MATHS QUESTION PAPER

Time : 2 Hrs. Max. Marks : 40

Q. 1 (A) Attempt any TWO of the following :

(i) Prepare the truth table of the statement pattern
$[p \to (q \to r)] \leftrightarrow [(p \wedge q) \to r]$ **(3)**

(ii) Construct a switching circuit represented by the statement
$[(p \wedge \sim q) \wedge r] \vee [p \wedge (\sim q \vee \sim r)]$ **(3)**

(iii) Write the negations of the following statements, using the rules of negations : **(3)**
(a) $\sim q \to p$ (b) $(p \wedge \sim q)$ (c) $p \vee \sim q$

(B) Attempt any ONE of the following :

(i) Find 'k' if the line $x - y + 3 = 0$ touches the parabola $y^2 = 4kx$. **(2)**

(ii) Find the eccentricity and the length of latus-rectum for the hyperbola
$\frac{x^2}{25} - \frac{y^2}{9} = 1$. **(2)**

Q. 2 (A) Attempt any TWO of the following :

(i) Using vector method, prove that the medians of a triangle are concurrent. **(3)**

(ii) If $\bar{a}, \bar{b}, \bar{c}$ are the conterminus edges of a parallelopiped, then prove that its volume is given by $[\bar{a}\ \bar{b}\ \bar{c}]$ **(3)**

(iii) Express $\bar{r} = 9\bar{i} + 2\bar{j} - 7\bar{k}$, as a linear combination of $\bar{a} = 3\bar{i} + 2\bar{j} - 7\bar{k}$, and $\bar{b} = \bar{i} + 2\bar{j} - 3\bar{k}$ **(3)**

(B) Attempt any ONE of the following :

(i) Find the equation of the tangents to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, making an angle of $60°$ with major axis. **(2)**

(ii) Find the equation of normal to the hyperbola $x^2 - 4y^2 = 36$, at the point (10, 4). **(2)**

Q. 3 (A) (a) Attempt any ONE of the following :

(i) Find the inverse of $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ using elementary transformation. **(3)**

(ii) Solve the equations $x + 2y + 3z = 9$, $2x + 3y + z = 4$, $4x + 5y + 4z = 15$ by using reduction method. **(3)**

(b) Attempt any ONE of the following :

(i) Show that the acute angle 'θ' between the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$, is given by
$\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$ **(3)**

(ii) Derive the equation of the tangle to the circle,
$x^2 + y^2 + 2gx + 2fy + c = 0$ at a point $P(x_1, y_1)$. **(3)**

(B) Attempt any ONE of the following :

(i) If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively such that $5\bar{a} + 7\bar{b} - 12\bar{c} = 0$, find the ratio in which the point A divides segment BC. **(2)**

(ii) If the vectors, $\bar{i} + \bar{j} + \bar{k}$, $2\bar{i} + \lambda\bar{j} + 3\bar{k}$, and $5\bar{i} + 9\bar{j} + 7\bar{k}$ are coplanar, find λ. **(2)**

Q. 4 (A) (a) Attempt any ONE of the following :

(i) Find the joint equation of pair of lines through the origin and perpendicular to the lines given by $5x^2 - 8xy + 13y^2 = 0$ **(3)**

(ii) Find the equation of the circle touching both the co-ordinate axes and passing through the point (1,2). **(3)**

(b) **Attempt any ONE of the following :**

(i) A four digit number is formed by using the digits 1, 2, 3, 4, 5, 6, 7. Find the probability that it is greater than 3000 if repetition of digits is not allowed. **(3)**

(ii) Three unbiased coins are tossed. Find the probability distribution of the number of heads occurring on the topmost faces. **(3)**

(B) Attempt any ONE of the following :

(i) Write the following L.P.P. in standard form for the Simplex method. **(2)**
Maximize $Z = 4x_1 + 6x_2$
Subject to $x_1 + 3x_2 \leq 240,\ 3x_1 + 4x_2 \leq 370,\ 2x_1 + x_2 \leq 180,\ x_1 \geq 0,\ x_2 \geq 0$.

(ii) Food X contains 6 units of Vitamin A and 7 unit of Vitamin B per gram and costs Rs. 12 per gram. Food Y contains 8 units of Vitamin A and 12 units of Vitamin B per gram and costs Rs. 20 per gram. The daily minimum requirements of Vitamin A and Vitamin B are 100 units and 120 units respectively. Formulate the above L.P.P. to minimize the cost. **(2)**

Q. 5 (A) (a) Attempt any ONE of the following :

(i) The line $x - y - 5 = 0$ touches the ellipse whose foci are $(\pm 3, 0)$. Find the equation of the ellipse. **(3)**

(ii) Show that the normal to the parabola $y^2 = 8x$ at the point (2, 4) meets the parabola again at (18, – 12). **(3)**

(b) Attempt any ONE of the following :

(i) Using vector method, find the equation of line passing through the points A (3, 2, – 1) and B (4, – 1, 3).
Also, write the equation in Cartesian form. **(3)**

(ii) Find the vector equation of the plane passing through the points (2, 2, – 1), (3, 4, 1) and (7, 0, 6). **(3)**

(B) Attempt any ONE of the following :

(i) Find 'k' if the length of the tangent segment from the point (8, – 3) to the circle $x^2 + y^2 - 2x + ky - 23 = 0$ is $\sqrt{10}$ units. **(2)**

(ii) Find 'k' if the slope of one of the lines given by $kx^2 + 4xy - y^2 = 0$ is 3 times the other. **(2)**