Maharashtra State Board
Class X Mathematics - Geometry
Board Paper – 2016 Solution

1.

i. $\Delta DEF \sim \Delta MNK$ ...(given)

$\therefore \frac{A(\Delta DEF)}{A(\Delta MNK)} = \frac{DE^2}{MN^2}$ ....(Areas of similar triangles)

$\therefore \frac{A(\Delta DEF)}{A(\Delta MNK)} = \frac{5^2}{6^2} = \frac{25}{36}$

ii. ΔABC is 30°-60°-90° triangle.

BC is the side opposite to 30°.

$\therefore BC = \frac{1}{2} \times \text{hypotenuse} = \frac{1}{2} \times AC = \frac{1}{2} \times 18 = 9 \text{ cm}$

iii. m(arc PMQ) = 130° ....(given)

$\therefore \angle PQS = \frac{1}{2} \times m(\text{arc}\,PMQ) = \frac{1}{2} \times 130^\circ = 65^\circ$

iv. $\cos(-\theta) = \cos\theta$

$\therefore \cos(-60°) = \cos 60°$

$\therefore \cos 60^\circ = \frac{1}{2}$

v. Inclination of the line = θ = 30°

$\therefore \text{slope} = \tan 30° = \frac{1}{\sqrt{3}}$

Thus, the slope of the line is $\frac{1}{\sqrt{3}}$.

vi. E = 30, F = 12

Euler's formula:

F + V = E + 2

$\therefore$ 12 + V = 30 + 2

$\therefore$ 12 + V = 32

$\therefore$ V = 32 – 12 = 20

2.

i. Seg RS bisects $\angle PRQ$ ....(given)

$\therefore \frac{PR}{QR} = \frac{PS}{SQ}$ ....(angle bisector property)

$\therefore \frac{15}{QR} = \frac{6}{8}$

$\therefore QR = \frac{15 \times 8}{6} = 20$

ii. PA is a tangent segment and PBC is the secant.

$\therefore PB \times PC = PA^2$

$\therefore 10 \times PC = 15^2$

$\therefore PC = \frac{225}{10} = 22.5$

Now, $PB + BC = PC$ ...$(P - B - C)$

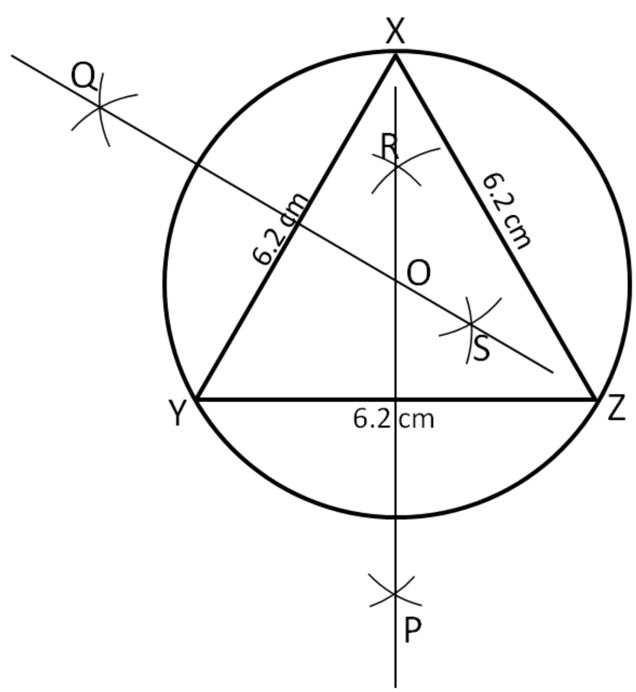
$\therefore 10 + BC = 22.5$

$\therefore BC = 22.5 - 10 = 12.5$

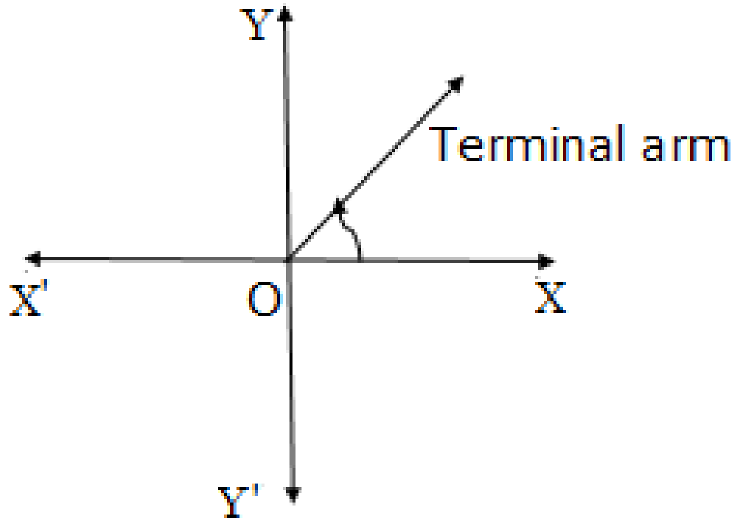
iii. Steps of construction:

1. Construct the equilateral ΔXYZ of side equal to 6.2 cm.
2. Draw the perpendicular bisectors PR and QS of sides $\overline{XY}$ and $\overline{YZ}$ respectively.
3. Mark the point of intersection as O.
4. Draw a circle with centre O and radius OX or OY or OZ.

   This is the required circumcircle.



iv. The initial arm rotates 25° in the anticlockwise direction.
The angle is positive and the measure of the angle is between 0° and 90°.
Thus, the terminal arm lies in quadrant I.



v. Length of an arc = 10 cm
Radius (r) = 5 cm

$$\text{Area of the sector} = \frac{r}{2} \times \text{length of an arc}$$
$$= \frac{5}{2} \times 10$$
$$= 25 \text{ cm}^2$$

Thus, the area of the sector is 25 cm^2.

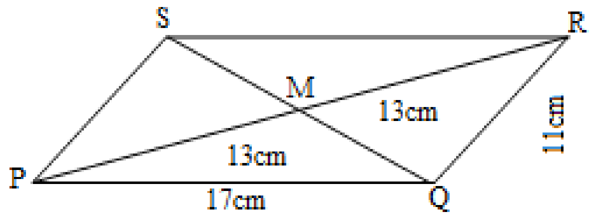
vi. Radius (r) of a sphere = 4.2 cm
Surface area of a sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (4.2)^2$$
$$= 221.76 \text{ cm}^2$$

Thus, the surface area of sphere is 221.76 cm^2.

3.

i. Let □PQRS be a parallelogram.



Then, PQ = 17 cm, QR = 11 cm and diagonal PR = 26 cm
The diagonals of a parallelogram bisect each other.
Point M is the point of intersection of diagonals PR and QS.

$\therefore PM = MR = \frac{1}{2}PR = \frac{1}{2} \times 26$

$\therefore PM = MR = 13 \text{ cm} \qquad ...(1)$

$QM = MS = \frac{1}{2}QS$

$\therefore QS = 2QM \qquad ...(2)$

In ΔPQR, QM is the median.

$PQ^2 + QR^2 = 2PM^2 + 2QM^2 \quad ...(\text{By Apollonius theorem})$

$(17)^2 + (11)^2 = 2(13)^2 + 2QM^2$

$\therefore 289 + 121 = 2(169) + 2QM^2$

$\therefore 410 = 2(169) + 2QM^2$

Diving by 2, we get

$205 = 169 + QM^2$

$\therefore QM^2 = 205 - 169 = 36$

$\therefore QM = 6$

$\therefore QS = 2QM = 2 \times 6 = 12 \text{ cm}$

Thus, the length of the other diagonal is 12 cm.

ii. Given: m(arc PCR) = 26°, m(arc QDS) = 48°
By Inscribed Angle Theorem,we get

(i) $\angle PQR = \frac{1}{2}m(\text{arc PCR}) = \frac{1}{2} \times 26^\circ = 13^\circ \; ...(1)$

(ii) $\angle SPQ = \frac{1}{2}m(\text{arc QDS}) = \frac{1}{2} \times 48^\circ = 24^\circ \; ...(2)$

(iii) In ΔAQR, by the Remote Interior Angle theorem,

$\angle RAQ + \angle AQR = \angle SRQ$

$\angle SRQ = \angle SPQ$ ...(Angles subtended by the same arc)

i.e. $\angle RAQ + \angle AQR = \angle SPQ$

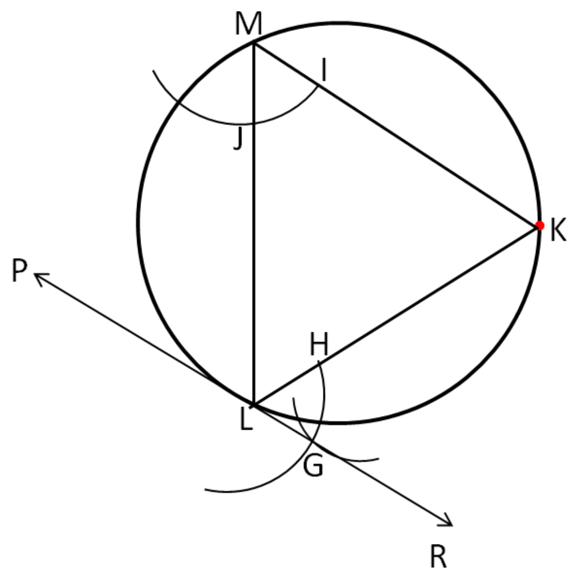
$\therefore m\angle RAQ = m\angle SPQ - m\angle AQR = 24^\circ - 13^\circ = 11^\circ$ ...[From (1) and (2)]

iii. Steps of construction:

1. Draw a circle of radius 3.5 cm. Take any point K on it.
2. Draw a chord KL through K. Take any point M on the major arc KL.
3. Join KM and ML.
4. Draw an arc of the same radius taking M and L as the centres. Taking H as the centre and radius equal to IJ, draw an arc intersecting the previous arc at G.

   $\therefore \angle KML = \angle KLR$

   Join LG and extend it on both the sides to draw PR which is the required tangent to the circle at K.



iv. Given that α is in quadrant IV, where x is positive and y is negative.

$\sec\alpha = \frac{r}{x} = \frac{2}{\sqrt{3}}$

Let $r = 2k$, then $x = \sqrt{3}k$

$r^2 = x^2 + y^2$

$\therefore (2k)^2 = \left(\sqrt{3}k\right)^2 + y^2$

$\therefore y^2 = 4k^2 - 3k^2 = k^2$

$\therefore y = \pm k$

Now, y is negative.

$\therefore y = -k$

$\operatorname{cosec}\alpha = \frac{r}{y} = \frac{2k}{-k} = -2.$

Substituting the value of $\operatorname{cosec}\alpha$, we get

$\frac{1 - \operatorname{cosec}\alpha}{1 + \operatorname{cosec}\alpha} = \frac{1 - (-2)}{1 + (-2)} = \frac{1 + 2}{1 - 2} = \frac{3}{-1}$

$\therefore \frac{1 - \operatorname{cosec}\alpha}{1 + \operatorname{cosec}\alpha} = -3$

v. Let (2, 3) ≡ (x_1, y_1) and (4, 7) ≡ (x_2, y_2).

The equation of a line passing through a pair of points is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\therefore \frac{y-3}{7-3} = \frac{x-2}{4-2}$$

$$\therefore \frac{y-3}{4} = \frac{x-2}{2}$$

$\therefore y - 3 = 2(x - 2)$ [Multiplying both the sides by (-4)]

$\therefore y - 3 = 2x - 4$

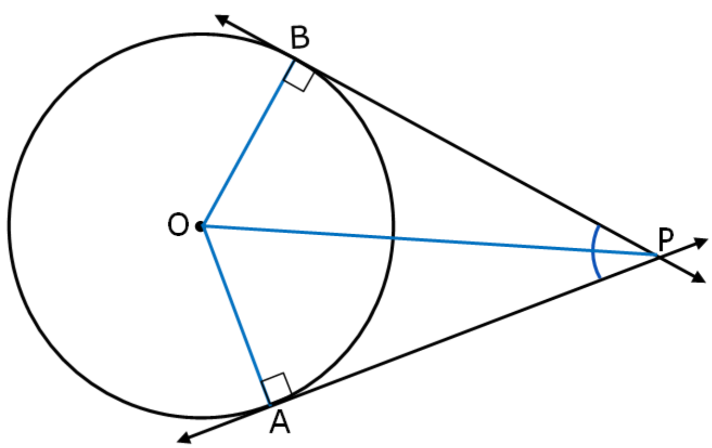
$\therefore y = 2x - 4 + 3$

$\therefore y = 2x - 1$

The equation of the line is $y = 2x - 1$

4.

i. Given: A circle with centre O and an external point P are given.
AP and BP are the two tangents drawn from an external point P.



To prove: AP = BP

Construction: Draw seg OA, seg OB and seg OP.

Proof: In ΔOBP and ΔOAP,

OA = OB … (Radii of the same circle)

OP = OP … (Side common to both the triangles)

∠OAP = ∠OBP = 90° … (tangent is perpendicular to the radius at the point of contact)

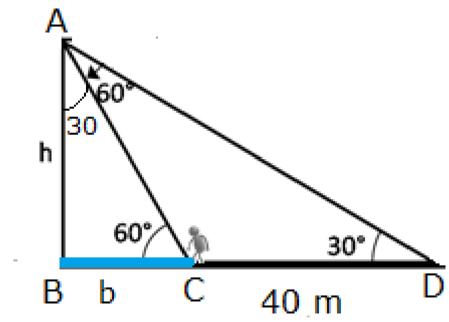
ΔOBP ≅ ΔOAP … (By R.H.S)

∴ AP = BP … (corresponding sides of congruent triangles)

Thus, the lengths of two tangent segments to a circle drawn from an external point are equal.

ii. Let AB = height of the tower = h metres

In the right angled ΔABC and right angled ΔABD,



$\tan 60° = \frac{h}{BC} = \sqrt{3} \quad \Rightarrow BC = \frac{h}{\sqrt{3}}$

$\tan 30° = \frac{h}{BD} = \frac{1}{\sqrt{3}} \quad \Rightarrow BD = h\sqrt{3}$

Now, BD – BC = 40

$\therefore h\sqrt{3} - \frac{h}{\sqrt{3}} = 40$

$\therefore \frac{3h - h}{\sqrt{3}} = 40$

$\therefore 2h = 40\sqrt{3}$

$\therefore h = 20\sqrt{3}$ metres

In right angled $\triangle ABC$,

$\tan 30° = \frac{1}{\sqrt{3}}$

$\therefore \frac{BC}{h} = \frac{1}{\sqrt{3}}$

$\therefore \frac{BC}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\therefore BC = b = 20$ m

Width of the river = BC = 20 m

Thus, the height of the tree is $20\sqrt{3}$ metres and width of the river is 20 metres.

iii. AD is the median

$\therefore CD = BD$ ...($\because$ D is the midpoint of BC)

Coordinates of D can be found by using section formula.

Let (x, y) be coordinates of the centre of the circle.

$(x, y) \equiv \left(\frac{1-3}{1+1}, \frac{-8-2}{1+1}\right) \equiv (-1, -5)$

$\therefore$ Coordinates of point D are $(-1, -5)$.

Let m be the slope of AD.

Coordinates of A(5, 4) and D(−1, −5)

$\therefore m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-5)}{5-(-1)} = \frac{9}{6} = \frac{3}{2}$

Equation of line is $y = mx + c$, where c is the y intercept.

$\therefore 4 = \frac{3 \times 5}{2} + c$ ...(Substituting the coordinates of A)

$\therefore c = \frac{-7}{2}$

$\therefore$ Equation of line of AD is $y = \frac{3x}{2} - \frac{7}{2}$

$\therefore 2y = 3x - 7$ is equation of the median AD.

The coordinates of $A(5,4)$ and $C(1,-8)$

Slope of AC $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8-4}{1-5} = \frac{-12}{-4} = 3$

Substituting the coordinates of $A(5,4)$ in the equation $y = mx + c$

$\therefore 4 = (3 \times 5) + c$

$\therefore c = -11$

$\therefore$ Line parallel to AC and passing through $B(-3,-2)$ has slope $= 3$

Substituting the coordinates of $B(-3,-2)$ in the equation $y = mx + c$

$\therefore -2 = (3 \times -3) + c$

$\therefore c = 7$

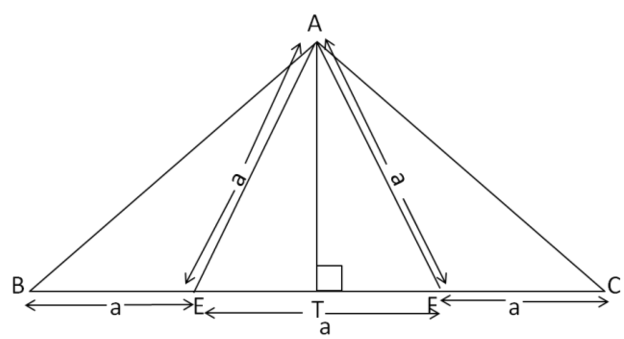
$\therefore$ Equation of line parallel to AC and passing through $B(-3,-2)$ is $y = 3x + 7$.

5.

i. Given: AE = EF = AF = BE = CF,

$AT \perp EF$

ΔAEF is equilateral triangle.



$\therefore ET = TF = \frac{a}{2}$

$\therefore BT = CT = a + \frac{a}{2}$ ....(1)

In right triangles, ΔATB and ΔATC,

AT = AT ... (Side common to both triangles)

∠ATB = ∠ATC ... (Right angles)

BT = CT .... (from 1)

∴ ΔATB ≅ ΔATC .....(by SAS)

∴ AB = AC

In ΔAEF, AE = AF = EF ...(Given)

∴ ΔAEF is an equilateral triangle.

$AT = \frac{\sqrt{3}}{2}a$ ...(Altitude of equilateral triangle)

In $\Delta ATB, (AB)^2 = (AT)^2 + (BT)^2$

$\therefore (AB)^2 = \left(\frac{\sqrt{3}}{2}a\right)^2 + \left(a + \frac{a}{2}\right)^2 = \frac{3a^2}{4} + \frac{9a^2}{4} = \frac{12a^2}{4} = 3a^2$

$\therefore AB = \sqrt{3}a$

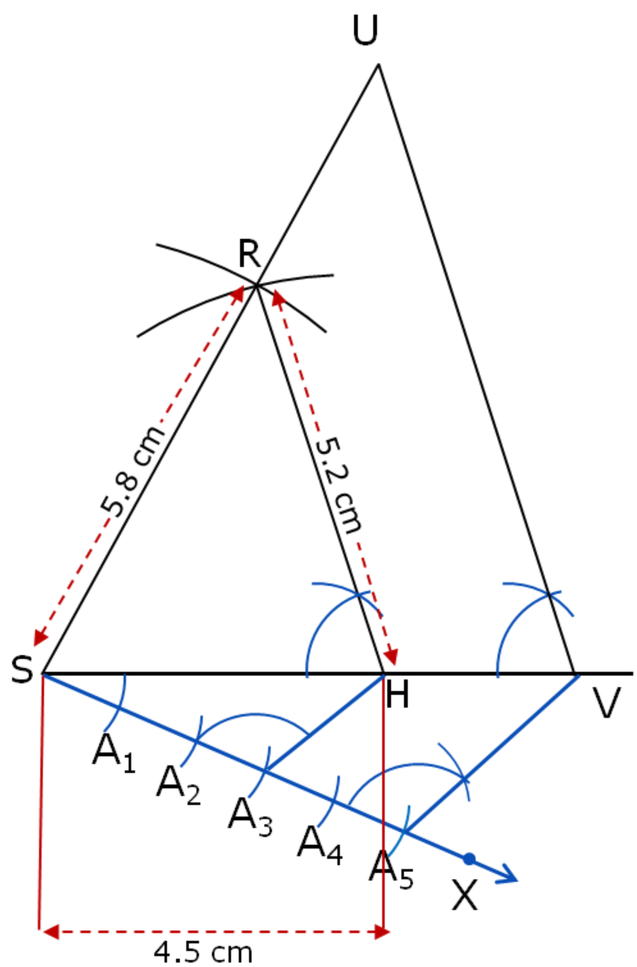
i.e., $AB = AC = \sqrt{3}a$

ii. Steps of construction:

1. Construct the Δ SHR with the given measurements. For this draw SH of length 4.5 cm.
2. Taking S as the centre and radius equal to 5.8 cm draw an arc above SH.
3. Taking H as the centre and radius equal to 5.2 cm draw an arc to intersect the previous arc. Name the point of intersection as R.
4. Join SR and HR. Δ SHR with the given measurements is constructed. Extend SH and SR further on the right side.
5. Draw any ray SX making an acute angle with SH on the side opposite to the vertex R.
6. Locate 5 points. (the ratio of old triangle to new triangle is $\frac{3}{5}$ and $5 > 3$)

   Locate A_1, A_2, A_3, A_4 and A_5 on AX so that $SA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$.
7. Join A_3H and draw a line through A_5 parallel to A_3H, intersecting the extended part of SH at V.
8. Draw a line VU through V parallel to HR.

   Δ SVU is the required triangle.



iii. Diameter of a pipe = 20 mm ....(given)

Radius of the pipe $= \frac{20}{2}$mm $= 10$ mm $= 1$ cm

Speed of water $= 15$ m/min $= 1500$ cm/min

Volume of cylinder $= \pi r^2 h$

$\therefore$ Volume of water that flows in pipe in 1 minute

$= \frac{22}{7} \times 1^2 \times 1500$

$= \frac{33000}{7}$ cm^3

Radius of conical vessel $= \frac{40}{2} = 20$ cm,

Depth $= 45$ cm ...(Given)

Capacity of the conical vessel $= \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 20 \times 20 \times 45$

$= \frac{396000}{21}$ cm^3

$\therefore$ Time required to fill the vessel

$= \frac{\text{Capacity of the vessel}}{\text{Volume of water flowing per minute}}$

$= \frac{\frac{396000}{21}}{\frac{33000}{7}}$

$= \frac{396}{99}$

$= 4$ minutes

Thus, the time required to fill the conical vessel is 4 minutes.