BOARD QUESTION PAPER : MARCH 2016
MATHEMATICS AND STATISTICS

Total Marks: 80

Time: 3 Hours

Note:

i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Graph of L.P.P. should be drawn on graph paper only.
iv. Answer to every new question must be written on a new page.
v. Answers to both the sections should be written in the same answer book.
vi. Use of logarithmic table is allowed.

SECTION – I

Q.1. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: (6) [12]

i. The negation of $p \wedge (q \rightarrow r)$ is

(A) $p \vee (\sim q \vee r)$ (B) $\sim p \wedge (q \rightarrow r)$
(C) $\sim p \wedge (\sim q \rightarrow \sim r)$ (D) $\sim p \vee (q \wedge \sim r)$

ii. If $\sin^{-1}(1 - x) - 2\sin^{-1}x = \frac{\pi}{2}$ then x is

(A) $-\frac{1}{2}$ (B) 1
(C) 0 (D) $\frac{1}{2}$

iii. The joint equation of the pair of lines passing through (2, 3) and parallel to the coordinate axes is

(A) $xy - 3x - 2y + 6 = 0$ (B) $xy + 3x + 2y + 6 = 0$
(C) $xy = 0$ (D) $xy - 3x - 2y - 6 = 0$

(B) Attempt any THREE of the following: (6)

i. Find $(AB)^{-1}$ if $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -2 \end{bmatrix}$

ii. Find the vector equation of the plane passing through a point having position vector $3\hat{i} - 2\hat{j} + \hat{k}$ and perpendicular to the vector $4\hat{i} + 3\hat{j} + 2\hat{k}$.

iii. If $\bar{p} = \hat{i} - 2\hat{j} + \hat{k}$ and $\bar{q} = \hat{i} + 4\hat{j} - 2\hat{k}$ are position vector (P.V.) of points P and Q, find the position vector of the point R which divides segment PQ internally in the ratio 2:1.

iv. Find k, if one of the lines given by $6x^2 + kxy + y^2 = 0$ is $2x + y = 0$.

v. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are at right angle then find the value of k.

Q.2. (A) Attempt any TWO of the following: (6)[14]

i. Examine whether the following logical statement pattern is tautology, contradiction or contingency.
$[(p \rightarrow q) \wedge q] \rightarrow p$

ii. By vector method prove that the medians of a triangle are concurrent.

iii. Find the shortest distance between the lines $\bar{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$ where λ and μ are parameters.

(B) Attempt any TWO of the following: **(8)**

i. In ΔABC with the usual notations prove that

$$(a-b)^2\cos^2\left(\frac{C}{2}\right)+(a+b)^2\sin^2\left(\frac{C}{2}\right)=c^2.$$

ii. Minimize $z = 4x + 5y$ subject to $2x + y \geq 7$, $2x + 3y \leq 15$, $x \leq 3$, $x \geq 0$, $y \geq 0$. Solve using graphical method.

iii. The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is ` 60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is ` 90 whereas the cost of 6 dozen pencils, 2 dozen pens and 3 dozen erasers is ` 70. Find the cost of each item per dozen by using matrices.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. Find the volume of tetrahedron whose coterminus edges are $7\hat{i}+\hat{k}$, $2\hat{i}+5\hat{j}-3\hat{k}$ and $4\hat{i}+3\hat{j}+\hat{k}$.

ii. Without using truth table show that

$\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

iii. Show that every homogeneous equation of degree two in x and y, i.e., $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through origin if $h^2 - ab \geq 0$.

(B) Attempt any TWO of the following: **(8)**

i. If a line drawn from the point A(1, 2, 1) is perpendicular to the line joining P(1, 4, 6) and Q(5, 4, 4) then find the co-ordinates of the foot of the perpendicular.

ii. Find the vector equation of the plane passing through the points $\hat{i}+\hat{j}-2\hat{k}$, $\hat{i}+2\hat{j}+\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$. Hence find the cartesian equation of the plane.

iii. Find the general solution of $\sin x + \sin 3x + \sin 5x = 0$.

SECTION – II

Q.4. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions: **(6)[12]**

i. If the function

$f(x) = k + x$, for $x < 1$

$\quad = 4x + 3$, for $x \geq 1$

is continuous at $x = 1$ then k =

(A) 7 (B) 8 (C) 6 (D) –6

ii. The equation of tangent to the curve $y = x^2 + 4x + 1$ at $(-1, -2)$ is

(A) $2x - y = 0$ (B) $2x + y - 5 = 0$

(C) $2x - y - 1 = 0$ (D) $x + y - 1 = 0$

iii. Given that X ~ B(n = 10, p). If E(X) = 8 then the value of p is

(A) 0.6 (B) 0.7 (C) 0.8 (D) 0.4

(B) Attempt any THREE of the following: **(6)**

i. If $y = x^x$, find $\frac{dy}{dx}$.

ii. The displacement 's' of a moving particle at time 't' is given by $s = 5 + 20t - 2t^2$. Find its acceleration when the velocity is zero.

iii. Find the area bounded by the curve $y^2 = 4ax$, X-axis and the lines $x = 0$ and $x = a$.

iv. The probability distribution of a discrete random variable X is:

$X = x$	1	2	3	4	5
$P(X = x)$	k	2k	3k	4k	5k

Find $P(X \leq 4)$.

v. Evaluate: $\int \frac{\sin x}{\sqrt{36-\cos^2 x}} dx$

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x then prove that $y = f(g(x))$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

ii. The probability that a person who undergoes kidney operation will recover is 0.5. Find the probability that of the six patients who undergo similar operations.

a. None will recover.

b. Half of them will recover.

iii. Evaluate: $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

(B) Attempt any TWO of the following: **(8)**

i. Discuss the continuity of the following functions. If the function have a removable discontinuity, redefine the function so as to remove the discontinuity.

$$\left.\begin{aligned} f(x) &= \frac{4^x - e^x}{6^x - 1}, \text{ for } x \neq 0 \\ &= \log\left(\frac{2}{3}\right), \text{ for } x = 0 \end{aligned}\right\} \text{at } x = 0$$

ii. Prove that:

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + c$$

iii. A body is heated at 110°C and placed in air at 10°C. After 1 hour its temperature is 60°C. How much additional time is required for it to cool to 35°C?

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Prove that: $\int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(2a - x)dx$

ii. Evaluate: $\int \frac{1 + \log x}{x(2 + \log x)(3 + \log x)} dx$

iii. If $y = \cos^{-1}\left(2x\sqrt{1-x^2}\right)$, find $\frac{dy}{dx}$

(B) Attempt any TWO of the following: **(8)**

i. Solve the differential equation $\cos(x + y)dy = dx$
Hence find the particular solution for $x = 0$ and $y = 0$.

ii. A wire of length l is cut into two parts. One part is bent into a circle and other into a square. Show that the sum of areas of the circle and square is the least, if the radius of circle is half the side of the square.

iii. The following is the p.d.f. (Probability Density Function) of a continuous random variable X:

$$\begin{aligned} f(x) &= \frac{x}{32}, \quad 0 < x < 8 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

a. Find the expression for c.d.f. (Cumulative Distribution Function) of X.

b. Also find its value at $x = 0.5$ and 9.