MATHS QUESTION PAPER

Time : 2 Hrs. Max. Marks : 40

Note :

(i) All questions are compulsory.

(ii) Figures to the right indicates full marks.

(iii) Graph paper is not necessary for LPP.

(iv) Answer to every question must be written on a new page.

Q. 1 (A) Attempt any two of the following : [8]

(i) If the statements p and q are true and statement r is false, then find the truth values of the following statement patterns -

(a) $\sim p \leftrightarrow (\sim q \wedge r)$

(b) $\sim q \wedge (p \rightarrow q)$

(c) $p \vee (\sim q \leftrightarrow \sim r)$ (3)

(ii) By using truth table, prove that $\sim (p \rightarrow \sim q) \equiv (p \wedge q)$ (3)

(iii) Construct the switching circuit for the following logical statement -

$(\sim p \wedge q) \vee (p \wedge \sim r)$ (3)

(B) Attempt any one of the following :

(i) A toy manufacturing firm produces toys T_1 and T_2, each of which must be processed through two machines M_1 and M_2. The maximum availability of machines M_1 and M_2 per day are 14 hours and 20 hours respectively. Manufacturing of toy T_1 requires 5 hours on machine M_1 and 3 hours on machine M_2, whereas toy T_2 requires 4 hours on machine M_1 and 6 hours on machine M_2. If the profit on manufacturing of toy T1 is Rs. 50 and profit on manufacturing of toy T_2 is Rs. 70, formulate this problem as L. P. P. in order to maximum the profit. (2)

(ii) Draw the graph and state only the vertices of feasi bleregion of the following inequalities - $3x + 5y \leq 15, 5x + 2y \leq 10, x \geq 0, y \geq 0$. (2)

Q. 2 (A) Attempt any two of the following : [8]

(i) If $\bar{a}$, $\bar{b}$ and $\bar{p}$ are position vectors of points A, B and P respectively and P divides the line AB in the ratio m : n externally, then prove that -

$$p = \frac{mb - na}{m - n}$$ (3)

(ii) By vector method, prove that in any circle, angle subtended on a semicircle is a right angle. (3)

(iii) Find x, if the points A (3, 2, 1), B (4, x, 5), C (4, 2, 2) and D (6, 5, − 1) are coplanar. (3)

(B) Attempt any one of the following :

(i) If a, b, c are non-zero, non co-planar vectors and

$$p \; \frac{\bar{b} \times \bar{c}}{[\bar{a}\ \bar{b}\ \bar{c}]} ; q = \frac{\bar{c} \times \bar{a}}{[\bar{a}\ \bar{b}\ \bar{c}]} ; r \; \frac{\bar{a} \times \bar{b}}{[\bar{a}\ \bar{b}\ \bar{c}]}$$

then prove that $(a + b) \cdot p\ (b + c) \cdot q\ (c + a) \cdot r = 3$. (2)

(ii) If p, q, r are position vectors of the points P, Q, R respectively,

where $\bar{p} = \bar{a} + 2\bar{b} + 5\bar{c}$, $\bar{q} = 3\bar{a} + 2\bar{b} + \bar{c}$, $\bar{r} = 2\bar{a} + 2\bar{b} + 3\bar{c}$

then show that the points P, Q, R are collinear. (2)

Q. 3 (A) (a) Attempt any one of the following : [8]

(i) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ by using elementary row transformations. (3)

(ii) Solve the following equations by reduction method -
$2x + 3y + 4z = -9, 3x + y + 2z = -12, 4x + 2y + 2z + 12 = 0.$ (3)

(b) Attempt any one of the following :

(i) Prove that the acute angle θ between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is given by $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$. (3)

(ii) Find the equations of the tangent and the normal to the circle $x^2 + y^2 = a^2$ at a point $P(x_1, y_1)$. (3)

(B) Attempt any one of the following :

(i) Find k if one of the lines given by $kx^2 + 10xy + 8y^2 = 0$ is perpendicular to $2x - y = 5$. (2)

(ii) Find the equation of the circle with centre at (3, 2) and touching the line $4x + 3y - 8 = 0$. (2)

Q. 4 (A) (a) Attempt any one of the following : [8]

(i) Find the value of k, if the following equation represents a pair of lines.
$3x^2 + 10xy + 3y^2 + 16y + k = 0$ (3)
Further find whether these lines are parallel or intersecting. (3)

(ii) Find the equation of the tangents to the circle $x^2 + y^2 + 6x + 4y + 3 = 0$ which are perpendicular to the line $3x + 9y = 8$. (3)

(b) Attempt any one of the following :

(i) If $P(A) = \frac{2}{3}$, $P(B) = \frac{3}{5}$ and $P(A \cap B) = \frac{2}{5}$, find $P(A' \cap B')$ and $P(A'/B)$ (3)

(ii) From a group of 4 men, 3 children and 3 women, 4 persons are selected at random. Find the probability that it contains exactly 2 children. (3)

(B) Attempt any one of the following :

(i) Find the equation of the tangent to the ellipse $4x^2 + 9y^2 = 36$ making equal intercepts on the co-ordinate axes. (2)

(ii) Find the equation of the tangent to the hyperbola $2x^2 - 3y^2 = 5$ at the point $(-2, -1)$. (2)

Q. 5 (A) (a) Attempt any one of the following: [8]

(i) Find the equation of the hyperbola in standard form whose distance between the directrices is $\frac{8}{3}$ and eccentricity is $\frac{3}{2}$. (3)

(ii) Find the equation of the locus of point, the tangents from which to the parabola $y^2 = 4ax$ are such that the sum of their slopes is 2. (3)

(b) Attempt any one of the following:

(i) By vector method, find the equation of the line passing through the point A $(2, -3, -4)$ and parallel to OB, where point O is origin and B is $(2, -2, -1)$. (3)

(ii) By vector method, find the equation of a plane through the point $(1, -1, 1)$ which is parallel to the vectors $2\bar{i} - \bar{j} - \bar{k}$ and $7i + j - 3\bar{k}$. (3)

(B) Attempt any one of the following :

(i) Find the value of k, if the line $y = 2x + k$ is tangent to the parabola $y^2 = 8x$. (2)

(ii) Find the eccentricity and foci of the ellipse $\frac{x^2}{5} + \frac{y^2}{2} = 1$. (2)