Maharashtra Board
Class X Mathematics - Geometry
Board Paper – 2014 Solution

Time: 2 hours **Total Marks: 40**

Note: - (1) All questions are compulsory.
(2) Use of calculator is not allowed.

1.

i. Ratio of the areas of two triangles with common or equal heights is equal to the ratio of their corresponding bases.

$$\frac{A(\Delta TRP)}{A(\Delta TPK)} = \frac{RP}{PK} = \frac{3}{2}$$

ii. If two circles touch internally, then distance between their centers is the difference of their radii.
The distance between their centers = 8 – 3 = 5 cm

iii. We know that, for any angle $\theta, \sin(-\theta) = -\sin\theta$

$$\therefore \sin(-60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$$

iv. $A \equiv (2,3) \equiv (x_1, y_1)$ and $B \equiv (4,7) \equiv (x_2, y_2)$

Slope of line AB = $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{7-3}{4-2} = \frac{4}{2} = 2$

v. Given radius of a circle r =7 cm.

Circumference of the circle = $2\pi r = 2 \times \frac{22}{7} \times 7 = 44 cm.$

vi. The sides of the triangle are 6 cm, 8 cm and 10 cm.
The longest side is 10 cm.

$(10)^2 = 100$ ....(i)

Now, the sum of the squares of the other two sides will be,

$(6)^2 + (8)^2 = 36 + 64$ $= 100$ ....(ii)

$(10)^2 = (6)^2 + (8)^2$ ..... from (i) and (ii)

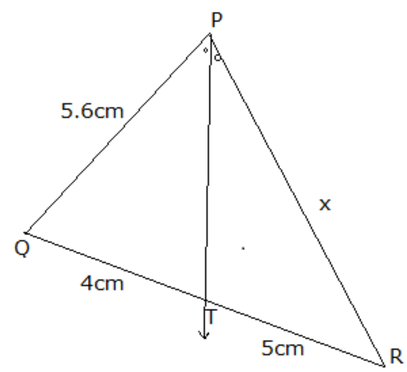
By the converse of Pythagoras theorem:
The given sides form a right angled triangle.

2.

i. Given: ray PT is bisector of $\angle QPR$.

PQ = 5.6 cm, QT = 4 cm and TR = 5 cm.

In a triangle, the angle bisector divides the side opposite to the angle in the ratio of the remaining sides.



$$\frac{QT}{TR} = \frac{PQ}{PR}$$

$$\frac{4}{5} = \frac{5.6}{x}$$

$$x = \frac{5.6 \times 5}{4}$$

$$x = \frac{28}{4} = 7$$

$$PR = 7\ cm$$

ii. Given: $\angle MPN = 40^\circ$

The line perpendicular to a radius of a circle at its outer end is a tangent to the circle.

$\angle PMQ = 90^\circ \quad and \quad \angle QNP = 90^\circ$

The sum of the measures of the angles of a quadrilateral is 360°.

$$\angle MPN + \angle PMQ + \angle QNP + \angle MQN = 360^\circ$$

$$40^\circ + 90^\circ + 90^\circ + \angle MQN = 360^\circ$$

$$220^\circ + \angle MQN = 360^\circ$$

$$\angle MQN = 360^\circ - 220^\circ$$

$$\angle MQN = 140^\circ$$

iii. Given equation is 2x – 3y – 4 = 0

$$-3y = -2x + 4$$

$$y = \frac{2}{3}x - \frac{4}{3}$$

Comparing given equation with y = mx + c, we get

$slope = m = \frac{2}{3}$ and y-intercept = c = $-\frac{4}{3}$

iv. Given $\cos\theta = \frac{1}{\sqrt{2}}$

$$\cos^2\theta = (\frac{1}{\sqrt{2}})^2 = \frac{1}{2}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{1}{2} + \sin^2\theta = 1$$

$$\sin^2\theta = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\sin\theta = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

v. $(4,-3) \equiv (x_1, y_1)$ is a point on the line AB and slope = m = -2

Equation of line AB in point slope form is

$$y - y_1 = m(x - x_1)$$

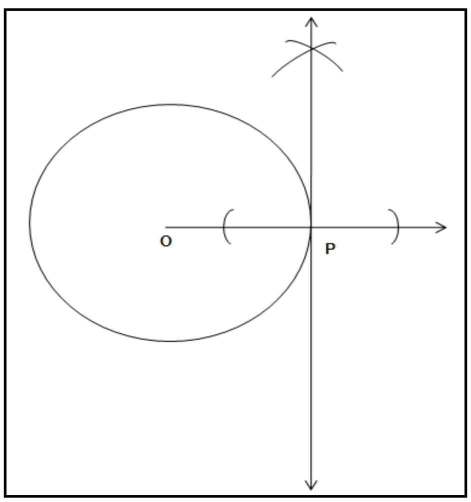
$$y - (-3) = -2(x - 4)$$

$$y + 3 = -2x + 8$$

$$2x + y - 5 = 0$$

The equation of the line AB is $2x + y - 5 = 0$.

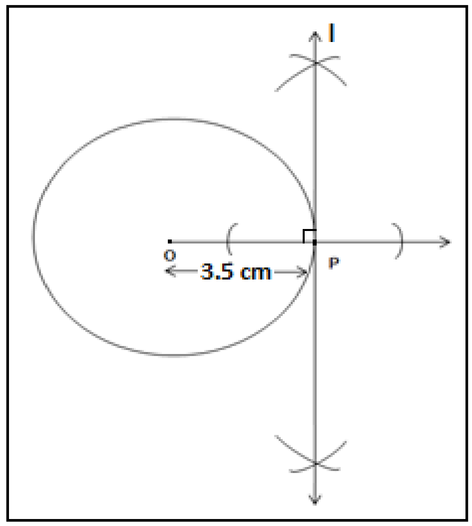
vi. Construction:



(Analytical figure)

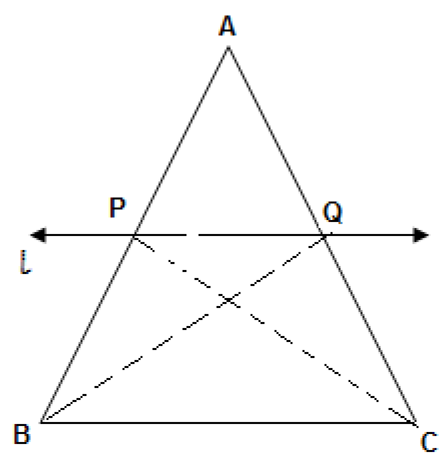
Steps of construction:

1. Draw a circle with centre O and radius 3.5cm.
2. Take any point 'P' on the circle and draw ray OP.
3. Draw a line perpendicular to ray OP at the point P. Name that line as 'l' which is tangent to the circle.



3.

i. Construction: Join seg PC and seg BQ.



Given: In ΔABC, line $l \parallel BC$

Line l intersects side AB and side AC in points P and Q, respectively.

A-P-B and A-Q-C

To prove: $\dfrac{AP}{BP} = \dfrac{AQ}{QC}$

Construction: Join seg BQ and seg CP

In ΔAPQ and ΔBPQ,

$$\frac{A(\Delta APQ)}{A(\Delta BPQ)} = \frac{AP}{BP} \quad \text{(i) ...(Triangles having equal height)}$$

In ΔAPQ and ΔCPQ,

$$\frac{A(\Delta APQ)}{A(\Delta CPQ)} = \frac{AQ}{CQ} \quad \text{(ii) ...(Triangles having equal height)}$$

$$A(\Delta BPQ) = A(\Delta CPQ) \quad \text{(iii) ...(Triangles with common base PQ and same height)}$$

$$\frac{A(\Delta APQ)}{A(\Delta BPQ)} = \frac{A(\Delta APQ)}{A(\Delta CPQ)} \quad \text{......From (i), (ii) and (iii)}$$

$$\frac{AP}{BP} = \frac{AQ}{QC}$$

ii. Given: In an isosceles triangle ΔABC side AB $\cong$ side AC,
Perimeter = 44 cm and base BC = 12 cm.
Perimeter of ΔABC = 44 cm

$AB + BC + AC = 44$

$AB + BC + AB = 44$ ..... (side AB $\cong$ side AC)

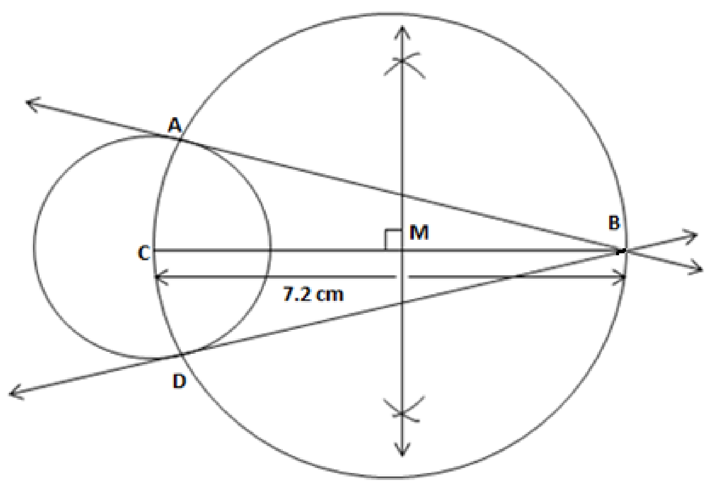
$2AB + 12 = 44$ .... (side BC = 12)

$2AB = 44 - 12 = 32$

$AB = \frac{32}{2} = 16$

$AB = AC = 16$

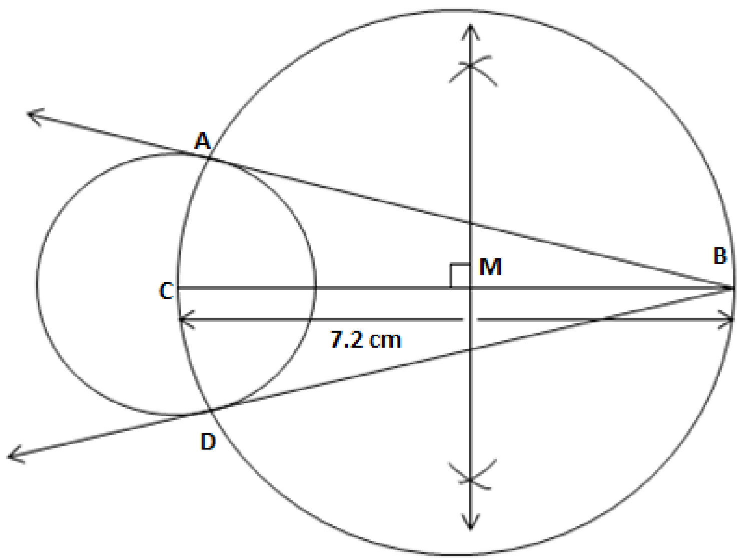
iii. Construction:



Analytical figure

Steps of Construction:

1. Construct a circle with centre C and radius 3.6cm. Take point B such that CB =7.2cm.
2. Obtain Midpoint M of seg CB. Draw a circle with centre M and radius MB.
3. Let A and D be the points of intersection of these two circles. Draw lines BA and BD which are the required tangents.



iv. To prove that: $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \times \operatorname{cosec}^2\theta$

L.H.S = $\sec^2\theta + \operatorname{cosec}^2\theta$

$= 1 + \tan^2\theta + 1 + \cot^2\theta$ $[\because \sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta]$

$= 2 + \tan^2\theta + \cot^2\theta$ ......(i)

R.H.S = $\sec^2\theta \times \operatorname{cosec}^2\theta$

$= (1 + \tan^2\theta) \times (1 + \cot^2\theta)$ $[\because \sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta]$

$= 1 + \cot^2\theta + \tan^2\theta + \tan^2\theta \times \cot^2\theta$

$= 1 + \cot^2\theta + \tan^2\theta + \tan^2\theta \times \dfrac{1}{\tan^2\theta}$ $(\because \cot^2\theta = \dfrac{1}{\tan^2\theta})$

$= 2 + \tan^2\theta + \cot^2\theta$ ......(ii)

From (i) and (ii)

$\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \times \operatorname{cosec}^2\theta$

v.

(1) The required equation of x-axis is y = 0 and y-axis is x = 0.

(2) Let P ≡ (0,0) ≡ (x_1, y_1) and Q ≡ (-3,5) ≡ (x_2,y_2)

The required equation is

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}$$

$$\frac{x-0}{0+3}=\frac{y-0}{0-5}$$

$$\frac{x}{3}=\frac{y}{-5}$$

$$5x+3y=0$$

(3) The equation of x-axis line is y = 0

Slope of the line = 0

Required line is parallel to X-axis we know that parallel lines have equal slopes.

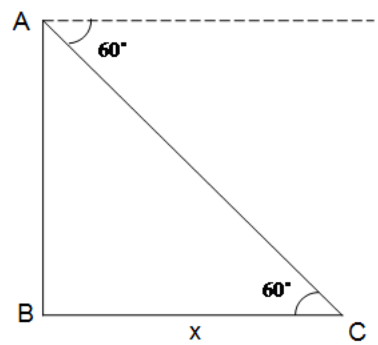
Slope of the required line = m= 0 and point (-3, 4) is on the line.

By point slope form of equation,

$$y-y_1=m(x-x_1)$$

$$y-4=0(x-(-3))$$

$y=4$ is the required equation.



4.

i. As shown in the figure, assume AB as the lighthouse and let A be the position of the observer and C be the position of the ship. Let the distance from the ship to the lighthouse be x.

Given, the height of the lighthouse is 90 m and the angle of depression to be 60°.

For the right-angled triangle ABC

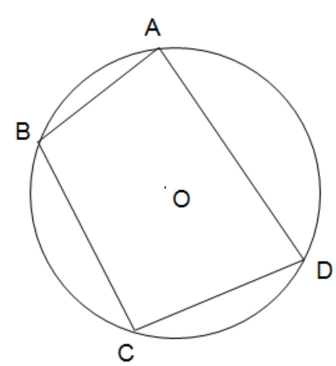
$$\tan 60^\circ=\frac{AB}{BC}$$

$$\tan 60^\circ=\frac{AB}{x}$$

$$x=\frac{AB}{\tan 60^\circ}=\frac{90}{\sqrt{3}}$$

$$x=52.02\approx 52\,\text{m}$$

The ship is at a distance of 52 m from the lighthouse.



ii. Consider the circle having centre O and a cyclic quadrilateral ABCD.

To prove: $\angle BAD + \angle BCD = 180°$ and $\angle ABC + \angle ADC = 180°$

arc BCD is intercepted by the inscribed $\angle BAD$.

$\angle BAD = \frac{1}{2}$ m(arc BCD) ....(1) (Inscribed angle theorem)

arc BAD is intercepted by the inscribed $\angle BCD$.

$\angle BCD = \frac{1}{2}$ m(arc DAB) ....(2) (Inscribed angle theorem)

From (1) and (2)

$$\angle BAD + \angle BCD = \frac{1}{2}[\text{m(arc BCD) + m(arc DAB)}]$$

$$= \frac{1}{2} \times 360^\circ$$

$$= 180^\circ$$

The sum of the measure of angles of a quadrilateral is $360°$

$\therefore \angle ADC + \angle ABC = 360° - (\angle BAD + \angle BCD) = 360° - 180° = 180°$

Hence the opposite angles of a cyclic quadrilateral are supplementary.

iii. Let length, breadth and height of the cuboid be l cm, b cm and h cm, respectively.

Given: l + b + h = 38 cm and diagonal = 22 cm.

To find: total surface area of the cuboid

Diagonal of cuboid = $\sqrt{l^2 + b^2 + h^2}$

$\sqrt{l^2 + b^2 + h^2} = 22$

$l^2 + b^2 + h^2 = 484$ .... (i)

As $l + b + h = 38$

$(l + b + h)^2 = 1444$

$l^2 + b^2 + h^2 + 2(lb + bh + lh) = 1444$

$484 + 2(lb + bh + lh) = 1444$ ...From (i)

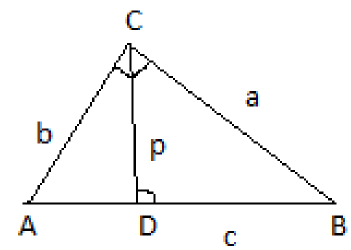
$2(lb + bh + lh) = 960$

The surface area of the cuboid is 960 cm^2.

5.

i.

1) Area of a triangle= $\frac{1}{2} \times$ Base $\times$ Height



$A(\Delta ABC)=\frac{1}{2} \times AB \times CD$

$A(\Delta ABC)=\frac{1}{2} \times cp$ ...(i)

Area of right angle triangle ABC=$A(\Delta ABC)=\frac{1}{2} \times AC \times BC$

$A(\Delta ABC)=\frac{1}{2} \times ba$ ...(ii)

From (i) and (ii)

$cp=ba \Rightarrow cp=ab$ ...(iii)

2) We have,

$cp=ab$ ...From (iii)

$p=\frac{ab}{c}$

Square both sides of the equation.

We get, $p^2=\frac{a^2b^2}{c^2}$

$\frac{1}{p^2}=\frac{c^2}{a^2b^2}$ ...(iv) ...[By invertendo]

In right angled triangle ABC,

$AB^2 = AC^2 + BC^2$ ....[By Pythagoras' theorem]

$c^2 = b^2+a^2$ ...(v)

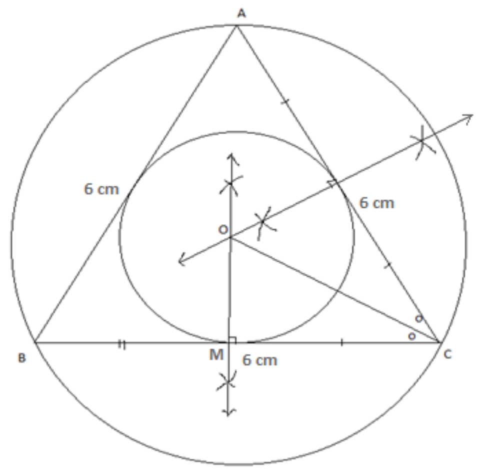
Substituting the value of c^2 in equation (iv), we get

$\frac{1}{p^2}=\frac{b^2+a^2}{a^2b^2}$

$\Rightarrow \frac{1}{p^2}=\frac{b^2}{a^2b^2}+\frac{a^2}{a^2b^2}$

$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

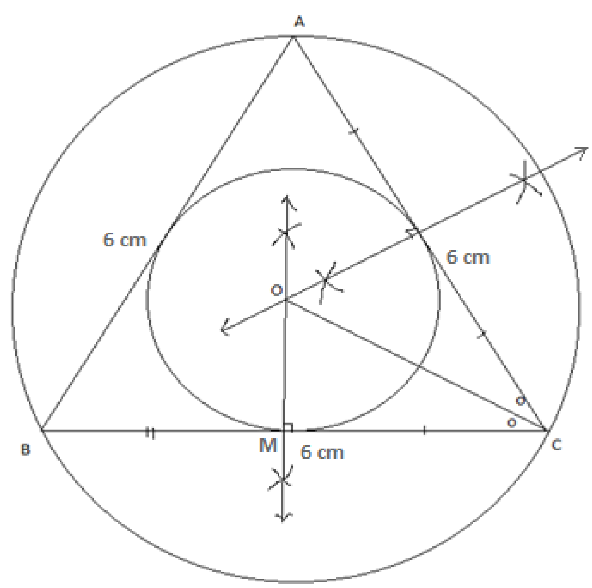
ii. Construction:



Analytical figure

Steps of Construction:

1. Construct an equilateral triangle ΔABC with side 6 cm.
2. Draw perpendicular bisectors of any two sides. Let O be the point of intersection.
3. Draw a circle with centre O and radius OA or OB or OC. This gives us circumcircle equilateral triangle ΔABC.
4. Draw the bisector of $\angle C$. It passes through centre of the circle O.
5. Draw a circle with radius OM. This gives us incircle of the equilateral triangle ΔABC.



In an equilateral triangle angle bisector and median are same, also circumcentre and incentre aresame. For equilateral triangle circumcentre divides the median in 2:1.

So the ratio of radii of circumcircle and incircle is 2:1.

iii. The 1^{st} stair-step=h_1=12 cm.

The 2^{nd} stair-step=h_2=24 cm.

The 3^{rd} stair-step=h_3=36 cm.

The total height=h= h_1+ h_2+ h_3= 12 + 24 + 36 = 72 cm.

Length and width will remain same.

Length=l=50 cm

Width=w=25 cm

The volume of the three stair-step=Length(l) × Width(w) × Height(h)

The volume of the three stair-step=50 × 25 × 72 cu.cm

Given: The volume of 1 brick = 12.5 × 6.25 × 4 cu.cm

Let the number of bricks required be n.

(12.5 × 6.25 × 4) × n = 50 × 25× 72

$$n = \frac{50 \times 25 \times 72}{12.5 \times 6.25 \times 4} = 288$$

288 bricks are used in three stair-steps.