Maharashtra Board
Class X Mathematics - Algebra
Board Paper – (2014) Solution

Time: 2 hours **Total Marks: 40**

Note: - (1) All questions are compulsory.
(2) Use of calculator is not allowed.

1.

i) Given:
$t_3=8$ and $t_4=12$
Since, $t_n=a+(n-1)d$
$t_3=a+(3-1)d$
$8=a+2d$... (i)
$t_4=a+(4-1)d$
$12=a+3d$ ... (ii)
Subtracting (i) from (ii), we get
$d=4$
The common difference is 4.

ii) $(x+5)(x-2)=0$
$x+5 = 0$ and $x-2 = 0$
$x=-5$ and $x=2$
-5 and 2 are the roots of the quadratic equation $(x+5)(x-2)=0$.

iii) Total number of students=140 + 100 + 70 + 40 + 10 = 360
Central angle (θ) for the mode of Transport 'Bus' =
$$\frac{\text{Number of students using Bus}}{\text{Total number of students}}\times 360° = \frac{100}{360}\times 360°$$
$$=100^o$$
Central angle (θ)=100^o

iv) Sample Space S={H, T}
n(S)=2

v) Mean = $\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$
$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{75}{15} = 5$$
Mean= $\overline{x}$ =5

vi) $3x^2-10-7=0$ is in the standard form.

2.

i) The given sequence is 1, 3, 6, 10, …………

Here $t_1 = 1$, $t_2 = 3$, $t_3 = 6$, $t_4 = 10$

Then,

$t_2 - t_1 = 3-1 = 2$

$t_2 - t_3 = 6-3 = 3$

$t_4 - t_3 = 10-6 = 4$

$t_2 - t_1 \neq t_2 - t_3 \neq t_4 - t_3$

Since the difference between two consecutive terms is not constant.

Therefore the given sequence is not an A.P.

ii) $9x^2-25=0$

$(3x)^2-(5)^2=0$

$(3x-5)(3x+5)=0$…….$[a^2-b^2=(a-b)(a+b)]$

$(3x-5)=0$ or $(3x+5)=0$

$3x = 5$ or $3x = -5$

$x = \frac{5}{3}$ or $x = -\frac{5}{3}$

$\therefore \left\{\frac{5}{3}, -\frac{5}{3}\right\}$ is the solution set of the given equation.

iii) Given:

$5x + ay = 19$

$(x, y) = (3, 2)$

The point (x, y) lies on the graph of the equation; hence it satisfies the equation.

Substitute x = 3 and y = 2 in the given equation,

We get

$5(3) + a(2) = 19$

$\Rightarrow 15 + 2a = 19$

$\Rightarrow 2a = 19 - 15$

$\Rightarrow 2a = 4$

$\Rightarrow a = \frac{4}{2} = 2$

Therefore the value of a is 2.

iv) The given equations are

$12x+13y=29 \ldots(i)$

$13x+12y=21 \ldots(ii)$

Add (i) and (ii), we get

$$\begin{array}{rcrl} 12x & + & 13y & =29 \\ \underline{13x} & \underline{+} & \underline{12y} & \underline{=21} \\ 25x & + & 25y & =50 \end{array}$$

$25(x+y) = 50$

$x + y = 2$

v) The sample space (S) is

$S = \{1, 2, 3, 4, 5, 6\}$

No. of sample points = $n(S) = 6$

Let A be the event of getting an even number.

$A= \{2, 4, 6\}$

$\Rightarrow n(A)=3$

vi) The inter-relation between the measures of central tendency is given by

Mean – Mode = 3(Mean-Median)

20 – 11 = 3 (20-Median)

9 = 3 (20-Median)

$\frac{9}{3}$ = 20-Median

3 = 20-Median

Median = 20-3

Median =17

3.

i) The given quadratic equation is $3y^2 + 7y + 4 = 0$.

Comparing the given equation with $ax^2 + bx + c = 0$ we get,

a = 3, b = 7 and c = 4.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(7) \pm \sqrt{(7)^2 - 4(3)(4)}}{2(3)}$$

$$y = \frac{-(7) \pm \sqrt{49 - 48}}{6}$$

$$y = \frac{-(7) \pm \sqrt{1}}{6}$$

$$y = \frac{-(7) \pm 1}{6}$$

$y = \frac{-(7)+1}{6}$ or $y = \frac{-(7)-1}{6}$

$y = \frac{-6}{6} = -1$ or $y = \frac{-8}{6} = \frac{-4}{3}$

$y = -1$ or $y = -\frac{4}{3}$

Therefore -1 and $-\frac{4}{3}$ are the roots of given equation.

ii) The given equations are

3x – y = 7…(i)

x + 4y = 11…(ii)

Equation (i) and (ii) are in standard form.

$$D = \begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix} = (3 \times 4) - (-1 \times 1) = 12 + 1 = 13 \neq 0$$

$$D_x = \begin{vmatrix} 7 & -1 \\ 11 & 4 \end{vmatrix} = (7 \times 4) - (-1 \times 11) = 28 + 11 = 39$$

$$D_y = \begin{vmatrix} 3 & 7 \\ 1 & 11 \end{vmatrix} = (3 \times 11) - (7 \times 1) = 33 - 7 = 26$$

By Cramer's rule, we get

$x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$

$x = \frac{39}{13}$ and $y = \frac{26}{13}$

x = 3 and y = 2

(x,y) = (3,2)

X = 3 and y = 2 is the solution to the given equation.

iii)

The sample space (S) is

S = {TT, HT,TH, HH}

n(s) = 4

a) Let A be the event of getting at least one head.

A = {HT, HH, TH}

n(A) = 3

b) Let B be the event of getting exactly one head.

B = {HT, TH}

n(B) = 2

iv) By Assumed Mean Method

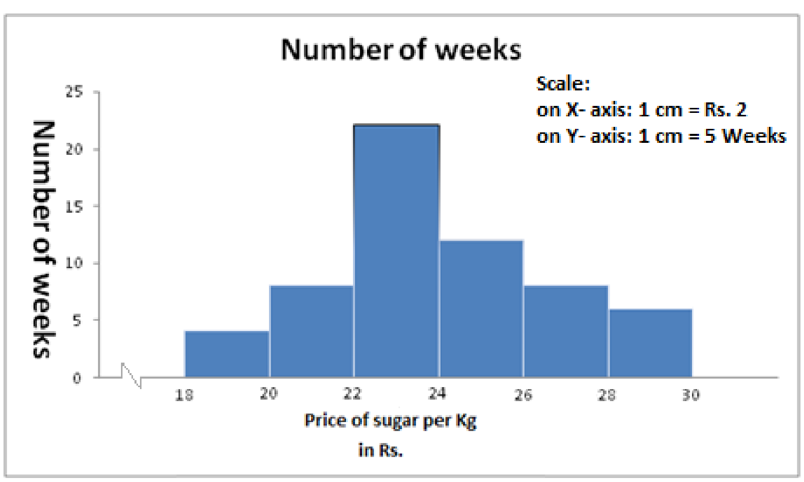
No. of trees	Class mark x_i	$d_i = x_i - A$	No of housing societies(f_i)	$f_i d_i$
10-15	12.5	-10	2	-20
15-20	17.5	-5	7	-35
20-25	22.5→A	0	9	0
25-30	27.5	5	8	40
30-35	32.5	10	6	60
35-40	37.5	15	4	60
Total	-	-	$\sum f_i = 36$	$\sum f_i d_i = 105$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{105}{36} = 2.916$$

Mean = $\bar{x}$ = A + $\bar{d}$ = 22.5 + 2.916 = 25.42

Therefore the mean number of trees planted by Housing Societies is 25.42.

v)



4.

i) As each installment being less that the preceding installment by Rs. 10 the installments are in A.P.

S_{12} = 1000 + 140 = 1140

n = 12, d = -10

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2a + (12-1)(-10)]$$

$$1140 = 6[2a + (11)(-10)]$$

$$1140 = 6[2a - 110]$$

$$\frac{1140}{6} = [2a - 110]$$

$$190 = 2a - 110$$

$$2a = 190 + 110$$

$$2a = 300$$

$$a = \frac{300}{2}$$

$$a = 150$$

The first installment = Rs. 150.

ii) Let the three boys be b_1, b_2, b_3 and the three girls be g_1 and g_2.

The Sample space(S) is

S = { $b_1 b_2$, $b_3 b_1$, $b_2 b_3$, $b_1 g_1$, $b_1 g_2$, $b_2 g_1$, $b_2 g_2$, $b_3 g_1$, $b_3 g_2$, $g_1 g_2$}

⇒ n(S)=10

a) Let B be event that the committee contains only one girls.

B = { $b_1 g_1$, $b_1 g_2$, $b_2 g_1$, $b_2 g_2$, $b_3 g_1$, $b_3 g_2$, $g_1 g_2$}

⇒n(B)=7

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{10}$$

b) Let C be the event that the committee contains one boy and one girl.

C = { $b_1 g_1$, $b_1 g_2$, $b_2 g_1$, $b_2 g_2$, $b_3 g_1$, $b_3 g_2$}

⇒ n(C)=7

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{10}$$

c) Let D be the event that the committee contains only boys.

D = { $b_1 b_2$, $b_3 b_1$, $b_2 b_3$}

⇒ n(D)=3

$$P(D) = \frac{n(D)}{n(S)} = \frac{3}{10}$$

iii) Given: Sales of salesman A = Rs. 18000

a) Sales of salesman A = Rs. 18000

Sales of salesman A= $\frac{Central\ angle}{360°} \times Total$ sales

18000 =$\frac{90}{360} \times Total\ sales$

Total sales = 18000 × 4=Rs. 72000

b) Sales of salesman B = $\frac{Central\ angle}{360°} \times Total$ sales

Sales of salesman B = $\frac{120}{360} \times 72000$

Sales of salesman B =Rs. 24000

Sales of salesman C = $\frac{Central\ angle}{360°} \times Total$ sales

Sales of salesman C = $\frac{80}{360} \times 72000$

Sales of salesman C = Rs. 16000

Sales of salesman D = $\frac{Central\ angle}{360°} \times Total$ sales

Sales of salesman D = $\frac{70}{360} \times 72000$

Sales of salesman D = Rs. 14000

c) Salesman B is the salesman with the highest sale.

d) Difference between the highest sale and the lowest sale

= Sales of salesman B - Sales of salesman D

= Rs. 24000 - Rs. 14000

= Rs. 10000

5.

i) Given

$t_m = [a+(m-1)d]$

$t_n = [a+(n-1)d]$

$m\ (t_m) = n\ (t_n)$

$m[a+(m-1)d] = n[a+(n-1)d]$

$\Rightarrow m[a+md-d] = n[a+nd-d]$

$\Rightarrow am+m^2d-md = an+n^2d-nd$

$\Rightarrow am+m^2d-md-an-n^2d+nd = 0$

$\Rightarrow am-an+ m^2d- n^2d-md+nd = 0$

$\Rightarrow a(m-n)+d(m^2- n^2) -d(m-n) = 0$

$\Rightarrow (m-n)[a+d(m+n)-d] = 0$ ... [Divide by (m−n)]

$\Rightarrow [a+d(m+n)-d] = 0$

$\Rightarrow a+(m+n-1)d = 0$

$\Rightarrow \mathbf{t_{m+n} = 0}$

ii) Let 5x, 5(x+1), 5(x+2) and 5(x+3) be four consecutive natural numbers, which are multiple of fives.

It is given that product of these consecutive numbers is 15000.

$5x \times 5(x+1) \times 5(x+2) \times 5(x+3) = 15000$

$625 \times x(x+1)(x+2)(x+3) = 15000$

$x(x+1)(x+2)(x+3) = \frac{15000}{625}$

$x(x+1)(x+2)(x+3) = 24$

When $x = 1$

Then,

$x(x+1)(x+2)(x+3$

$1(1+1)(1+2)(1+3$

$1(2)(3)(4) = 24$

Hence, the four consecutive natural numbers are

$5x = 5 \times 1 = 5$

$5(x+1) = 5 \times (1+1) = 5 \times 2 = 10$

$5(x+2) = 5 \times (1+2) = 5 \times 3 = 15$

$5(x+3) = 5 \times (1+3) = 5 \times 4 = 20$

Therefore four consecutive natural numbers are 5, 10, 15 and 20.

iii) The given simultaneous equations are 4x + 3y = 24 and 4x - 3y = -24.

4x+3y =24 ...(i)

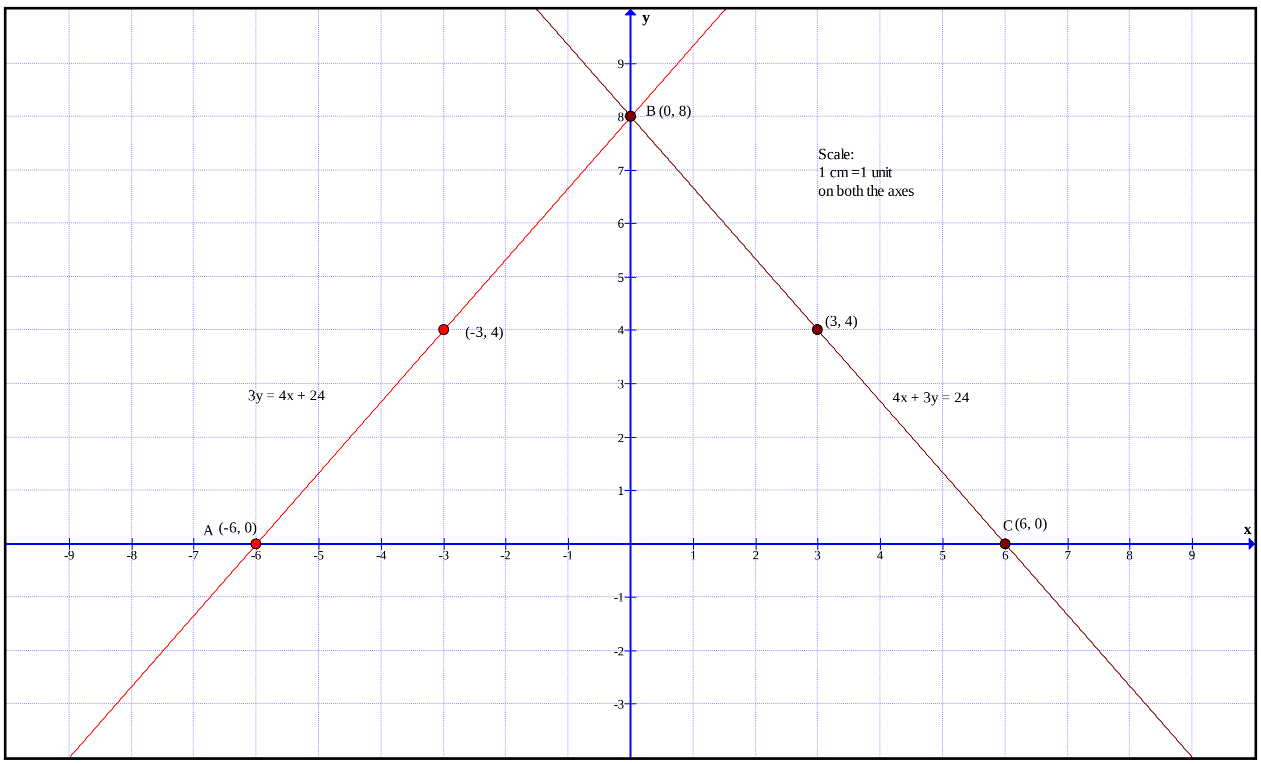
$y = \frac{24-4x}{3}$

x	0	3	6
y	8	4	0
(x,y)	(0, 8)	(3, 4)	(6, 0)

4x-3y =-24 ...(ii)

$y = \frac{4x+24}{3}$

x	0	-3	-6
y	8	4	0
(x,y)	(2, 8)	(-3, 4)	(-6, 0)



From the graph A (-6, 0), B(0, 8) C (6, 0) and AC = 12 units.

Height = h= 8 units

Base = b = 12 units

Area of triangle= $\frac{1}{2} \times Base \times Height$

A(ΔABC)= $\frac{1}{2} \times 12 \times 8$

A(ΔABC)=48 sq units