Maharashtra State Board
Class X Mathematics - Algebra
Board Paper – 2017
Solution

Note: - (1) All questions are compulsory.
(2) Use of calculator is not allowed.

1.

i. 3, 6, 12, 24....

Let, 'a' be the first term of the given sequence and 'd' be the common difference.

Also, t_2, t_3, t_4 be the $2^{nd}, 3^{rd}, 4^{th}$ terms respectively.

Consider,

$t_2 - a = 6 - 3$

$= 3$

and

$t_3 - t_2 = 12 - 6$

$= 6$

Here, we can see that difference between two successive terms is not constant.

Hence, it is not an Arithmetic Progression.

ii. One root of the quadratic equation is given to be $3 - 2\sqrt{5}$.

The other root will be the conjugate of $3 - 2\sqrt{5}$.

Conjugate of $3 - 2\sqrt{5} = 3 + 2\sqrt{5}$

iii. Given that there are 15 tickets bearing the numbers from 1 to 15 in a bag.

Hence, sample space can be written as:

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$n(S) = 15$

iv. Class mark for the given class 35 - 39 is :

$\frac{35 + 39}{2} = \frac{74}{2}$

$= 37$

v. The first term, $a = 3$

common difference, $d = 4$

So, the next two terms would be $a + d, a + 2d$.

That is, the next two terms are 7 and 11.

vi. Given quadratic equation is $2x^2 = x + 3$.

Writing this equation in standard form, we get

$2x^2 - x - 3 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$a = 2, b = -1, c = -3$

2.

i. To find the first term, substitute n = 1 in $S_n = n^2(n+2)$

$\Rightarrow S_1 = 1^2(1+2) = 1(3) = 3$

Now S_1 is the sum of the first term itself, which is also the first term.

So, the first term = 3

To find the second term, substitute n = 2 in $S_n = n^2(n+2)$

$\Rightarrow S_2 = 2^2(2+2) = 4(4) = 16$

Now S_2 is the sum of the first two terms.

$\Rightarrow$ the first term + the second term =16

$\Rightarrow$ 3 + the second term =16

$\Rightarrow$ the second term = 13

So, the first term = 3 and the second term = 13.

ii. Given quadratic equation is $x^2 + 5x + 1 = 0$.

$a = 1, b = 5, c = 1$

The discriminant $(\Delta) = b^2 - 4ac = 5^2 - 4(1)(1) = 25 - 4 = 21$

iii. The equation of the X-axis is y = 0.

To find the point of intersection of the equation $x + y = 3$ with the X - axis, substitute $y = 0$ in $x + y = 3$.

$\Rightarrow x + 0 = 3$

$\Rightarrow x = 3$

So, the point of intersection will be (3,0.)

iv. We have,

$A = 1200,\ \sum f_i d_i = 700$ and $\sum f_i = 100 = N$

$$\overline{X} = A + \frac{1}{N}\sum_{i=1}^{n} f_i d_i$$

$$\Rightarrow \overline{X} = 1200 + \frac{1}{100}(700)$$

$$\Rightarrow \overline{X} = 1200 + 7$$

$$\Rightarrow \overline{X} = 1207$$

v. Since two coins are tossed simultaneously,

$S = \{HT, TH, HH, TT\}$

$\Rightarrow n(S) = 4$

$A = \{HT, TH, HH\}$

$\Rightarrow n(A) = 3$

vi. The given equations are

$kx + 2y = 6$ ......(i)

and $9x + 6y = 18$

$\Rightarrow 3\ (3x + 2y) = 18$

$\Rightarrow 3x + 2y = 6$ .....(ii)

From (i) and (ii), we get $k = 3$

3.

i. The first three digit natural number divisible by 2 is 100.

Common difference $d = 2$

Last three digit natural number divisible by 2 is 998

We know that,

$t_n = a + (n - 1)\ d$

$\Rightarrow 998 = 100 + (n - 1)\ 2$

$\Rightarrow 898 = 2\ (n - 1)$

$\Rightarrow 449 = n - 1$

$\Rightarrow n = 450$

Hence, there are 450 three digit natural numbers divisible by 2.

ii. $3x^2 - 22x + 40 = 0$

$\Rightarrow 3x^2 - 12x - 10x + 40 = 0$

$\Rightarrow 3x\ (x - 12) - 10(x - 12) = 0$

$\Rightarrow (x - 12)(3x - 10) = 0$

$\Rightarrow x - 12 = 0$ or $3x - 10 = 0$

$\Rightarrow x = 12 \text{ or } x = \frac{10}{3}$

iii. Consider,

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2 \neq 0$$

$$x = \frac{\begin{vmatrix} 4 & 2 \\ 6 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{16 - 12}{-2} = -2$$

$$y = \frac{\begin{vmatrix} 1 & 4 \\ 3 & 6 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}} = \frac{6 - 12}{-2} = 3$$

So, $x = -2$ and $y = 3$.

iv. S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\Rightarrow$n(S) = 36

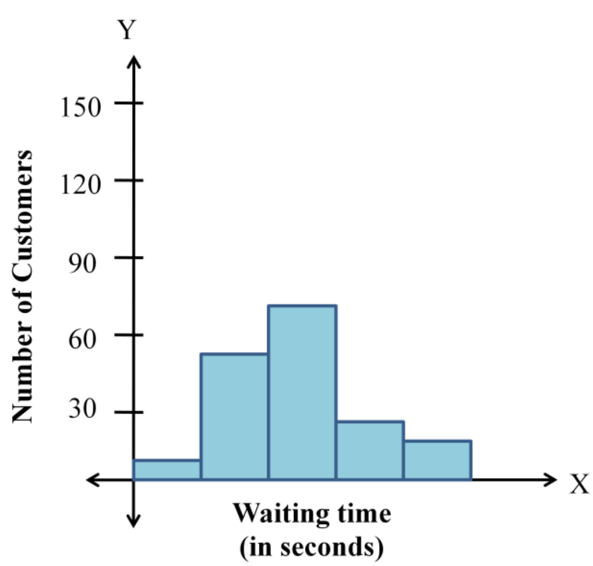
Let A = event that the product of numbers on their upper faces is 12

$\Rightarrow$A = {(2,6), (3, 4), (4,3), (6,2)}

$\Rightarrow$n(A) = 4

$$\Rightarrow \text{Probability of the event A} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{4}{36} = \frac{1}{9}$$

v.



4.

i.

$\Pr(A) = 2\Pr(B)$, and $\Pr(B) = 3\Pr(C)$

Hence, $\Pr(A) = 6\Pr(C)$

But $\Pr(A) + \Pr(B) + \Pr(C) = 1$

Consequently, $6\Pr(C) + 3\Pr(C) + \Pr(C) = 1$

So, $\Pr(C) = \frac{1}{10}$;

Since $\Pr(B) = 3\Pr(C)$

$\Rightarrow \Pr(B) = \frac{3}{10}$

and since $\Pr(A) = 6\Pr(C)$

$\Rightarrow \Pr(A) = \frac{6}{10}$

ii. Calculation of the Median size of farms.

Size of Farms	**f**	**cf**
5 - 15	7	7
15 – 25	12	19
25 - 35	17	36
35 - 45	25	61
45 – 55	31	92
55 - 65	5	97
65 - 75	3	100

We have $N = 100 \Rightarrow \frac{N}{2} = 50$

The cumulative frequency just greater than $\frac{N}{2}$ is 61 and the corresponding class is 35 - 45.

Thus, 35 - 45 is the median class such that $l = 35, f = 25, cf = 36, h = 10$.

$$\Rightarrow \text{Median} = l + \frac{\frac{N}{2} - cf}{f} \times h$$

$$= 35 + \frac{50 - 36}{25} \times 10$$

$$= 35 + 5.6$$

$$= 40.6$$

iii. We compute the central angle for each crop as shown in the following table.

Sector	**Measure of central angle**	**Amount (in crores)**
Agriculture	120°	$\frac{120}{360}\times 180 = \text{Rs.}60$
Dairy	40°	Rs. 20
Industry	360 - (120° + 40°) = 200°	$\frac{200}{360}\times 180 = \text{Rs.}100$
Total	360°	Rs. 180

(a)

$$\frac{40^\circ}{360^\circ}\times \text{Total} = 20$$

$$\Rightarrow \text{Total} = \frac{20\times 360}{40}$$

$$\Rightarrow \text{Total} = \text{Rs.}180 \text{ crores}$$

Hence, the total loan disbursed is Rs.180 crores.

(b)

Total loan for the agriculaure sector :

$$\frac{120}{360}\times 180 = \text{Rs. } 60$$

(c)

Total loan for industrial sector :

$$\frac{200}{360}\times 180 = \text{Rs.}100$$

The additional amount the industrial sector received than the agriculture sector

= Rs.100 - Rs.60

= Rs.40

5.

(i)

Let x be the original cost of a dozen bananas.

For Rs.600 let us one gets y dozens.

$xy = 600 \quad .....(1)$

$\Rightarrow y = \frac{600}{x}$

$(x+10)(y-3) = 600 \quad .....(2)$

Substituting the y value in (2), we get,

$(x+10)\left(\frac{600}{x} - 3\right) = 600$

$\Rightarrow (x+10)\left(\frac{600-3x}{x}\right) = 600$

$\Rightarrow (10+x)(600-3x) = 600x$

$\Rightarrow 6000 + 570x - 3x^2 = 600x$

$\Rightarrow 6000 - 30x - 3x^2 = 0$

$\Rightarrow 2000 - 10x - x^2 = 0$

$\Rightarrow x^2 + 10x - 2000 = 0$

$\Rightarrow (x+50)(x-40) = 0$

$\Rightarrow x = -50 \text{ or } 40$

Since cost of bananas cannot be negative, x = 40.

So, the original cost of one dozen of bananas is Rs. 40.

(ii)

To show : $S_{p+q} = 0$

that is, to show : $\frac{p+q}{2}\left(2a + (p+q-1)d\right) = 0$

Given that $S_p = S_q$

Let a be the first term of the AP and d be the common difference.

$\Rightarrow \frac{p}{2}\left(2a + (p-1)d\right) = \frac{q}{2}\left(2a + (q-1)d\right)$

$\Rightarrow p\left(2a + (p-1)d\right) = q\left(2a + (q-1)d\right)$

$\Rightarrow 2ap + (p-1)dp = 2aq + (q-1)dq$

$\Rightarrow 2ap - 2aq + (p-1)dp - (q-1)dq = 0$

$\Rightarrow 2ap - 2aq + (p-1)dp - (q-1)dq = 0$

$\Rightarrow 2a(p-q) + d\left[p^2 - p - q^2 + q\right] = 0$

$\Rightarrow 2a(p-q) + d\left[p^2 - q^2 - p + q\right] = 0$

$\Rightarrow 2a(p-q) + d\left[(p-q)(p+q) - (p-q)\right] = 0$

$\Rightarrow 2a(p-q) + d\left[(p-q)[(p+q)-1]\right] = 0$

Dividing throughout by $p-q$, since $p \neq q$.

$\Rightarrow 2a + ((p+q)-1)d = 0$

$\Rightarrow 2a + (p+q-1)d = 0$

$S_{p+q} = \frac{p+q}{2}\left(2a + (p+q-1)d\right) = \frac{p+q}{2}(0) = 0$

Hence proved.

(iii)

$\frac{1}{3x} - \frac{1}{4y} + 1 = 0$ and $\frac{1}{5x} + \frac{1}{2y} = \frac{4}{15}$

$\frac{1}{3x} - \frac{1}{4y} = -1$ and $\frac{1}{5x} + \frac{1}{2y} = \frac{4}{15}$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

$\Rightarrow \frac{a}{3} - \frac{b}{4} = -1$ and $\frac{a}{5} + \frac{b}{2} = \frac{4}{15}$

$\Rightarrow 4a - 3b = -12$ and $2a + 5b = \frac{40}{15}$

$\Rightarrow 4a - 3b = -12$ and $2a + 5b = \frac{8}{3}$

$\Rightarrow 4a - 3b = -12$ and $3a + 15b = 8$

Solving the two equations, we get $a = -2$ and $b = \frac{4}{3}$.

Resubstituting $\frac{1}{x} = a$ and $\frac{1}{y} = b$,

$\Rightarrow x = -\frac{1}{2}$ and $y = \frac{3}{4}$