MATHS QUESTION PAPER

Time : 2 Hrs. **Max. Marks : 40**

Q. 1 (a) Attempt any ONE of the following : **[8]**

(i) If y is a differentiable function of u and u is a differentiable function of x, then prove that $\frac{dy}{dx} = \frac{dy}{dx} \times \frac{du}{dx}$. (3)

(ii) If x & y are differentiable functions t so that y is a function of x, then prove that $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$, $\frac{dx}{dt} \neq 0$. (3)

(b) Attempt any ONE of the following :

(i) If u and v are functions of x, then prove that - (3)

$$\int u \cdot v\, dx = u \int v\, dx - \int \left\{ \frac{du}{dx} \int v\, dx \right\} dx.$$

(ii) Prove that :

$$\int_{-a}^{a} f(x)\, dx = 2 \int_{0}^{a} f(x)\, dx, \quad \text{if } f(x) \text{ is an even function.}$$

$$= 0, \quad \text{if } f(x) \text{ is an odd function.}$$ (2)

(B) Attempt any ONE of the following :

(i) For all x in B, prove that $x = x \cdot x$ where B is Boolean Algebra. (2)

(ii) Write down the Boolean Function for the expression $x_1 \cdot (x_1 + x_2)$ in tabular form. (2)

Q. 2 (A) (a) Attempt any ONE of the following : **[8]**

(i) Show that $\Delta \log [f(x)] = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$ (3)

(ii) If f (x) is a polynomial of degree 2 in x and if f(0) = 8, f (1) = 12, f (2) = 18, find f (x). (3)

(b) Attempt any ONE of the following :

(i) Solve : $\left(y + x\frac{dy}{dx}\right) \cdot \sin(xy) = \cos x$ by putting $xy = u$. (3)

(ii) Solve : $\frac{dy}{dx} = \frac{x+y}{x-y}$ (3)

(B) Attempt any ONE of following :

(i) Evaluate : $\int \frac{dx}{2x + 3x \log x}$. (2)

(ii) Evaluate : $\int \frac{dx}{\sin^2 x \cdot \cos^2 x}$ (2)

Q. 3 (A) (a) Attempt any ONE of the following : **[8]**

(i) Evaluate : $\int \frac{e^x}{\sqrt{2e^{2x} + 7e^x - 5}}\, dx$. (3)

(ii) Evaluate : $\int \frac{x^2 + 37}{(x^2 - 7)(x^2 + 4)}\, dx$ (3)

(b) Attempt any ONE of the following :

(i) Evaluate : $\int_{0} \log(1 + \tan x)\, dx$. (3)

(ii) Show that : $\int^{3} \frac{dx}{x(x^3 - 1)} = \frac{1}{3} \log\left(\frac{208}{189}\right)$. (3)

(B) Attempt any ONE of the following :

(i) If $y = \sqrt{x}$; find $\frac{dy}{dx}$ using first principles. (2)

(ii) Find $\frac{dy}{dx}$; if $y = \tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$ (2)

Q. 4 (A) Attempt any TWO of the following : **[8]**

(i) If $y = \log\left(\frac{x + \sqrt{x^2 + a^2}}{\sqrt{x^2 + a^2} - x}\right)$; find $\frac{dy}{dx}$. (3)

(ii) Find the approximate value of $\tan^{-1}(0.999)$. (3)

(iii) If $y = x \cdot e^{xy}$ show that $\frac{dy}{dx} = \frac{y(1 + xy)}{x(1 - xy)}$. (3)

(B) Attempt any ONE of the following :

(i) Simplify and show that $x \cdot [(x' + z) \cdot y] = x \cdot y \cdot z)$ using Boolean Algebra. (2)

(ii) For the Boolean function $f(x_1, x_2) = (x_1 \cdot x_2') + (x_1' \cdot x_2)$ write down its value in tabular form. (2)

Q. 5 (A) Attempt any TWO of the following : **[8]**

(i) Evaluate : $\lim_{x \to 0} \frac{\sin(x + a) - \sin(x - a) - 2\sin a}{x \cdot \sin x}$ (3)

(ii) Evaluate : $\lim_{x \to 1} \frac{ab^x - ba^x}{x - 1}$ (3)

(iii) Find k if the function $f(x) = 3x - 4$, for $0 \leq x \leq 2$

$= 2x + k$, for $2 x \leq 4$.

is continuous at $x = 2$. (3)

(B) Attempt any ONE of the following :

(i) Solve : $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$. (2)

(ii) Verify that $y = Ae^x + Be^{-2x}$ is a solution of the Differential Equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$. (2)