MATHS QUESTION PAPER

Time : 2 Hrs.
Max. Marks : 40

Q. 1 (a) Attempt any ONE of the following :

(i) If \( y \) is a differentiable function of \( u \) and \( u \) is a differentiable function of \( x \), then prove that
\[
\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}.
\]

(ii) If \( x \) & \( y \) are differentiable functions so that \( y \) is a function of \( x \), then prove that
\[
\frac{dy}{dx} = \left( \frac{dy}{dt} \right) \left( \frac{dt}{dx} \right).
\]

(b) Attempt any ONE of the following :

(i) If \( u \) and \( v \) are functions of \( x \), then prove that
\[
\int u \cdot v \, dx = u \int v \, dx - \int \left( \frac{du}{dx} \int v \, dx \right) \, dx.
\]

(ii) Prove that:
\[
\int_{-a}^{a} f(x) \, dx = 2 \int_{0}^{a} f(x) \, dx, \quad \text{if} \ f(x) \ \text{is an even function.}
\]
\[
= 0, \quad \text{if} \ f(x) \ \text{is an odd function.}
\]

(B) Attempt any ONE of the following :

(i) For all \( x \) in \( B \), prove that \( x = x \cdot x \) where \( B \) is Boolean Algebra.

(ii) Write down the Boolean Function for the expression \( x_1 \cdot (x_1 + x_2) \) in tabular form.

Q. 2 (A) (a) Attempt any ONE of the following :

(i) Show that \( \Delta \log [f(x)] = \log \left[ 1 + \frac{\Delta f(x)}{f(x)} \right] \).

(ii) If \( f(x) \) is a polynomial of degree 2 in \( x \) and \( f(0) = 8, f(1) = 12, f(2) = 18 \), find \( f(x) \).

(b) Attempt any ONE of the following :

(i) Solve:
\[
\left( y + x \frac{dy}{dx} \right) \cdot \sin(xy) = \cos x \ \text{by putting} \ xy = u.
\]

(ii) Solve:
\[
\frac{dy}{dx} = \frac{x+y}{x-y}.
\]

(B) Attempt any ONE of following :

(i) Evaluate:
\[
\int \frac{dx}{2x + 3x \log x}.
\]

(ii) Evaluate:
\[
\int \frac{dx}{\sin^2 x - \cos^2 x}.
\]

Q. 3 (A) (a) Attempt any ONE of the following :

(i) Evaluate:
\[
\int \sqrt{2e^{2x} + 7e^x - 5} \, dx.
\]

(ii) Evaluate:
\[
\int \frac{dx}{(x^2 - 7)(x^2 + 4)}.
\]

(b) Attempt any ONE of the following :

(i) Evaluate:
\[
\int_{0}^{3} \log (1 + \tan x) \, dx.
\]

(ii) Show that:
\[
\int_{0}^{\frac{1}{3}} \frac{dx}{x(x^3 - 1)} = \frac{1}{3} \log \left( \frac{208}{189} \right).
\]
(B) Attempt any ONE of the following:

(i) If \( y = \sqrt{x} \); find \( \frac{dy}{dx} \) using first principles.

(ii) Find \( \frac{dy}{dx} \); if \( y = \tan^{-1} \left( \frac{\sin x}{1 + \cos x} \right) \).

Q. 4 (A) Attempt any TWO of the following:

(i) If \( y = \frac{x + \sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2} - x} \); find \( \frac{dy}{dx} \).

(ii) Find the approximate value of \( \tan^{-1} (0.999) \).

(iii) If \( y = x \cdot e^y \) show that \( \frac{dy}{dx} = \frac{y (1 + xy)}{x (1 - xy)} \).

(B) Attempt any ONE of the following:

(i) Simplify and show that \( x \cdot [(x' + z) \cdot y] = x \cdot y \cdot z \) using Boolean Algebra.

(ii) For the Boolean function \( f(x_1, x_2) = (x_1 \cdot x_2') + (x_1' \cdot x_2) \) write down its value in tabular form.

Q. 5 (A) Attempt any TWO of the following:

(i) Evaluate: \( \lim_{x \to 0} \frac{\sin (x + a) - \sin (x - a) - 2 \sin a}{x \cdot \sin x} \).

(ii) Evaluate: \( \lim_{x \to 1} \frac{ab^x - ba^x}{x - 1} \).

(iii) Find \( k \) if the function \( f(x) = \begin{cases} 3x - 4, & \text{for } 0 \leq x \leq \frac{2}{3} \\ 2x + k, & \text{for } 2x \leq 4. \end{cases} \) is continuous at \( x = 2 \).

(B) Attempt any ONE of the following:

(i) Solve: \( \frac{dy}{dx} = \frac{1 + y^2}{1 + x^2} \).

(ii) Verify that \( y = Ae^x + Be^{-2x} \) is a solution of the Differential Equation \( \frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0 \).