Board Question Paper: October 2014
Mathematics and Statistics

Time: 3 Hours. **Total Marks: 80**

Note:

i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Solution of L.P.P. should be written on graph paper only.
iv. Answers to both the sections should be written in the same answer book.
v. Answer to every new question must be written on a new page.

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. If $\left[\bar{a} \quad \bar{b} \quad \bar{c}\right] \neq 0$ and $\bar{p} = \dfrac{\bar{b}\times\bar{c}}{\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]}$, $\bar{q} = \dfrac{\bar{c}\times\bar{a}}{\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]}$, $\bar{r} = \dfrac{\bar{a}\times\bar{b}}{\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]}$, then $\bar{a}.\bar{p} + \bar{b}.\bar{q} + \bar{c}.\bar{r}$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

ii. The inverse of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$ (D) $-\frac{1}{2}\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iii. Direction cosines of the line passing through the points A(–4, 2, 3) and B(1, 3, –2) are

(A) $\pm\frac{1}{\sqrt{51}}, \pm\frac{5}{\sqrt{51}}, \pm\frac{1}{\sqrt{51}}$ (B) $\pm\frac{5}{\sqrt{51}}, \pm\frac{1}{\sqrt{51}}, \pm\frac{-5}{\sqrt{51}}$

(C) $\pm 5, \pm 1, \pm 5$ (D) $\pm\sqrt{51}, \pm\sqrt{51}, \pm\sqrt{51}$

(B) Attempt any THREE of the following: **(6)**

i. Write truth values of the following statements:

a. $\sqrt{5}$ is an irrational number but $3 + \sqrt{5}$ is a complex number

b. $\exists n \in N$ such that $n + 5 > 10$

ii. If $\bar{c} = 3\bar{a} - 2\bar{b}$, then prove that $\left[\bar{a} \quad \bar{b} \quad \bar{c}\right] = 0$

iii. Find the vector equation of the plane which is at a distance of 5 units from the origin and which is normal to the vector $2\hat{i}+\hat{j}+2\hat{k}$.

iv. The Cartesian equations of the line are:
$3x + 1 = 6y - 2 = 1 - z$.
Find its equation in vector form.

v. Find the direction ratios of a vector perpendicular to the two lines whose direction ratios are $-2, 1, -1$ and $-3, -4, 1$.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. Using truth table, prove the following logical equivalence $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$.

ii. Find the joint equation of the pair of lines through the origin each of which is making an angle of 30° with the line $3x + 2y - 11 = 0$.

iii. Show that $2\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{24}{7}\right)$.

(B) Attempt any TWO of the following: **(8)**

i. Solve the following equations by the method of reduction:
$2x - y + z = 1,\ x + 2y + 3z = 8,\ 3x + y - 4z = 1$.

ii. Show that volume of parallelopiped with coterminous edges as $\bar{a}, \bar{b}$ and $\bar{c}$ is $\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right]$, hence find the volume of the parallelopiped whose coterminous edges are $\hat{i}+\hat{j},\ \hat{j}+\hat{k},\ \hat{k}+\hat{i}$.

iii. Solve the following LPP by using graphical method.
Maximize: $Z = 6x + 4y$,
Subject to $x \le 2,\ x + y \le 3,\ -2x + y \le 1,\ x \ge 0,\ y \ge 0$.
Also find maximum value of Z.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. In ΔABC with usual notations, prove that
$$2\left\{a\sin^2\frac{C}{2} + c\sin^2\frac{A}{2}\right\} = (a + c - b)$$

ii. If p : It is a day time, q : It is warm, write the compound statements in verbal form denoted by–

a. $p \wedge \sim q$

b. $\sim p \rightarrow q$

c. $q \leftrightarrow p$

iii. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other, then find the value of k.

(B) Attempt any TWO of the following: **(8)**

i. Parametric form of the equation of the plane is
$\bar{r} = \left(2\hat{i}+\hat{k}\right) + \lambda\hat{i} + \mu\left(\hat{i}+2\hat{j}-3\hat{k}\right)$.
λ and μ are parameters. Find normal to the plane and hence equation of the plane in normal form. Write its cartesian form.

ii. If the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines $2x^2 - 5xy + 3y^2 = 0$, then show that $100(h^2 - ab) = (a + b)^2$.

iii. Find the general solution of $\sin x \tan x = \tan x - \sin x + 1$.

SECTION – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following sub-questions: (6)[12]

i. The differential equation of the family of curves $y = c_1e^x + c_2e^{-x}$ is

(A) $\frac{d^2y}{dx^2} + y = 0$ (B) $\frac{d^2y}{dx^2} - y = 0$

(C) $\frac{d^2y}{dx^2} + 1 = 0$ (D) $\frac{d^2y}{dx^2} + 1 = 0$

ii. If X is a random variable with probability mass function

$P(x) = kx$, for $x = 1, 2, 3$

$= 0$, otherwise

then k = ……

(A) $\frac{1}{5}$ (B) $\frac{1^\circ}{4}$

(C) $\frac{1}{6}$ (D) $\frac{2}{3}$

iii. If $\sec\left(\frac{x+y}{x-y}\right) = a^2$, then $\frac{d^2y}{dx^2} = ….$

(A) y (B) x

(C) $\frac{y}{x}$ (D) 0

(B) Attempt any THREE of the following: (6)

i. If $y = \sin^{-1}(3x) + \sec^{-1}\left(\frac{1}{3x}\right)$, find $\frac{dy}{dx}$.

ii. Evaluate: $\int x \log x \, dx$.

iii. If $\int_0^h \frac{1}{2+8x^2} dx = \frac{\pi}{16}$, then find the value of h.

iv. The probability that a certain kind of component will survive a check test is 0.5. Find the probability that exactly two of the next four components tested will survive.

v. Find the area of the region bounded by the curve $y = \sin x$, the lines $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$ and X-axis.

Q.5. (A) Attempt any TWO of the following: (6)[14]

i. Examine the continuity of the following function at given point:

$f(x) = \frac{\log x - \log 8}{x - 8}$, for $x \neq 8$

$= 8$, for $x = 8$ at $x = 8$

ii. If $x = \phi(t)$ is a differentiable function of 't', then prove that

$\int f(x)dx = \int f[\phi(t)]\phi'(t)dt$.

iii. Solve $3e^x \tan y \, dx + (1 + e^x) \sec^2 y \, dy = 0$. Also, find the particular solution when $x = 0$ and $y = \pi$.

(B) Attempt any TWO of the following: **(8)**

i. A point source of light is hung 30 feet directly above a straight horizontal path on which a man of 6 feet in height is walking. How fast will the man's shadow lengthen and how fast will the tip of shadow move when he is walking away from the light at the rate of 100 ft/min.

ii. Evaluate: $\int \frac{\log x}{(1+\log x)^2} dx$

iii. If x = f(t), y = g(t) are differentiable functions of parameter 't' then prove that y is a differentiable function of 'x' and

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \frac{dx}{dt} \neq 0$$

Hence find $\frac{dy}{dx}$ if $x = a \cos t$, $y = a \sin t$ t.

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Show that the function defined by $f(x) = |\cos x|$ is continuous function.

ii. Solve the differential equation $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$.

iii. Given X ~ B(n, p). If n = 20, E(X) = 10,
find p, Var. (X) and S.D.(X).

(B) Attempt any TWO of the following: **(8)**

i. A bakerman sells 5 types of cakes. Profits due to the sale of each type of cake is respectively ` 3, ` 2.5, ` 2, ` 1.5, ` 1. The demands for these cakes are 10%, 5%, 25%, 45% and 15% respectively. What is the expected profit per cake?

ii. Verify Lagrange's mean value theorem for the function

$f(x) = x + \frac{1}{x}, x \in [1, 3]$

iii. Prove that: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Hence evaluate: $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} dx$.