MATHS QUESTION PAPER

Time : 2 Hrs. Max. Marks : 40

Q. 1 (A) (a) Attempt any TWO of the following : [8]

(i) If $y = f(x)$ and $x = g(y)$, where g is the inverse function of f and if $\frac{dy}{dx}$ and $\frac{dx}{dy}$ both exist and $\frac{dx}{dy} \neq 0$, then prove that

$$\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

(ii) If x and y are differentiable functions of t, so that y is a function of x, then prove that

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}; \text{ if } \frac{dx}{dt} \neq 0$$ (3)

(b) Attempt any ONE of the following :

(i) If u and v are functions of x, then prove that

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$$ (3)

(ii) Prove that,

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx, \text{ if } f(x) \text{ is an even function.}$$

(B) Attempt any ONE of the following :

(i) If B is a Boolean algebra and $x_1, x_2 \in B$, then show that $x_1 \cdot (x'_1 + x_2) = x_1 \cdot x_2$ (2)

(ii) If B is a Boolean algebra and $x, y, z \in B$, write the duals of the following :

(a) $(x' + y' + z')' + (x \cdot y \cdot z') + (x' \cdot y \cdot z) = y \cdot (x + z)$

(b) If $x + y = 0$, then $x = 0 = y$ (2)

Q. 2 (A) (a) Attempt any ONE of the following : [8]

(i) Show that,

$\Delta^4 f(x) = f(x + 4h) - 4f(x + 3h) + 6f(x + 2h) - 4f(x + h) + f(x)$ (3)

(ii) If $f(x) = e^x$, show that

$f(x), \Delta f(x), \Delta^2 f(x), \ldots\ldots, \Delta^n f(x)$ are in Geometric Progression. (3)

(b) Attempt any ONE of the following :

(i) Solve the differential equation

$e^x \cdot \tan^2 y\,dx + (e^x - 1)\sec^2 y\,dy = 0$ (3)

(ii) Solve the differential equation

$\frac{dy}{dx} = \frac{y + \sqrt{x^2 - y^2}}{x}$, by putting $y = v \cdot x$ (3)

(B) Attempt any ONE of the following :

(i) Form the differential equation by eliminating the arbitrary constant from the equation

$y = c^2 + \frac{c}{x}$ (2)

(ii) Determine the order and the degree of the differential equation : $\frac{d^2y}{dx^2} = \sqrt[3]{1 - \left(\frac{dy}{dx}\right)^4}$ (2)

Q. 3 (A) (a) Attempt any ONE of the following : [8]

(i) Evaluate : $\int \frac{5x + 2}{x^3 - 3x + 2}\,dx$ (3)

(ii) Evaluate : $\int \operatorname{cosec}^3 x\,dx$ (3)

(b) **Attempt any ONE of the following :**

(i) Evaluate $\int_0^1 x^2 (1-x)^{\frac{5}{2}} dx$ (3)

(ii) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{2 + 2\sin^2 x + \cos^2 x} dx$ (3)

(B) Attempt any ONE of the following :

(i) Evaluate : $\int \frac{\sec^2 (\log x)}{x} dx$ (2)

(ii) Evaluate : $\int \frac{dx}{1 + \cos 2x}$ (2)

Q. 4 (A) (a) Attempt any TWO of the following : **[8]**

(i) Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ (3)

(ii) If $y = e^{m \cdot \tan^{-1} x}$ then show that,

$(1+x^2)\frac{d^2y}{dx^2} + (2x - m)\frac{dy}{dx} = 0$ (3)

(iii) Find approximately $\sin 31°$, given that $1° = 0.0175^C$ and $\cos 30° = 0.8660$ (3)

(B) Attempt any ONE of the following :

(i) Divide 20 into two parts so that their product is maximum. (2)

(ii) If $x = \cos (xy)$, find $\frac{dy}{dx}$. (2)

Q. 5 (A) Attempt any ONE of the following : **[8]**

(i) Evaluate : $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\pi - 4x}$ (3)

(ii) Evaluate : $\lim_{x \to e} \frac{\log x - 1}{x - e}$ (3)

(iii) Find the value of 'k',

if the function $f(x) = \frac{1 - \cos kx}{x \cdot \sin x}$, for $x \neq 0$

$= 2$, for $x = 0$ (3)

is continuous at $x = 0$.

(B) Attempt any ONE of the following :

(i) If B is a Boolean algebra and $x \in B$, then show that $x + x = x$ (2)

(ii) Draw the switching circuit of the Boolean expression : $[a \cdot b' \cdot (c + d)] + [a' \cdot (b + c)]$ (2)