Maharashtra State Board
ClassX Mathematics - Algebra
Board Paper – 2016 Solution

1.

i. $t_n = 3n - 4$
For n = 1, $t_1 = 3 \times 1 - 4 = 3 - 4 = -1$
For n = 2, $t_2 = 3 \times 2 - 4 = 6 - 4 = 2$
Hence, the first two terms of the sequence are –1 and 2.

ii. Given equation is $2x^2 - x - 3 = 0$.
Comparing the given equation with general form of quadratic equation
$ax^2 + bx + c$, we have
a = 2, b = −1 and c = −3

iii. Let the roots be $\alpha = -2$ and $\beta = -3$.
$\therefore \alpha + \beta = (-2) + (-3) = -5$ and $\alpha\beta = (-2)(-3) = 6$
Hence, the required quadratic equation is
$x^2 - (\alpha + \beta)x + \alpha\beta = 0$
i.e. $x^2 - (-5)x + 6 = 0$
i.e. $x^2 + 5x + 6 = 0$

iv. $\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} = (4 \times 1) - (-2 \times 3) = 4 + 6 = 10$

v. The sample space 'S' for selecting a day randomly of the week is given by
S = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

vi. $\text{Class Mark} = \dfrac{\text{Upper Limit} + \text{Lower Limit}}{2}$

$\therefore \text{Class Mark of the class } 20 - 30 = \dfrac{30 + 20}{2} = \dfrac{50}{2} = 25$

$\text{Class Mark of the class } 30 - 40 = \dfrac{40 + 30}{2} = \dfrac{70}{2} = 35$

2.

i. a = 4, d = −3
Hence,
$t_1 = 4$
$t_2 = t_1 + d = 4 + (-3) = 4 - 3 = 1$
$t_3 = t_2 + d = 1 + (-3) = 1 - 3 = -2$
Thus, the first three terms of the A.P. are 4, 1 and –2.

ii. $x^2 + 7x + 10 = 0$

Splitting the middle term 7x as 2x + 5x, we get

$x^2 + 2x + 5x + 10 = 0$

$\therefore x(x + 2) + 5(x + 2) = 0$

$\therefore (x + 2)(x + 5) = 0$

$\therefore x + 2 = 0$ or $x + 5 = 0$

$\therefore x = -2$ or $x = -5$

iii. $\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix} = 31$

$\therefore (m \times 7) - (-5 \times 2) = 31$

$\therefore 7m - (-10) = 31$

$\therefore 7m + 10 = 31$

$\therefore 7m = 21$

$\therefore m = 3$

iv. When a die is thrown, the sample space (S) is given by

S = {1, 2, 3, 4, 5, 6}

$\therefore$ n(S) = 6

Let A be the event of getting an odd number on the upper surface of the die.

Then A = {1, 3, 5}

$\therefore$ n(A) = 3

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

Let B be the event of getting a perfect square on the upper surface of the die.

Then B = {1, 4}

$\therefore$ n(B) = 2

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

v.

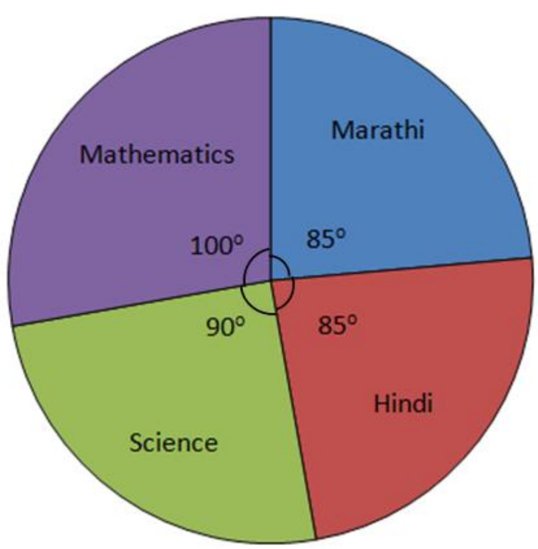
(Number of words) Class intervals	Class Mark x_i	(Number of candidates) Frequency f_i	$f_i x_i$
600 – 800	700	8	5600
800 – 1000	900	22	19800
1000 – 1200	1100	40	44000
1200 – 1400	1300	18	23400
1400 – 1600	1500	12	18000
Total		$\sum f_i = 100$	$\sum f_i x_i = 110800$

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{110800}{100} = 1108$$

$\therefore$ Mean number of words written in an essay is 1108.

vi. Central angle for each subject is computed in the following table:

Subject	Marks	Measure of central angle
Marathi	85	$\frac{85}{360} \times 360° = 85°$
Hindi	85	$\frac{85}{360} \times 360° = 85°$
Science	90	$\frac{90}{360} \times 360° = 90°$
Mathematics	100	$\frac{100}{360} \times 360° = 100°$
Total	360	$360°$



3.

i. Given quadratic equation is $5m^2 + 5m - 1 = 0$

Comparing with general form $ax^2 + bx + c = 0$, we have

$a = 5$, $b = 5$ and $c = -1$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(5)(-1)}}{2 \times 5}$$

$$= \frac{-5 \pm \sqrt{25 + 20}}{10}$$

$$= \frac{-5 \pm \sqrt{45}}{10}$$

$$= \frac{-5 \pm 3\sqrt{5}}{10}$$

$\therefore \frac{-5 + 3\sqrt{5}}{10}$ and $\frac{-5 - 3\sqrt{5}}{10}$ are the roots of the given equation.

ii. Here, there are three boys B_1, B_2, B_3 and two girls G_1, G_2.

A committee of two is to be formed.

Thus, the sample space (S) is given by

$S = \{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$

$\therefore n(S) = 10$

A is the event that the committee contains at least one girl.

Then, $A = \{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$

$\therefore n(A) = 7$

$$\therefore P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{7}{10}$$

B is the event that the committee contains one boy and one girl.

Then, $B = \{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2\}$

$\therefore n(B) = 6$

$$\therefore P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{6}{10}$$

$$= \frac{3}{5}$$

iii. Let A be the assumed mean.

A is taken as the class mark of the middle class.

Hence, let us take 40 as the assumed mean.

Then A = 40 and deviation $d_i = x_i - A = x_i - 40$

Diameter (in mm)	Class marks x_i	Deviations $d_i = x_i - A$ $d_i = x_i - 40$	Number of Screws f_i	$f_i d_i$
33-35	34	−6	10	−60
36-38	37	−3	19	−57
39-41	40 = A	0	23	0
42-44	43	3	21	63
45-47	46	6	27	162
Total	...	...	$\sum f_i = 100$	$\sum f_i d_i = 108$

Here, $\sum f_i d_i = 108$, $\sum f_i = 100$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{108}{100} = 1.08$$

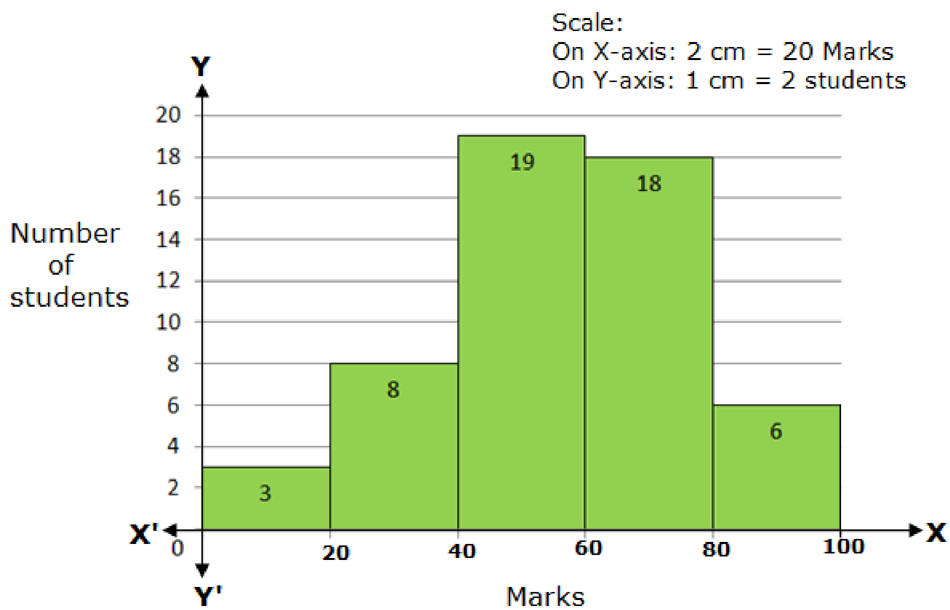
$$\bar{x} = A + \bar{d}$$

$$= 40 + 1.08$$

$$= 41.08$$

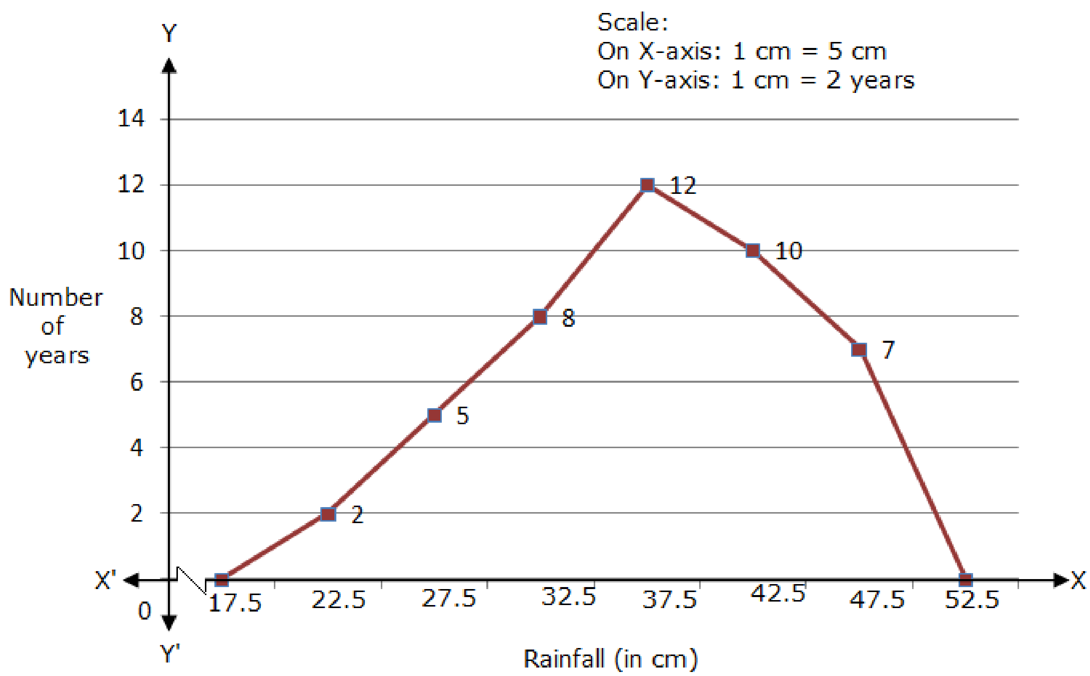
Thus, the mean diameter of the screw heads is 41.08 mm.

iv.



v.

Class mark	No. of Years
17.5	0
22.5	2
27.5	5
32.5	8
37.5	12
42.5	10
47.5	7
52.5	0



4.

i.

(a) Let 'a' be the first term and 'd' the common difference of the given A.P.

For $t_{11} = 16,\ n = 11$, we have

$t_{11} = a + (11 - 1)d$

$\therefore 16 = a + 10d$ ... (1)

For $t_{21} = 29,\ n = 21$, we have

$t_{21} = a + (21 - 1)d$

$\therefore 29 = a + 20d$ ... (2)

Subtracting (1) from (2), we get

$13 = 10d \Rightarrow d = 1.3$

Substituting $d = 1.3$ in (1), we get

$16 = a + 10(1.3)$

$\therefore 16 = a + 13 \Rightarrow a = 3$

Thus, the first term is 3 and the common difference is 1.3.

(b) For 34^{th} term, $n = 34,\ a = 3,\ d = 1.3$

$t_n = a + (n-1)d$

$\therefore t_{34} = 3 + (34-1)(1.3)$

$= 3 + 33 \times 1.3$

$= 3 + 42.9$

$= 45.9$

Thus, the 34^{th} term is 45.9.

(c) $t_n = 55,\ a = 3,\ d = 1.3$

$\therefore t_n = a + (n-1)d$

$\therefore 55 = 3 + (n-1)(1.3)$

$\therefore 55 - 3 = (n-1)(1.3)$

$\therefore (n-1)(1.3) = 52$

$\therefore n - 1 = \dfrac{52}{1.3}$

$\therefore n - 1 = 40$

$\therefore n = 40 + 1$

$\therefore n = 41$

ii.

$\dfrac{7}{2x+1} + \dfrac{13}{y+2} = 27 \quad ....(1)$

$\dfrac{13}{2x+1} + \dfrac{7}{y+2} = 33 \quad ....(2)$

Substituting $\dfrac{1}{2x+1} = m$ and $\dfrac{1}{y+2} = n$ in equations (1) and (2), we get

$7m + 13n = 27 \quad ....(3)$

and $13m + 7n = 33 \quad ....(4)$

Adding equations (3) and (4), we get

$20m + 20n = 60$

$\therefore m + n = 3 \quad ....(5)$

Subtracting equation (3) from equation (4), we get

$6m - 6n = 6$

$\therefore m - n = 1 \quad ....(6)$

Adding equations (5) and (6), we get

$2m = 4$

$\therefore m = 2$

Substituting $m = 2$ in equation (5), we get

$2 + n = 3$

$\therefore n = 1$

Resubstituting the values of m and n, we get

$\frac{1}{2x+1} = 2$ and $\frac{1}{y+2} = 1$

$\therefore 4x + 2 = 1$ and $y + 2 = 1$

$\therefore 4x = -1$ and $y = 1 - 2$

$\therefore x = -\frac{1}{4}$ and $y = -1$

iii. Given: P(A) = 2P(B) and P(B) = 2P(C)

$\therefore$ P(A) = 2[2P(C)] = 4P(C)

Now, P(A) + P(B) + P(C) = 1

$\therefore$ 4P(C) + 2P(C) + P(C) = 1

$\therefore$ 7P(C) = 1

$\therefore P(C) = \frac{1}{7}$

$\therefore P(A) = 4P(C) = 4 \times \frac{1}{7} = \frac{4}{7}$ and $P(B) = 2P(C) = 2 \times \frac{1}{7} = \frac{2}{7}$

5.

i. Dividend = 6123

Now, divisor and quotient are same

Let divisor = quotient = d

Now, remainder $= \frac{\text{divisor}}{2} = \frac{d}{2}$

Since dividend = divisor × quotient + remainder, we have

$6123 = d^2 + \frac{d}{2}$

$\therefore 12246 = 2d^2 + d$

$\therefore 2d^2 + d - 12246 = 0$

Comparing with $ax^2 + bx + c$, we get

$a = 2, b = 1, c = -12246$

$$\therefore d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 - 4(2)(-12246)}}{4}$$

$$= \frac{-1 \pm \sqrt{97969}}{4}$$

$$= \frac{-1 \pm 313}{4}$$

$\therefore d = \frac{-1 + 313}{4}$ or $d = \frac{-1 - 313}{4}$

$\therefore d = 78$ or $d = -78.5$

Ignoring the negative value, the divisor is 78.

ii. The numbers from 50 to 150 which are divisible by 6 are 54, 60, 66, ……, 348.

$\therefore$ First term = a = t_1 = 54, d = 6 and t_n = 348

$t_n = a + (n-1)d$

$\therefore 348 = 54 + (n-1)6$

$\therefore 294 = (n-1)6$

$\therefore 49 = n - 1$

$\therefore n = 50$

$S_n = \frac{n}{2}(t_1 + t_n)$

$\therefore S_{50} = \frac{50}{2}(54 + 348)$

$= 25 \times 402$

$= 10050$

$t_{15} = 54 + 14(6) = 54 + 84 = 138$

Thus, the sum of all numbers from 50 to 350, which are divisible by 6, is 10050 and the 15^{th} term of this A.P. is 138.

iii. Let the three-digit number be xyz.

Its numerical value = 100x + 10y + z

According to first information provided in the question,

100x + 10y + z = 17(x + y + z)

$\therefore$ 100x + 10y + z = 17x + 17y + 17z

$\therefore$ 83x − 7y − 16z = 0 ....(1)

Number obtained by reversing the digits: zyx

Its numerical value = 100z + 10y + x

According to second information provided in the question,

(100x + 10y + z) + 198 = 100z + 10y + x

$\therefore$ 99z − 99x = 198

$\therefore$ z − x = 2

$\therefore$ z = x + 2 ....(2)

According to third information provided in the question,

x + z = y − 1

$\therefore$ x + x + 2 = y − 1 ....[from (2)]

$\therefore$ y = 2x + 3

Substituting the values of z and y in equation (1),

83x − 7(2x + 3) − 16(x + 2) = 0

$\therefore$ 83x − 14x − 21 − 16x − 32 = 0

$\therefore$ 53x − 53 = 0

$\therefore$ 53x = 53

$\therefore$ x = 1

$\therefore$ y = 2x + 3 = 2(1) + 3 = 2 + 3 = 5

$\therefore$ z = x + 2 = 1 + 2 = 3

Thus, the three-digit number is 153.