

MATHS QUESTION PAPER

Time : 2 Hrs.

Max. Marks : 40

Q. 1 (A) Attempt any TWO of the following :

[8]

(i) Which of the following sentences are statements ?

(3)

(a) A cyclic trapezium has its non-parallel sides congruent.

(b) Do you like Mathematics ?

(c) The sun rises in the east.

(ii) Determine whether the following statement pattern is a 'tautology' or a 'contradiction' or 'neither' of the two : $(\sim p \vee q) \rightarrow p \wedge (q \vee \sim q)$

(3)

(iii) Using the truth table, show that

$$\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$$

(3)

(B) Attempt any ONE of the following :

(i) Find the values of K if the lines represented by $K(x^2 + y^2) = 8xy$ are co-incident.

(2)

(ii) Find the lengths of intercepts made by the circle

$$x^2 + y^2 + 2x - 2y - 3 = 0$$

(2)

Q. 2 (A) Attempt any TWO of the following :

[8]

(i) Find 'P' if the vectors $\vec{i} + \vec{p}\vec{j} + 3\vec{k}$, $2\vec{i} + \vec{j} - 4\vec{k}$ and $\vec{i} - \vec{j} + \vec{k}$ are co-planar.

(3)

(ii) Using Vector method, show that the "medians of a triangle are concurrent."

(3)

(iii) If \vec{a} , \vec{b} , \vec{c} are three non-zero, non-coplanar vectors, then prove that and vector \vec{r} in the space can be uniquely expressed as a linear combination $x\vec{a} + y\vec{b} + z\vec{c}$, where x , y and z are scalars.

(3)

(B) Attempt any ONE of the following :

(i) If D is the mid-point of seg AB and E is the mid-point of seg CD,

$$\text{then show that } \vec{AC} + \vec{BD} = 2\vec{DE}.$$

(2)

(ii) Find the volume of the parallelepiped formed by the vectors

$$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}, \vec{b} = 2\vec{i} + \vec{j} + 4\vec{k} \text{ and } \vec{c} = 3\vec{i} + 4\vec{j} - \vec{k}$$

Q. 3 (A) (a) Attempt any ONE of the following :

(i) If $A = \begin{bmatrix} 4 & 3 & 3 \\ -1 & 0 & -1 \\ -4 & -4 & -3 \end{bmatrix}$, then find A^{-1} by Adjoint method.

(3)

(ii) Solve the following equations by Reduction method.

$$x - y + z = 2; 2x + y - z = 7; x + 2y + z = 8$$

(3)

(b) Attempt any ONE of the following :

(i) If ' θ ' is the measure of acute angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$, then

$$\text{show that } \tan \theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

(3)

(ii) Find equation of a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at the point $P(x_1, y_1)$.

(3)

(B) Attempt any ONE of the following :

(i) A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5,760 to invest and has space for 20 items only. A fan costs Rs. 360 and a sewing machine costs Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that he can sell all items that he can buy, formulate this problem as a L.P.P. to maximize his profit.

(2)

(ii) Two food products A and B are to be purchased. Their contents and price per unit are given in the following table.

(2)

Product	A	B
Calories	2	3
Vitamins	2	1
Price	3	4

Minimum requirements of calories and vitamins are 6 and 14 units respectively. Formulate this problem as a L.P.P. to minimize the cost.

Q. 4 (A) (a) Attempt any ONE of the following :

[8]

(i) Find the values of p and q if the equation $12x^2 + 7xy - py^2 + 8x + qy + 6 = 0$ represents a pair of perpendicular lines.

(3)

(ii) Find the equation of the circle passing through the points $(1, -4)$ $(5, 2)$ and whose centre lies on the line $x - 2y + 9 = 0$. (3)

(b) Attempt any ONE of the following :

(i) If A and B are two events of a sample space S such that $P(A) = \frac{5}{8}$, $P(B) = \frac{3}{4}$

and $P(A \cap B) = \frac{1}{2}$, then find : (a) $P(A \cap B)$, (b) $P(A/B)$, (c) $P(A \cup B)$. (3)

(ii) If a coin is tossed three times and X = number of heads, then find $E(X)$ and $V(X)$. (3)

(B) Attempt any ONE of the following :

(i) Find the equation of a parabola whose vertex is at the origin, having X axis as the axis and passing through the point $(3, 6)$. (2)

(ii) Find the co-ordinates of the foci and length of latus rectum of the ellipse $3x^2 + 4y^2 = 12$. (2)

Q. 5 (A) (a) Attempt any ONE of the following : [8]

(i) Find the equation of the tangents to the hyperbola $16x^2 - 25y^2 = 400$ passing through the point $(1, -4)$. (3)

(ii) Find the equation of common tangents to the parabolas $y^2 = 4x$ and $x^2 = 4y$. (3)

(b) Attempt any ONE of the following :

(i) Find the equation of a line in the Cartesian form passing through the point $(3, 2, -1)$ and perpendicular to the vectors $3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ and $\mathbf{i} - \mathbf{j} + \mathbf{k}$. (3)

(ii) Find the Cartesian equation of the plane passing through the points $(1, 1, 1)$, $(2, 4, 3)$ and $(3, 9, 7)$ using Vector method. (3)

(B) Attempt any ONE of the following :

(i) Find the equation of a normal to the ellipse $2x^2 + 3y^2 = 30$ at the point $(-3, 2)$. (2)

(ii) If the line $y = mx + \sqrt{a^2 m^2 - b^2}$ touches the hyperbola $b^2 x^2 - a^2 y^2 = a^2 b^2$ at the point $P(a \sec \theta, b \tan \theta)$, then show that $\sin \theta = \frac{b}{am}$. (2)