MATHS QUESTION PAPER

Time : 2 Hrs. **Max. Marks : 40**

Q. 1 (A) Attempt any TWO of the following : **[8]**

(i) Evaluate : $\lim_{x \to \frac{\pi}{2}} \frac{3^{(x - \frac{\pi}{2})} - 6^{(x - \frac{\pi}{2})}}{\cos x}$ (3)

(ii) Given : $f(x) \frac{\log x - \log 3}{x - 3}$, for $x \neq 3$, If $f(x)$ is continuous at $x = 3$, find $f(3)$. (3)

(iii) Test the continuity of the function f at $x = 0$, where

$f(x) = x^2 \sin\left(\frac{1}{x}\right)$, for $x \neq 0$

$= 1$ for $x = 0$ (3)

(B) Attempt any ONE of the following :

(i) Evaluate : $\int \frac{\sin x}{\cos (x - a)} dx$ (2)

(ii) Evaluate : $\int \log x \, dx$ (2)

Q. 2 (A) Attempt any TWO of the following : **[8]**

(i) $\sin^{-1}\left(\frac{5 \sin x + 4 \cos x}{\sqrt{41}}\right)$ w. r. t. x. (3)

(ii) Find $\frac{dy}{dx}$, if $x^y = 2^{x - y}$ (3)

(iii) Examine the function $f(x) = 2x^3 - 9x^2 + 12x + 5$. For maxima and minima. (3)

(B) Attempt any ONE of the following :

(i) Solve the differential equation : $\frac{dy}{dx} = e^{x + y} + x^2 e^y$ (2)

(ii) Show that $y = \cos (x + 5)$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$ (2)

Q. 3 (A) (a) Attempt any ONE of the following : **[8]**

(i) Evaluate : $\int \frac{dx}{(x - 1)^2 (x + 1)}$ (3)

(ii) Evaluate : $\int \frac{dx}{3 \sin x + 4 \cos x + 5}$ (3)

(b) Attempt any ONE of the following :

(i) Evaluate : $\int_0 x^2 e^x \, dx$ (3)

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(ii) Evaluate : $\int e^x (1 + \tan x + \tan^2 x) \, dx$ (3)

(B) Attempt any ONE of the following :

(i) Differentiate x^5 w. r. t. 5^x (2)

(ii) Find $\frac{dy}{dx}$, if $y = \log_2 x + \log_3 x$ (2)

Q. 4 (A) (a) Attempt any ONE of the following : **[8]**

(i) If the interval of differencing is 1 show that

$f(5) = f(4) + \Delta f(3) + \Delta^2 f(2) + \Delta^3 f(1) + \Delta^4 f(1)$ (3)

(ii) With usual notations prove that : $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\Delta}{\nabla}$ (3)

(b) Attempt any ONE of the following :

(i) Form the differential equation by eliminating arbitrary constants A and B from the relation $y = Ae^{3x} + Be^{-2x}$ (3)

(ii) Solve the differential equation : $(x + y)^2 \frac{dy}{dx} = a^2$, by using $x + y = u$. (3)

(B) Attempt any ONE of the following :

(i) If B is a Boolean algebra, for $x \in B$ prove that (a) $x + 1 = 1$ (b) $x \cdot 0 = 0$ (2)

(ii) If B is a Boolean algebra, for $x, y \in B$ prove that : $x \cdot (x + y) = x$ (2)

Q. 5 (A) (a) Attempt any ONE of the following : [8]

(i) If x and y are differentiable functions of t so that y is a function of x,

then prove that $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$, where $\frac{dx}{dt} \neq 0$ (3)

(ii) If y is a differentiable function of u and u is a differentiable function Of x, then prove that $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ (3)

(b) Attempt any ONE of the following :

(i) If $x = \phi(t)$ is a differentiable function of t, then prove that $\int f(x)\, dx = \int f[\phi(t)]\, \phi'(t)\, dt$ (3)

(ii) Prove that $\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$ (3)

(B) Attempt any ONE of the following :

(i) Construct input/output table for the Boolean function 'f' given by $f(x_1, x_2) = x_1 \cdot x'_2$ (2)

(ii) Determine the Booleanexpression for the following switching circuit (2)