Board Question Paper: March 2014
Mathematics and Statistics

Time: 3 Hours **Total Marks: 80**

Note:
i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Solution of L.P.P. should be written on graph paper only.
iv. Answers to both the sections should be written in the same answer book.
v. Answer to every new question must be written on a new page.

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following: **(6)[12]**

i. Which of the following represents direction cosines of the line?

(A) $0, \frac{1}{\sqrt{2}}, \frac{1}{2}$ (B) $0, \frac{-\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$

(C) $0, \frac{\sqrt{3}}{2}, \frac{1}{2}$ (D) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

ii. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and A (adj A) = KI, then the value of 'K' is _______.

(A) 2 (B) – 2
(C) 10 (D) – 10

iii. The general solution of the trigonometric equation $\tan^2\theta = 1$ is _______.

(A) $\theta = n\pi \pm \frac{\pi}{3}, n \in Z$ (B) $\theta = n\pi \pm \frac{\pi}{6}, n \in Z$

(C) $\theta = n\pi \pm \frac{\pi}{4}, n \in Z$ (D) $\theta = n\pi, n \in Z$

(B) Attempt any THREE of the following: **(6)**

i. If $\bar{a}$, $\bar{b}$, $\bar{c}$ are the position vectors of the points A, B, C respectively and $2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$, then find the ratio in which the point C divides the line segment AB.

ii. The cartesian equation of a line is $\frac{x-6}{2} = \frac{y+4}{7} = \frac{z-5}{3}$, find its vector equation.

iii. Equation of a plane is $\bar{r} \cdot \left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) = 8$. Find the length of the perpendicular from the origin to the plane.

iv. Find the acute angle between the lines whose direction ratios are 5, 12, – 13 and 3, – 4, 5.

v. Write the dual of the following statements:
a. $(p \vee q) \wedge T$
b. Madhuri has curly hair and brown eyes.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of k.

ii. Prove that three vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$ are coplanar, if and only if, there exists a non-zero linear combination $x\bar{a} + y\bar{b} + z\bar{c} = \bar{0}$.

iii. Using truth table, prove that
$\sim p \wedge q \equiv (p \vee q) \wedge \sim p$.

(B) Attempt any TWO of the following: **(8)**

i. In any ΔABC, with usual notations, prove that
$b^2 = c^2 + a^2 - 2ca \cos B$.

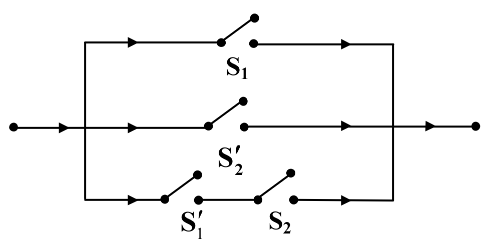
ii. Show that the equation $x^2 - 6xy + 5y^2 + 10x - 14y + 9 = 0$ represents a pair of lines. Find the acute angle between them. Also find the point of intersection of the lines.

iii. Express the following equations in the matrix form and solve them by the method of reduction:
$2x - y + z = 1$, $x + 2y + 3z = 8$, $3x + y - 4z = 1$.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. Prove that a homogeneous equation of degree two in x and y (i.e., $ax^2 + 2hxy + by^2 = 0$), represents a pair of lines through the origin if $h^2 - ab \geq 0$.

ii. Find the symbolic form of the following switching circuit, construct its switching table and interpret it.



iii. If A, B, C, D are (1, 1, 1), (2, 1, 3), (3, 2, 2), (3, 3, 4) respectively, then find the volume of the parallelopiped with AB, AC and AD as the concurrent edges.

(B) Attempt any TWO of the following: **(8)**

i. Find the equation of the plane passing through the line of intersection of the planes
$2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z-3}{5}$.

ii. Minimize: $Z = 6x + 4y$
Subject to: $3x + 2y \geq 12$,
$x + y \geq 5$,
$0 \leq x \leq 4$,
$0 \leq y \leq 4$.

iii. Show that: $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$.

SECTION – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following: (6)[12]

i. If $y = 1 - \cos\theta$, $x = 1 - \sin\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is

(A) -1 (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

ii. The integrating factor of linear differential equation

$\frac{dy}{dx} + y \sec x = \tan x$ is

(A) $\sec x - \tan x$ (B) $\sec x \cdot \tan x$

(C) $\sec x + \tan x$ (D) $\sec x \cdot \cot x$

iii. The equation of tangent to the curve $y = 3x^2 - x + 1$ at the point (1, 3) is

(A) $y = 5x + 2$ (B) $y = 5x - 2$

(C) $y = \frac{1}{5}x + 2$ (D) $y = \frac{1}{5}x - 2$

(B) Attempt any THREE of the following: (6)

i. Examine the continuity of the function

$$f(x) = \begin{cases} \sin x - \cos x, & \text{for } x \neq 0 \\ -1, & \text{for } x = 0 \end{cases}$$

at the point $x = 0$.

ii. Verify Rolle's theorem for the function

$f(x) = x^2 - 5x + 9$ on [1, 4].

iii. Evaluate: $\int \sec^n x \cdot \tan x \, dx$

iv. The probability mass function (p.m.f.) of X is given below:

$X = x$	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Find $E(X^2)$.

v. Given that $X \sim B(n = 10, p)$. If $E(X) = 8$, find the value of p.

Q.5. (A) Attempt any TWO of the following: (6)[14]

i. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then prove that $y = f[g(x)]$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

ii. Obtain the differential equation by eliminating the arbitrary constants A, B from the equation:

$y = A\cos(\log x) + B\sin(\log x)$

iii. Evaluate: $\int \frac{x^2}{(x^2 + 2)(2x^2 + 1)} dx$

(B) Attempt any TWO of the following: **(8)**

i. An open box is to be made out of a piece of a square cardboard of sides 18 cms by cutting off equal squares from the corners and turning up the sides. Find the maximum volume of the box.

ii. Prove that: $\int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(2a - x)dx$

iii. If the function f(x) is continuous in the interval [–2, 2], find the values of a and b, where

$$f(x) = \frac{\sin ax}{x} - 2, \quad \text{for } -2 \le x < 0$$
$$= 2x + 1, \quad \text{for } 0 \le x \le 1$$
$$= 2b\sqrt{x^2 + 3} - 1, \quad \text{for } 1 < x \le 2$$

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Solve the differential equation: $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$.

ii. A fair coin is tossed 8 times. Find the probability that it shows heads at least once.

iii. If $x^p y^q = (x + y)^{p+q}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

(B) Attempt any TWO of the following: **(8)**

i. Find the area of the sector of a circle bounded by the circle $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant.

ii. Prove that:

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$$

iii. A random variable X has the following probability distribution:

$X = x$	0	1	2	3	4	5	6
$P[X = x]$	k	3k	5k	7k	9k	11k	13k

a. Find k.
b. Find P(0 < X < 4).
c. Obtain cumulative distribution function (c.d.f.) of X.