MATHS QUESTION PAPER

Time : 2 Hrs. Max. Marks : 40

Note :

(i) All questions are compulsory. (ii) Figure to right indicates full marks.
(iii) Graph paper is not necessary for LPP. (iv) Answer to every question must be written on a new page.

Q. 1 (A) Attempt any two of the following : [8]

(i) Evaluate : $\lim_{x \to 2} (x - i)^{\frac{1}{3x-6}}$ (3)

(ii) Evaluate : $\lim_{x \to 1} \frac{4^{x-1} - 2^x + 1}{(x-1)^2}$ (3)

(iii) If a function 'f' is continuous at x = 0 where,

$f(x) = \frac{\sin 3x}{5x} + a$, for $x < 0$

$= x + 4 - b$, for $x \geq 0$. find the value of a + b. (3)

(B) Attempt any one of the following :

(i) Evaluate : $\int \frac{dx}{x + \sqrt{x}}$ (2)

(ii) Evaluate : $\int \frac{dx}{1 + \sin x}$ (2)

Q. 2 (A) Attempt any two of the following : [8]

(i) If $y = \tan^{-1}\left(\frac{5x+1}{3 - x - 6x^2}\right)$ show that $\frac{dy}{dx} = \frac{3}{1 + (3x+2)^2} + \frac{2}{1 + (2x-1)^2}$ (3)

(ii) If $2y = \sqrt{x+1} + \sqrt{x-1}$, show that $4(x^2 - 1)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - y = 0$ (3)

(iii) Find the approximately value of $\tan^{-1}(0.999)$ (3)

(B) Attempt any one of the following: (2)

(i) Differentiate $(x^x + a^a)$ w.r.t. x (2)

(ii) Find $\frac{dy}{dx}$, if $y = \tan(x.e^x)$ (2)

Q. 3 (A) (a) Attempt any one of the following : [8]

(i) Evaluate : $\int \sin(\log x)\, dx$ (3)

(ii) Evaluate : $\int \frac{x\, dx}{(x-1)(x^2+1)}$ (3)

(b) Attempt any one of the following:

(i) Evalute : $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$ (3)

(ii) Show that : $\int_0^1 \frac{dx}{\sqrt{x^2 - x + 1}} - \log 3$ (3)

B) Attempt any one of the following : (2)

(i) Form the differentiate equation by eliminating the arbitrary constants a and b from the relation $y = ae^{2x} + be^{-2x}$. (2)

(ii) Solve the differential equation : $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$

Q. 4 (A) (a) Attempt any one of the following : [8]

(i) Prove that $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{E(e^x)}{\Delta^2(e^x)} = e^x$. (3)

(ii) Using the relation between Δ and E, estimate the missing term in the following table : (3)

x	0	1	2	3	4
f (x)	−5	−2	7		91

(b) **Attempt any one of the following :**

(i) Solve the differential equation : (3)

$\frac{dy}{dx} = (9x + y + 2)^2$, by using $9x + y + 2 = u$.

(ii) Find the particular solution of the differential equation : (3)

$y(1 + \log x)\frac{dx}{dy} - x \log x = 0$, when $x = e$ and $y = e^2$.

B) Attempt any one of the following :

(i) In a Boolean algebra, prove that, the zero element '0' and unit element '1' are unique. (2)

(ii) If B is a Boolean algebra, for $x \in B$ prove that : (2)

(a) $x + x = x$ (b) $x \cdot x = x$.

). 5 (A) (a) Attempt any one of the following : [8]

(i) If y is a differentiable function of u, and u is a differentiable function of x, then show that, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ (3)

(ii) If $y = f(x)$ is a differentiable function of x such that, the inverse function $x = f^{-1}(y)$ is defined.

Then prove that $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$, where $\frac{dy}{dx} \neq 0$. (3)

(b) **Attempt any one of the following :**

(i) Prove that : (3)

$$\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2} \log\left(x + \sqrt{x^2 + a^2}\right) + c.$$

(ii) Prove that : $\int_{-a}^{a} f(x)\, dx = 2\int_{0}^{a} f(x)\, dx$, if f (x) an even function.

$= 0$, if f (x) is an odd function. (3)

3) Attempt any one of the following : (2)

(i) Draw the switching circuit of the Boolean expression $a\,[b \cdot (c + a')]$. (2)

(ii) Simplify the switching circuit given below : (2)