1.

i. The ratio of the angles of the triangle is 2 : 2 : 5.
Let the measures of the angles be 2x°, 2x° and 5x°.
Then 2x° + 2x° + 5x° = 180°
\[9x° = 180°\]
\[x° = 20°\]
\[2x° = 2 \times 20° = 40°\] and \[5x° = 5 \times 20 = 100°\]
Since two angles of the triangle are equal, it is an isosceles triangle.

ii. AB + BC > CA, BC + AC > AB, AC + AB > BC
Yes, they form a triangle as sum of two sides is greater than the third side

iii. AM = \(\frac{AB}{2} = \frac{24}{2} = 12\) cm and AO = \(\frac{AD}{2} = \frac{30}{2} = 15\) cm
In \(\triangle AOM\),
\[AO^2 = AM^2 + OM^2\] ....(Pythagoras theorem)
\[\therefore (15)^2 = (12)^2 + OM^2\]
\[\therefore 225 = 144 + OM^2\]
\[\therefore OM^2 = 225 - 144 = 81\]
\[\therefore OM = 9\] cm
Thus, the distance of AB from the centre of the circle is 9 cm.

iv. Let \(J \equiv (x_1, y_1)\) and \(L \equiv (x_2, y_2)\)
Here, \(x_1 = -8, y_1 = -4, x_2 = 1, y_2 = 2, m = 1, n = 2\)
Let \(P \equiv (x, y)\) divides JL externally in the given ratio.
Then, by section formula for external division, we have
\[x = \frac{mx_2 - nx_1}{m - n} = \frac{1(1) - 2(-8)}{1 - 2} = \frac{1 + 16}{-1} = -17\]
\[y = \frac{my_2 - ny_1}{m - n} = \frac{1(2) - 2(-4)}{1 - 2} = \frac{2 + 8}{-1} = -10\]
Thus, the co-ordinates of the point \(P \equiv (-17, -10)\).
v. \cot 45^\circ = 1, \sec 60^\circ = 2, \cosec 30^\circ = 2 \text{ and } \cot 90^\circ = 0\\
\therefore 4\cot^2 45^\circ - \sec^2 60^\circ + \cosec^2 30^\circ + \cot 90^\circ\\
= 4(1)^2 - (2)^2 + (2)^2 + 0\\
= 4 - 4 + 4\\
= 4\\

vi. \text{Radius of the circle } = r = 28 \text{ cm}\\
\therefore \text{Diameter of the circle } = d = 2r = 2 \times 28 = 56 \text{ cm}\\

\text{Perimeter of the semicircle } = \pi r + d\\
= \frac{22}{7} \times 28 + 56\\
= 22 \times 4 + 56\\
= 88 + 56\\
= 144 \text{ cm}\\

2.\\

i. \text{Measure of the given angle } = \frac{3}{5} \times \text{right angle}\\
= \frac{3}{5} \times 90^\circ\\
= 3 \times 18^\circ\\
= 54^\circ\\

\text{Thus, the measure of its supplementary angle } = 180^\circ - 54^\circ = 126^\circ\\

ii. \Delta CAB \sim \Delta FDE\\
\therefore \frac{CA}{FD} = \frac{AB}{DE} = \frac{CB}{FE} \quad \text{(Corresponding sides are proportional)}\\
\therefore \frac{13}{n} = \frac{12}{m} = \frac{5}{2}\\
\therefore \frac{12}{m} = \frac{5}{2}\\
\therefore m = \frac{12 \times 2}{5} = \frac{24}{5}\\
\therefore m = 4.8\\
Also, \frac{13}{n} = \frac{5}{2}\\
\therefore \frac{n}{13} = \frac{26}{5}\\
\therefore n = 5.2
iii. In $\triangle ABC$, $\angle A + \angle B + \angle C = 180^\circ$

$\therefore 40^\circ + 80^\circ + \angle C = 180^\circ$

$\therefore \angle C = 180^\circ - 120^\circ = 60^\circ$

The descending order of the measures of the angles is $\angle B > \angle C > \angle A$.

$\therefore$ side $AC >$ side $AB >$ side $BC$ ...(Sides opposite to the angles)

Thus, the shortest side is side $BC$ and the longest side is side $AC$.

iv. $BD = 3DC$ ....(given)

Let $DC = x$

Then, $BD = 3x$ ....(1)

Now, $BC = BD + DC = 3x + x = 4x$

$\therefore BC^2 = 16x^2$ ....(2)

In right-angled $\triangle ADB$, by Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ ....(3)

Similarly, in right-angled $\triangle ADC$,

$AC^2 = AD^2 + DC^2$

$\therefore AD^2 = AC^2 - DC^2$ ....(4)

From (3) and (4), we have

$AB^2 = AC^2 - DC^2 + BD^2$

$= AC^2 - x^2 + (3x)^2$

$= AC^2 - x^2 + 9x^2$

$= AC^2 + 8x^2$

$= AC^2 + \frac{1}{2} \times 16x^2$ ....(5)

$\therefore AB^2 = AC^2 + \frac{1}{2}BC^2$ .... [From (5) and (2)]

v. Let the length of one side of the parallelogram be $x$ cm.

Then, the length of other side is $(25 + x)$ cm.

Perimeter of the parallelogram is 150 cm.

$\therefore x + (25 + x) + x + (25 + x) = 150$

$\therefore 4x + 50 = 150$

$\therefore 4x = 100$

$\therefore x = 25$ cm

$\therefore 25 + x = 25 + 25 = 50$ cm

Thus, the lengths of the sides of the parallelogram are 25 cm, 50 cm, 25 cm and 50 cm.
vi. \[
\frac{\tan^2 60^\circ + 4 \cos^2 45^\circ + \sec^2 30^\circ + 5 \cos 90^\circ}{\csc 30^\circ + \sec 60^\circ - \cot 30^\circ}
\]
\[=
\frac{(\sqrt{3})^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{3}}\right)^2 + 5 \times 0}{2 + 2 - (\sqrt{3})^2}
\]
\[=
\frac{3 + \frac{4 \times 1}{2} + \frac{4}{3} + 0}{4 - 3}
\]
\[=
\frac{3 + \frac{4}{3}}{1}
\]
\[=
\frac{9 + 6 + 4}{3}
\]
\[=
\frac{19}{3}
\]

3.

i. PQ is a straight line.

\[\therefore a + b = 180^\circ \quad \text{...(Linear pair of angles)}(1)\]

Now, \(a - b = 80^\circ \quad \text{...(given)}(2)\)

Adding equation (1) and (2), we get

\[a + b + a - b = 180^\circ + 80^\circ\]

\[\therefore 2a = 260^\circ\]

\[\therefore a = 130^\circ\]

Substituting \(a = 130^\circ\) in equation (1), we get

\[130^\circ + b = 180^\circ\]

\[\therefore b = 180^\circ - 130^\circ\]

\[\therefore b = 50^\circ\]

\[\therefore \angle POR = 130^\circ \text{ and } \angle ROQ = 50^\circ\]

ii.

(a) In \(\triangle ABC\) and \(\triangle CDE\),

\[m\angle B = m\angle D = 90^\circ\]

hypotenuse \(AC \cong \text{hypotenuse CE} \quad \text{...(Given)}\)

seg \(BC \cong \text{seg ED} \quad \text{...(Given)}\)

\[\therefore \triangle ABC \cong \triangle CDE \quad \text{...(Hypotenuse-side theorem)}\]

(b) As \(\triangle ABC \cong \triangle CDE\)

\[\therefore \angle BAC \cong \angle DCE \quad \text{...(1)(c.a.c.t.)}\]
(c) In $\triangle ABC$,
\[ \angle BAC + \angle ACB = 90^\circ \quad (2) \text{(Acute angles of a right-angled triangle)} \]

From (1) and (2),
\[ \angle DCE + \angle ACB = 90^\circ \quad \text{....(3)} \]

Now, \[ \angle ACB + \angle ACE + \angle DCE = 180^\circ \]
\[ \therefore \angle ACB + \angle DCE + \angle ACE = 180^\circ \]
\[ \therefore 90^\circ + \angle ACE = 180^\circ \quad \text{....[From (3)]} \]
\[ \therefore \angle ACE = 90^\circ \]

iii. $\triangle ABC$ is an isosceles triangle. \quad \text{....(given)}

\[ \therefore AB = AC \]

Now, D, E and F are the mid-points of sides AB, AC and BC respectively.

Then, by midpoint theorem, we have

\[ EF = \frac{1}{2} AB \text{ and } DF = \frac{1}{2} AC \]

Since, \[ AB = AC \]
\[ \frac{1}{2} AB = \frac{1}{2} AC \]
\[ \therefore EF = DF \]

Now, in $\triangle DEF$, EF = DF

Hence, DEF is an isosceles triangle.

iv. \[ A \equiv (-3, 0) \text{ and } B \equiv (3, 0) \]

Let \[ C \equiv (x, y) \]

Since $\triangle ABC$ is an equilateral triangle, we have

\[ AB = BC = AC \]
\[ \therefore AB^2 = BC^2 = AC^2 \]

Consider, \[ AB^2 = BC^2 \]
\[ \therefore [3 - (-3)]^2 + (0)^2 = (x - 3)^2 + (y - 0)^2 \]
\[ \therefore 36 = x^2 - 6x + 9 + y^2 \]
\[ \therefore x^2 - 6x + y^2 = 27 \quad \text{....(1)} \]

Consider, \[ BC^2 = AC^2 \]
\[ \therefore (x - 3)^2 + (y - 0)^2 = [x - (-3)]^2 + (y - 0)^2 \]
\[ \therefore x^2 - 6x + 9 + y^2 = (x + 3)^2 + y^2 \]
\[ \therefore x^2 - 6x + 9 + y^2 = x^2 + 6x + 9 + y^2 \]
\[ \therefore 12x = 0 \]
\[ \therefore x = 0 \]
Substituting $x = 0$ in equation (1), we get

$$(0)^2 - 6(0) + y^2 = 27$$

$\therefore y^2 = 27$

$\therefore y = \pm 3\sqrt{3}$

Thus, the coordinates of $C$ are $(0, 3\sqrt{3})$ or $(0, -3\sqrt{3})$.

v. In $\triangle ABC$, $\angle B = 90^\circ$, $\angle ACB = x$

$\therefore \angle BAC = 90^\circ - x$

In $\triangle ADC$, $\angle D = 90^\circ$, $\angle ACD = y$

$\therefore \angle DAC = 90^\circ - y$

$$\tan x = \frac{AB}{BC}$$

$$\cot(90^\circ - y) = \cot \angle DAC = \frac{AD}{DC}$$

$$\sec y = \frac{AC}{CD}$$

$$\sin(90^\circ - x) = \sin \angle BAC = \frac{BC}{AC}$$

$$\csc(90^\circ - y) = \csc \angle DAC = \frac{AC}{DC}$$

$$\cos(90^\circ - x) = \cos \angle BAC = \frac{AB}{AC}$$

4.

i. Let $\triangle PQR$ be an isosceles triangle.

(a) Let $PQ = PR = x$ cm \hspace{1cm} \ldots(1)

$QR = 1.5x$ \hspace{1cm} \ldots(2)$

Now, perimeter of $\triangle PQR = 42$ cm \hspace{1cm} \ldots(given)

$\therefore PQ + PR + QR = 42$

$\therefore x + x + 1.5x = 42$ \hspace{1cm} [From (1) and (2)]

$\therefore 3.5x = 42$

$\therefore x = 12$

$\therefore PQ = PR = 12$ cm

$QR = 1.5x = 1.5 \times 12 = 18$ cm

Thus, the length of congruent sides of a triangle, $PQ = PR = 12$ cm.
(b) In an isosceles triangle, the perpendicular to the base bisects the base.

Let PS \perp QR.

Le PS = h

\[ QS = \frac{1}{2} \times QR = \frac{1}{2} \times 18 = 9 \text{ cm} \]

In \( \triangle PQS \), using Pythagoras theorem,

\[ PQ^2 = QS^2 + PS^2 \]

\[ \therefore (12)^2 = (9)^2 + h^2 \]

\[ \therefore 144 = 81 + h^2 \]

\[ \therefore h^2 = 144 - 81 = 63 \]

\[ \therefore h = 3\sqrt{7} \text{ cm} \]

Thus, the height of a triangle, PS = 3\sqrt{7} \text{ cm}.

(c) Area of \( \triangle PQR = \frac{1}{2} \times \text{base} \times \text{height} \)

\[ \therefore A(\triangle PQR) = \frac{1}{2} \times QR \times PS = \frac{1}{2} \times 18 \times 3\sqrt{7} = 27\sqrt{7} \text{ cm}^2 \]

Thus, the area of a triangle is 27\sqrt{7} \text{ cm}^2.

ii. Let the measure of \( \angle D \) be \( x^\circ \).

Then, \( 4\angle S = 3\angle D = 3x^\circ \)

\[ \therefore \angle S = \frac{3x^\circ}{4} \]

Also, \( 6\angle R = 3\angle D = 3x^\circ \)

\[ \therefore \angle R = \frac{3x^\circ}{6} = \frac{x^\circ}{2} \]

Now, in \( \triangle DSR \),

\[ \angle D + \angle S + \angle R = 180^\circ \] \ldots (Angle sum property of a triangle)

\[ \therefore x^\circ + \frac{3x^\circ}{4} + \frac{x^\circ}{2} = 180^\circ \]

\[ \therefore 4x^\circ + 3x^\circ + 2x^\circ = 180^\circ \times 4 \]

\[ \therefore 9x^\circ = 180^\circ \times 4 \]

\[ \therefore x^\circ = \frac{180^\circ \times 4}{9} \]

\[ \therefore x^\circ = 80^\circ \]

\[ \therefore \angle S = \frac{3x^\circ}{4} = \frac{3 \times 80^\circ}{4} = 3 \times 20^\circ = 60^\circ \]

\[ \angle R = \frac{x^\circ}{2} = \frac{80^\circ}{2} = 40^\circ \]

\[ \therefore \angle D = 80^\circ, \angle S = 60^\circ \text{ and } \angle R = 40^\circ \]
iii. Steps of construction:

1. Draw a line \( l \), take any point \( M \) outside it.

2. Taking \( M \) as the centre and any radius, draw two arcs of circle on line \( l \).

3. Name the points of intersection of the arcs and line \( l \) as \( P \) and \( Q \) respectively.

4. Taking \( P \) and \( Q \) as the centres, and radius more than half of \( PQ \), draw arcs of circle above and below line \( l \).

5. Name the points of intersection as \( E \) and \( D \).

6. Draw line \( n \perp l \) by joining the points \( E \) and \( D \).

7. Taking \( M \) as the centre and any radius, draw arcs of circle above and below point \( M \) on line \( n \). Name the points of intersection as \( R \) and \( S \).

8. Taking \( R \) and \( S \) as the centres and radius more than half of \( RS \), draw arcs of circle on both sides of line \( n \). Name the points of intersection as \( A \) and \( B \).

9. Draw line \( m \perp n \) by joining the points \( A \) and \( B \).

Now, line \( l \perp n \) and line \( m \perp n \).

Hence, line \( m \) and line \( l \) are parallel to each other.
5.

i. \( \text{l}(AB) + \text{l}(BC) = \text{l}(AC) \)  \( \text{A-B-C} \)
\[ \therefore \text{l}(AB) + 5 = 8 \]
\[ \therefore \text{l}(AB) = 3 \text{ units} \quad \ldots(1) \]
seg AC ≅ seg BD \( \ldots \) \( \text{given} \)
\[ \therefore \text{l}(BD) = 8 \]
\( \text{l}(BC) + \text{l}(CD) = \text{l}(BD) \)  \( \text{B-C-D} \)
\[ \therefore 5 + \text{l}(CD) = 8 \]
\[ \therefore \text{l}(CD) = 3 \text{ units} \quad \ldots(2) \]
seg BD ≅ seg CE \( \ldots \) \( \text{given} \)
\[ \therefore \text{l}(CE) = 8 \]
\( \text{l}(CD) + \text{l}(DE) = \text{l}(CE) \)  \( \text{C-D-E} \)
\[ \therefore 3 + \text{l}(DE) = 8 \]
\[ \therefore \text{l}(DE) = 5 \text{ units} \quad \ldots(3) \]

(a) \( \text{l}(BC) = \text{l}(DE) = 5 \text{ units} \quad \ldots[\text{From (3) and given that } \text{l}(BC) = 5] \)
\[ \therefore \text{seg BC} \cong \text{seg DE} \]
(b) \( \text{l}(AB) = \text{l}(CD) = 3 \text{ units} \quad \ldots[\text{From (1) and (2)}] \)
\[ \therefore \text{seg AB} \cong \text{seg CD} \]

ii. In \( \Delta ABM \) and \( \Delta PQN \),
\[ \angle AMB = \angle PNQ \quad \ldots(\text{each right angle}) \]
\( AB = PQ \quad \ldots(\text{given}) \)
\( AM = PN \quad \ldots(\text{given}) \)
\[ \therefore \Delta ABM \cong \Delta PQN \quad \ldots(\text{hypotenuse-side congruence of right triangles}) \]
\[ \therefore BM = QN \quad \ldots(1)(\text{c.p.c.t.}) \]
In \( \Delta AMC \) and \( \Delta PNR \),
\[ \angle AMC = \angle PNR \quad \ldots(\text{each right angle}) \]
\( AC = PR \quad \ldots(\text{given}) \)
\( AM = PN \quad \ldots(\text{given}) \)
\[ \therefore \Delta AMC = \Delta PNR \quad \ldots(\text{hypotenuse-side congruence of right triangles}) \]
\[ \therefore MC = NR \quad \ldots(2)(\text{c.p.c.t.}) \]
Adding (1) and (2), we get
\( BM + MC = QN + NR \)
\[ \therefore BC = QR \]
Now, in \( \Delta ABC \) and \( \Delta PQR \),
\[ AB = PQ \quad \text{...(given)} \]
\[ AC = PR \quad \text{...(given)} \]
\[ BC = QR \quad \text{...(proved above)} \]
\[ \therefore \Delta ABC \cong \Delta PQR \]

iii. Let \( \square PQRS \) be a rhombus with \( \angle PSR = 60^\circ \).

In \( \triangle PSR \),
\[ PS = SR \quad \text{...(sides of a rhombus)} \]
\[ \therefore \angle SPR = \angle SRP \quad \text{...(1)(angles opposite to equal sides are equal)} \]
Now, in \( \triangle PSR \),
\[ \angle PSR + \angle SPR + \angle SRP = 180^\circ \]
\[ \therefore 2\angle SPR + 60^\circ = 180^\circ \quad \text{....[From (1)]} \]
\[ \therefore 2\angle SPR = 120^\circ \]
\[ \therefore \angle SPR = 60^\circ = \angle SRP \]
\[ \therefore \triangle PSR \text{ is an equilateral triangle.} \]
Since in a parallelogram, the opposite angles are equal, we have
\[ \angle PQR = 60^\circ \]
In \( \triangle PQR \),
\[ PQ = QR \quad \text{...(sides of a rhombus)} \]
\[ \therefore \angle QPR = \angle QRP \quad \text{...(2)(angles opposite to equal sides are equal)} \]
Now, in \( \triangle PQR \),
\[ \angle PQR + \angle QPR + \angle QRP = 180^\circ \]
\[ \therefore 2\angle QPR + 60^\circ = 180^\circ \quad \text{....[From (2)]} \]
\[ \therefore 2\angle APR = 120^\circ \]
\[ \therefore \angle QPR = 60^\circ = \angle QRP \]
\[ \therefore \triangle PQR \text{ is an equilateral triangle.} \]
Thus, in a rhombus with an angle of 60\(^\circ\), the shorter diagonal divides the rhombus into two equilateral triangles.