP = 180°

∴ 2∠SPR + 60° = 180° ….[From (1)]

∴ 2∠SPR = 120°

∴ ∠SPR = 60° = ∠SRP

∴ ΔPSR is an equilateral triangle.

Since in a parallelogram, the opposite angles are equal, we have

∠PQR = 60°

In ΔPQR,

PQ = QR ….(sides of a rhombus)

∴ ∠QPR = ∠QRP ….(2)(angles opposite to equal sides are equal)

Now, in ΔPQR,

∠PQR + ∠QPR + ∠QRP = 180°

∴ 2∠QPR + 60° = 180° ….[From (2)]

∴ 2∠APR = 120°

∴ ∠QPR = 60° = ∠QRP

∴ ΔPQR is an equilateral triangle.

Thus, in a rhombus with an angle of 60°, the shorter diagonal divides the rhombus into two equilateral triangles.