WHAT WE HAVE DISCUSSED

1. In a right angle triangle ABC, right angle is at B,

$$\sin A = \frac{\text{Side opposite to angle A}}{\text{Hypotenuse}}, \quad \cos A = \frac{\text{Side adjacent to angle A}}{\text{Hypotenuse}}$$

2. $\operatorname{cosec} A = \frac{1}{\sin A}$; $\sec A = \frac{1}{\cos A}$; $\tan A = \frac{\sin A}{\cos A}$; $\tan A = \frac{1}{\cot A}$

3. If one of the trignometric ratios of an acute angle is known, the remaining trignometric ratios of the angle can be determined.

4. The trignometric ratios for angle 0^o, 30^o, 45^o, 60^o and 90^o.

5. The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than or equal to 1.

6. $\sin(90^o - A) = \cos A$, $\cos(90^o - A) = \sin A$

   $\tan(90^o - A) = \cot A$, $\cot(90^o - A) = \tan A$

   $\sec A(90^o - A) = \operatorname{cosec} A$, $\operatorname{cosec}(90^o - A) = \sec A$

7. $\sin^2 A + \cos^2 A = 1$

   $\sec^2 A - \tan^2 A = 1$ for $0^o \le A \le 90^o$

   $\operatorname{cosec}^2 A - \cot^2 A = 1$ for $(0^o \le A \le 90^o)$

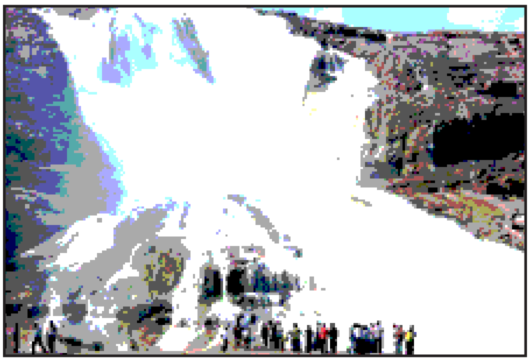
CHAPTER 12
Applications of Trigonometry

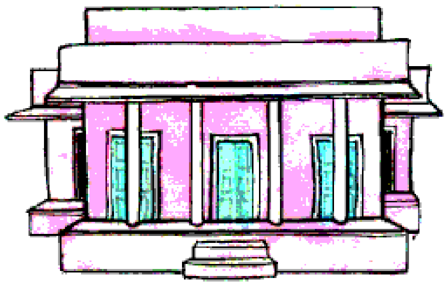
12.1 Introduction

You have studied in social studies that the highest mountain peak in the world is Mount Everest and its height is 8848 meters.

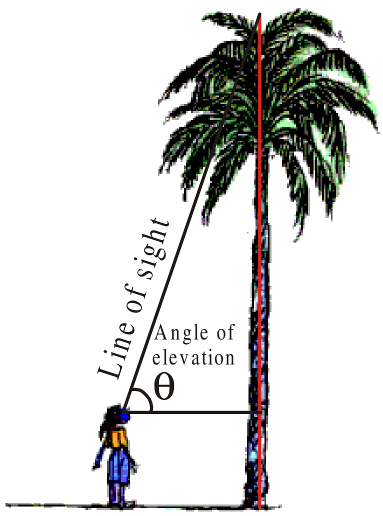
Kuntala waterfall in Adilabad district is the highest natural waterfall in Andhra Pradesh. Its height is 147 feet.

How were these heights measured? Can you measure the height of your school building or the tallest tree in or around your school?





Let us understand through some examples. Vijaya wants to find the height of a palm tree. She tries to locate the top most point of the tree. She also imagines a line joining the top most point and her eye.

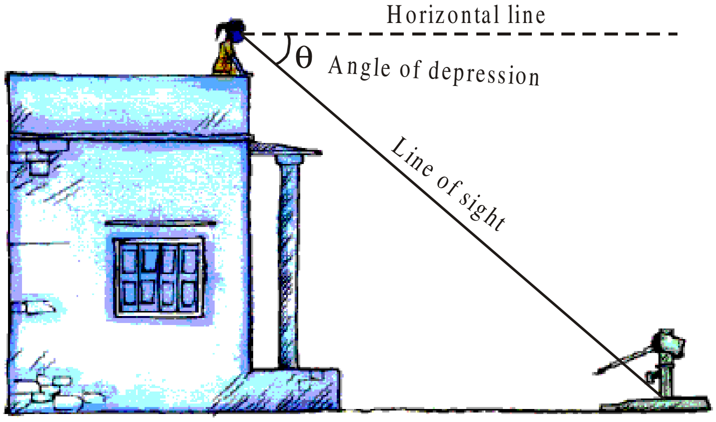


This line is called "line of sight". She also imagines a horizontal line, parallel to earth, from her eye to the tree.

Here, "the line of sight", "horizontal line" and "the tree" form a right angle triangle.

To find the height of the tree, she needs to find a side and an angle in this triangle.

"The line of sight is above the horizontal line and angle between the line of sight and the horizontal line is called **angle of elevation**".



Suppose you are standing on the top of your school building and you want to find the distance of borewell from the building on which you are standing. For that, you have to observe the base of the borewell.

Then, the line of sight from your eye to the base of borewell is below the horizontal line from your eye.

Here, "the angle between the line of sight and horizantal line is called **angle of depression**."

> Trigonometry has been used by surveyors for centuries. They use Theodolites to measure angles of elevation or depression in the process of survey. In nineteenth century, two large Theodolites were built by British India for the surveying project "great trigonometric survey". During the survey in 1852, the highest mountain peak in the world was discovered in the Himalayas. From the distance of 160 km, the peak was observed from six different stations and the height of the peak was calculated. In 1856, this peak was named after Sir George Everest, who had commissioned and first used the giant Theodolites. Those theodolites are kept in the museum of the Survey of India in Dehradun for display.

12.2 Drawing Figures to Solve Problems

When we want to solve the problems of heights and distances, we should consider the following:

(i) All the objects such as towers, trees, buildings, ships, mountains etc. shall be considered as linear for mathematical convenience.

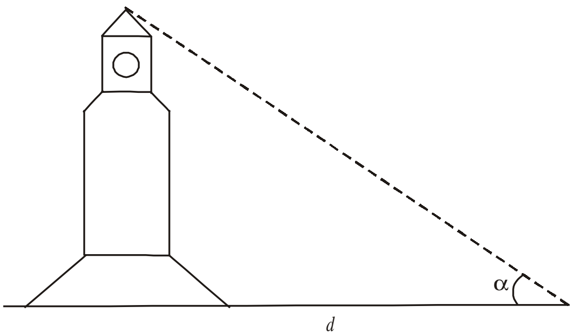
(ii) The angle of elevation or angle of depression is considered with reference to the horizontal line.

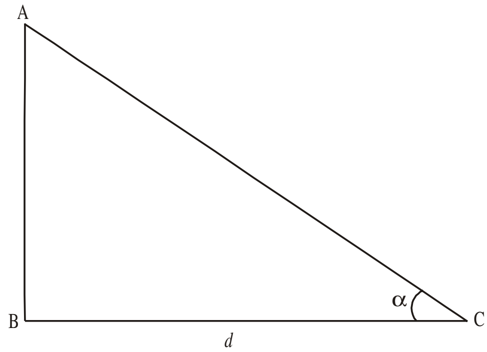
(iii) The height of the observer is neglected, if it is not given in the problem.

When we try to find heights and distances at an angle of elevation or depression, we need to visualise geometrically. To find heights and distances, we need to draw figures and with the help of these figures we can solve the problems. Let us see some examples.

Example-1. The top of a clock tower is observed at angle of elevation of α^o and the foot of the tower is at the distance of d meters from the observer. Draw the diagram for this data.

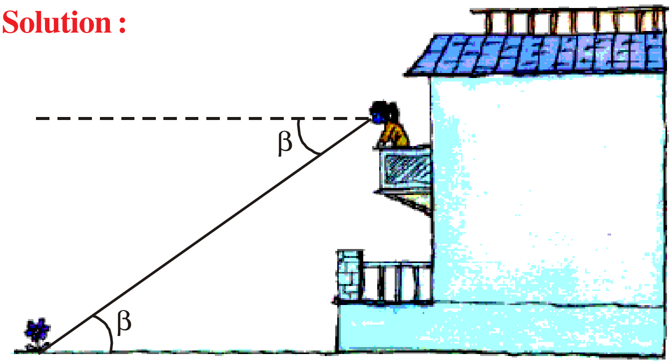
Solution : The diagrams are as shown below :

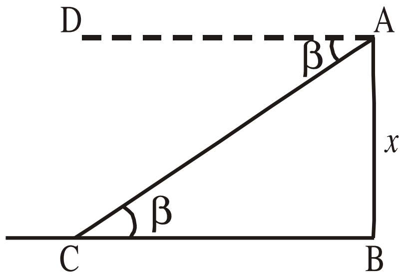




Example-2. Rinky observes a flower on the ground from the balcony of the first floor of a building at an angle of depression β^{o}. The height of the first floor of the building is x meters. Draw the diagram for this data.

Solution :



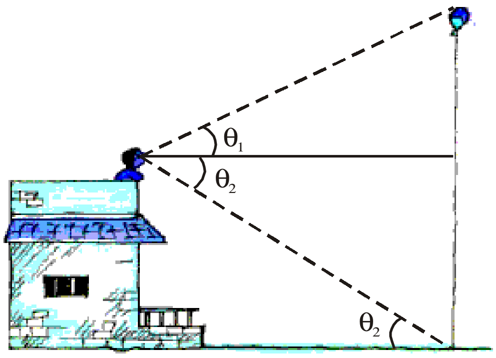


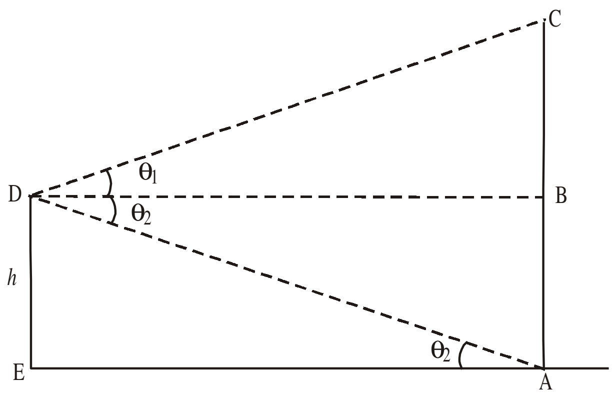
Here $\angle DAC = \angle ACB = \beta$ (why?)

Example-3. A large balloon has been tied with a rope and it is floating in the air. A person has observed the balloon from the top of a building at angle of elevation of θ_1 and foot of the rope at an angle of depression of θ_2. The height of the building is h feet. Draw the diagram for this data.

Solution : We can see that

$\angle BDA = \angle DAE$. (Why?)



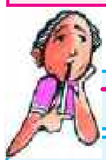


DO THIS

1. Draw diagram for the following situations :

   (i) A person is flying a kite at an angle of elevation α and the length of thread from his hand to kite is 'ℓ'.

   (ii) A person observes two banks of a river at angles of depression θ_1 and θ_2 ($\theta_1 < \theta_2$) from the top of a tree of height h which is at a side of the river. The width of the river is 'd'.



THINK - DISCUSS

1. You are observing top of your school building at an angle of elevation α from a point which is at d meter distance from foot of the building.

   Which trigonometric ratio would you like to consider to find the height of the building?

2. A ladder of length x meter is leaning against a wall making angle θ with the ground. Which trigonometric ratio would you like to consider to find the height of the point on the wall at which the ladder is touching?

Till now, we have discussed how to draw diagrams as per the situations given. Now, we shall discuss how to find heights and distances.

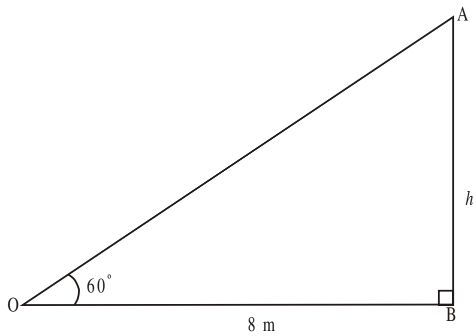
Example-4. A boy observed the top of an electric pole at an angle of elevation of 60º when the observation point is 8 meters away from the foot of the pole. Find the height of the pole.

Solution : From the figure, in triangle OAB

OB = 8 meters

$\angle AOB = 60^\circ$

Let, height of the pole = AB = h meters

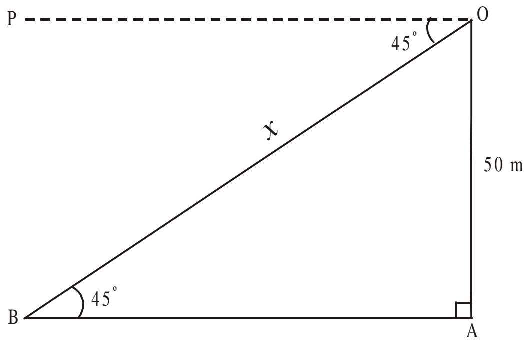


(we know the adjacent side and we need to find the opposite side of $\angle AOB$ in the triangle ΔOAB. Hence we need to consider the trigonometric ratio "tan" to solve the problem).

$$\tan 60^\circ = \frac{AB}{OB}$$

$$\sqrt{3} = \frac{h}{8} \qquad h = 8\sqrt{3}m.$$

Example-5. Rajender observes a person standing on the ground from a helicopter at an angle of depression 45°. If the helicopter flies at a height of 50 meters from the ground, what is the distance of the person from Rajender?



Solution : From the figure, in triangle OAB

OA = 50 meters

$\angle POB = \angle OAB = 45°$ (why ?)

OB = distance of the person from Rajender = x.

(we know the opposite side of $\angle OBA$ and we need to find hypotenuse OB in the triangle OAB. Hence, we need to consider the ratio "sin".)

$$\sin 45° = \frac{OA}{OB}$$

$$\frac{1}{\sqrt{2}} = \frac{50}{x}$$

$$x = 50\sqrt{2} \text{ meters}$$

(The distance from the person to Rajendar is $50\sqrt{2}$ m)

Exercise - 12.1

1. A tower stands vertically on the ground. From a point which is 15 meter away from the foot of the tower, the angle of elevation of the top of the tower is 45°. What is the height of the tower?

2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground by making 30° angle with the ground. The distance between the foot of the tree and the top of the tree on the ground is 6m. Find the height of the tree before falling down.

3. A contractor wants to set up a slide for the children to play in the park. He wants to set it up at the height of 2 m and by making an angle of 30° with the ground. What should be the length of the slide?

4. Length of the shadow of a 15 meter high pole is $5\sqrt{3}$ meters at 7 o'clock in the morning. Then, what is the angle of elevation of the Sun rays with the gound at the time?

5. You want to erect a pole of height 10 m with the support of three ropes. Each rope has to make an angle 30° with the pole. What should be the length of the rope?

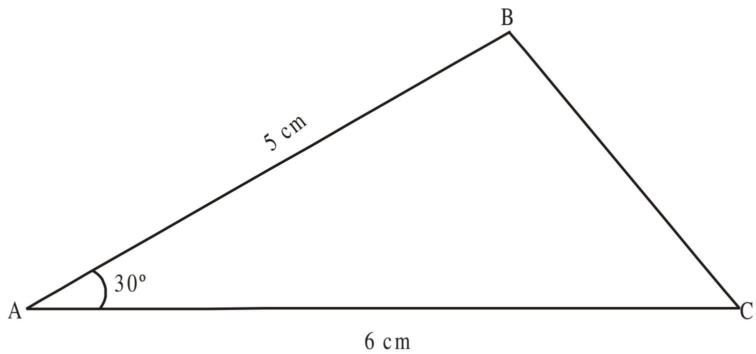
6. Suppose you are shooting an arrow from the top of a building at an height of 6 m to a target on the ground at an angle of depression of 60º. What is the distance between you and the object?

7. An electrician wants to repair an electric connection on a pole of height 9 m. He needs to reach 1.8 m below the top of the pole to do repair work. What should be the length of the ladder which he should use, when he climbs it at an angle of 60º with the ground? What will be the distance between foot of the ladder and foot of the pole?

8. A boat has to cross a river. It crosses the river by making an angle of 60º with the bank of the river due to the stream of the river and travels a distance of 600m to reach the another side of the river. What is the width of the river?

9. An observer of height 1.8 m is13.2 m away from a palm tree. The angle of elevation of the top of the tree from his eyes is 45º. What is the height of the palm tree?

10. In the adjacent figure, AC = 6 cm, AB = 5 cm and $\angle BAC$ = 30º. Find the area of the triangle.

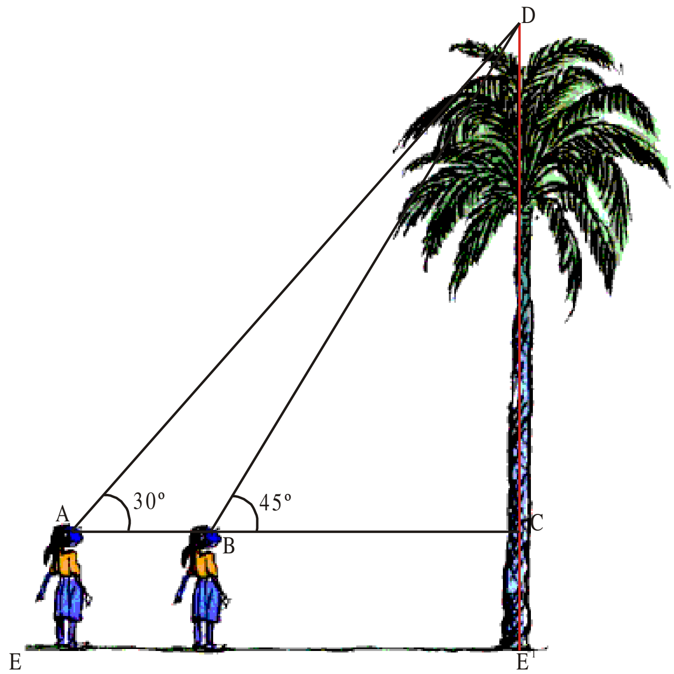


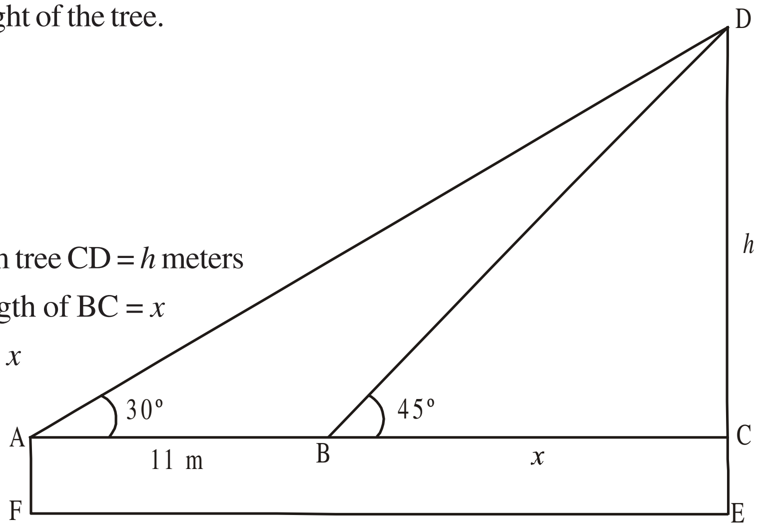
12.3 Solution for Two Triangles

We have discussed the solution of a one triangle problem. What will be the solution if there are two triangles?

Suppose you are standing on one side of a tree. You want to find the height of a tree and you want to observe the tree from different points of observations.

How can you do this? Suppose you are observing the top of the palm tree at an angle of elevation 45º. The angle of elevation changes to 30º when you move 11 m away from the tree.





Let us see how we can find height of the tree.

From figure, we have

$AB = 11$ m

$\angle DAC = 30^\circ$

$\angle DBC = 45^\circ$

Let the height of the palm tree CD = h meters

and length of BC = x

AC = 11 + x

from triangle BDC

$$\tan 45^\circ = \frac{DC}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = h\sqrt{3} \qquad \ldots(1)$$

from triangle ADC

$$\tan 30^\circ = \frac{DC}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{11+x}$$

$$h = \frac{11+x}{\sqrt{3}}$$

$$h = \frac{11}{\sqrt{3}} + \frac{h}{\sqrt{3}}$$

$$h - \frac{h}{\sqrt{3}} = \frac{11}{\sqrt{3}}$$

$$h\frac{\left(\sqrt{3}-1\right)}{\sqrt{3}} = \frac{11}{\sqrt{3}}$$

$$h = \frac{11}{\left(\sqrt{3}-1\right)} \text{ meters.}$$

Note **:** Total height of the palm tree is CD + CE where CE = AF, which is the height of the girl.

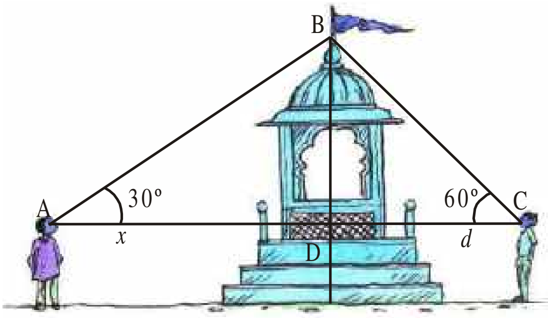
Example-6. Two men on either side of a temple of 30 meter height observe its top at the angles of elevation 30º and 60º respectively. Find the distance between the two men.

Solution : Height of the temple BD = 30 meter.

Angle of elevation of one person $\angle BAD = 30^\circ$

Angle of elevation of another person $\angle BCD = 60^\circ$

Let the distance between the first person and the temple, AD = x and distance between the second person and the temple, CD = d



From ΔBAD

$$\tan 30^\circ = \frac{BD}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{30}{x}$$

$$x = 30\sqrt{3} \text{ .... (1)}$$

From ΔBCD

$$\tan 60^\circ = \frac{BD}{d}$$

$$\sqrt{3} = \frac{30}{d}$$

$$d = \frac{30}{\sqrt{3}} \text{ .... (2)}$$

from (1) and (2) distance between the persons = BC + BA = $x + d$

$$= 30\sqrt{3} + \frac{30}{\sqrt{3}} = \frac{30\times 4}{\sqrt{3}} = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ meter}$$

Example-7. A straight highway leads to the foot of a tower. Ramaiah standing at the top of the tower observes a car at an angle of depression 30º. The car is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60º. Find the time taken by the car to reach the foot of the tower from this point.

Solution :

Let the distance travelled by the car in 6 seconds = AB = x meters

Heights of the tower CD = h meters

The remaining distance to be travelled by the car BC = d meters

and AC = AB + BC = $(x + d)$ meters

$\angle PDA = \angle DAP = 30^\circ$ (why?)

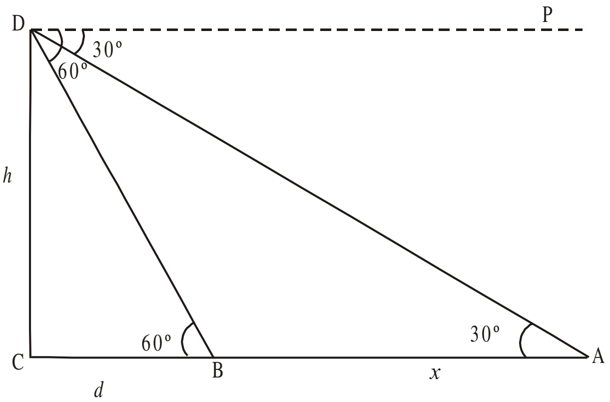
$\angle PDB = \angle DBP = 60^\circ$ (why?)

From ΔBCD

$$\tan 60^\circ = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{h}{d}$$

$$h = \sqrt{3}d \qquad ...(1)$$



From ΔACD

$$\tan 30^\circ = \frac{CD}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{(x+d)}$$

$$h = \frac{(x+d)}{\sqrt{3}} \qquad ...(2)$$

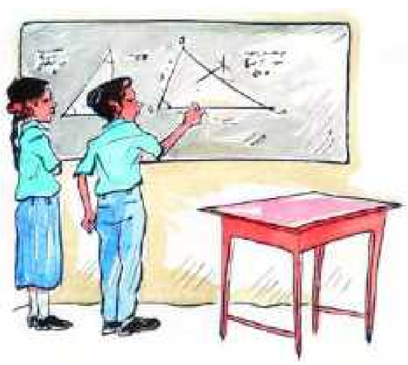
From (1) & (2), we have

$$\frac{x+d}{\sqrt{3}} = \sqrt{3}d$$

$$x + d = 3d$$

$$x = 2d$$

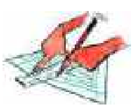
$$d = \frac{x}{2}$$



Time taken to travel 'x' meters = 6 seconds.

Time taken to travel the distance of 'd' meters

i.e., $\frac{x}{2}$ meters = 3 seconds.



Exercise - 12.2

1. A TV tower stands vertically on the side of a road. From a point on the other side directly opposite to the tower, the angle of elevation of the top of tower is 60^0. From another point 10 m away from this point, on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30º. Find the height of the tower and the width of the road.

2. A 1.5 m tall boy is looking at the top of a temple which is 30 meter in height from a point at certain distance. The angle of elevation from his eye to the top of the crown of the temple increases from 30º to 60º as he walks towards the temple. Find the distance he walked towards the temple.

3. A statue stands on the top of a 2m tall pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60º and from the same point, the angle of elevation of the top of the pedestal is 45º. Find the height of the statue.

4. From the top of a building, the angle of elevation of the top of a cell tower is 60º and the angle of depression to its foot is 45º. If distance of the building from the tower is 7m, then find the height of the tower.

5. A wire of length 18 m had been tied with electric pole at an angle of elevation 30º with the ground. Because it was convering a long distance, it was cut and tied at an angle of elevation 60º with the ground. How much length of the wire was cut?

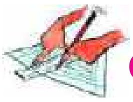
6. The angle of elevation of the top of a building from the foot of the tower is 30º and the angle of elevation of the top of the tower from the foot of the building is 60º. If the tower is 30 m high, find the height of the building.

7. Two poles of equal heights are standing opposite to each other on either side of the road, which is 120 feet wide. From a point between them on the road, the angles of elevation of the top of the poles are 60º and 30º respectively. Find the height of the poles and the distances of the point from the poles.

8. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m, find the height of the tower from the base of the tower and in the same straight line with it are complementary.

9. The angle of elevation of a jet plane from a point A on the ground is 60º. After a flight of 15 seconds, the angle of elevation changes to 30º. If the jet plane is flying at a constant height of $1500\sqrt{3}$ meter, find the speed of the jet plane. $\left(\sqrt{3}=1.732\right)$

10. Clinky observes a tower PQ of height 'h' from a point A on the ground. She moves a distance 'd' towards the foot of the tower and finds that the angle of elevation has direction and finals that the angle of elevation is '3' times at A. Prove that $36h^2 = 35d^2$.



Optional Exercise

[This exercise his not meant for examination]

1. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60º. After some time, the angle of elevation reduces to 30º. Find the distance travelled by the balloon during the interval.

2. The angle of elevation of the top of a tower from the foot of the building is 30º and the angle of elevation of the top of the building from the foot of the tower is 60º. What is the ratio of heights of tower and building.

3. The angles of elevation of the top of a lighthouse from 3 boats A, B and C in a straight line of same side of the light house are a, $2a$, $3a$ respectively. If the distance between the boats A and B is x meters. Find the hight of light house?

4. Inner part of a cupboard is in the cuboidical shape with its length, breadth and height in the ratio $1 : \sqrt{2} : 1$. What is the angle made by the longest stick which can be inserted cupboard with its base inside.

5. An iron sperical ball of volume 232848 cm^3 has been melted and converted into a cone with the vertical angle of 120^o. What are its height and base?

What We Have Discussed

In this chapter, we have studied the following points :

1. (i) The line of sight is the line drawn from the eye of an observer to a point on the object being viewed by the observer.

   (ii) The angle of elevation of the object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.

   (iii) The angle of depression of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.

2. The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.

CHAPTER 13 Probability

13.1 Introduction

Kumar and Sudha were walking together to play a carroms match:

Kumar : Do you think we will win?

Sudha : There are 50 percent chances of that. We may win.

Kumar : How do you say 50 percent?

Do you think Sudha is right in her statement?

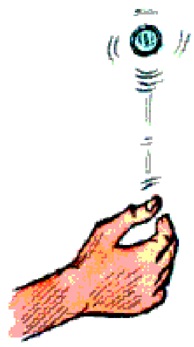
Is her chance of wining 50%?

In this chapter, we study about such questions. We also discuss words the like 'probably', 'likely', 'possibly', etc. and how to quantify these. In class IX we studied about events that are extremely likely and in fact, are almost certain and those that are extremely unlikely and hence almost impossible. We also talked about chance, luck and the fact that an event occurs one particular time does not mean that it would happen each time. In this chapter, we try to learn how the likelihood of an event can be quantified.

This quantificatioin into a numerical measure is referred to as finding 'Probability'.

13.1.1 What is Probability

Consider an experiment: A normal coin was tossed 1000 times. Head turned up 455 times and tail turned up 545 times. If we try to find the likelihood of getting heads we may say it is 455 out of 1000 or $\frac{455}{1000}$ or 0.455.



This estimation of probability is based on the results of an actual experiment of tossing a coin 1000 times. These estimates are called experimental or empirical probabilities. In fact, all experimental probabilities are based on the results of actual experiments and an adequate recording of what happens in each of the events. These probabilities are only 'estimations'. If we perform the same experiment for another 1000 times, we may get slightly different data, giving different probability estimate.

Many other persons from different parts of the world have done this kind of experiment and recorded the number of heads that turned up.

For example, the eighteenth century French naturalist Comte de Buffon, tossed a coin 4040 times and got 2048 heads. The experimental probability of getting a head, in this case, was $\frac{2048}{4040}$ i.e., 0.507.

J.E. Kerrich, from Britain, recorded 5067 heads in 10000 tosses of a coin. The experimental probability of getting a head, in this case, was $\frac{5067}{10000} = 0.5067$. Statistician Karl Pearson spent some more time, making 24000 tosses of a coin. He got 12012 heads, and thus, the experimental probability of a head obtained by him was 0.5005.

Now, suppose we ask, 'What will be the experimental probability of getting a head, if the experiment is carried on up to, say, one million times? Or 10 million times? You would intuitively feel that as the number of tosses increases, the experimental probability of a head (or a tail) may settle down closer and closer to the number 0.5 , i.e., $\frac{1}{2}$. This matches the theoretical probability of getting a head (or getting a tail), we will learn how to find the theoretical probability.

This chapter is an introduction to the theoretical (also called classical) probability of an event, Now we discuss simple problems based on this concept.

13.2 Probability - A Theoretical Approach

Let us consider the following situation: Suppose a ‘fair’ coin is tossed at random.

When we speak of a coin, we assume it to be 'fair', that is, it is symmetrical so that there is no reason for it to come down more often on one side than the other. We call this property of the coin as being 'unbiased'. By the phrase 'random toss', we mean that the coin is allowed to fall freely without any bias or interference. (Here we dismiss the possibility of its 'landing' on its edge, which may be possible, for example, if it falls on sand). We refer to this by saying that the outcomes, head and tail, are equally likely.

For basic understanding of probability, in this chapter, we will assume that all the experiments have equally likely outcomes.

Now, we know that the experimental or empirical probability P(E) of an event E is

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$$

Do This

a. Outcomes of which of the following experiments are equally likely?

1. Getting a digit 1, 2, 3, 4, 5 or 6 while throwing a die.
2. Picking a different colour ball from a bag of 5 red balls, 4 blue balls and 1 black ball.
3. Winning in a game of carrom.
4. Units place of a two digit number selected may be 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9.
5. Picking a different colour ball from a bag of 10 red balls, 10 blue balls and 10 black balls.
6. Raining on a particular day of July.

b. Are the outcomes of every experiment equally likely?

c. Give examples of 5 experiments that have equally likely outcomes and five more examples that do not have equally likely outcomes.

Activity

(i) Take any coin, toss it, 50 times, 100 times, 150 times and count the number of times a head and a tail come up seperately. Record your observations in the following table:-

S. No.	Number of experiments	Number of heads	Probability of head	Number of tails	Probability of tails
1.	50				
2.	100				
3.	150				

What do you observe? Obviously, as the number of experiments are more and more, probability of head or tail reaches 50% or $\frac{1}{2}$. This empirical interpretation of probability can be applied to every event associated with an experiment that can be repeated a large number of times.

Probability and Modelling

The requirement of repeating an experiment has some limitations, as it may be very expensive or unfeasible in many situations. Of course, it worked well in coin tossing or die throwing experiments. But how about repeating the experiment of launching a satellite in order to compute the empirical probability of its failure during launching, or the repetition of the phenomenon of an earthquake to compute the empirical probability of a multi-storeyed building getting destroyed in an earthquake? For finding these probabilities we calculate models of behaviour and use them to estimate behaviour and likely outcomes. Such models are complex and are validated by predictions and outcomes. Forecast of weather, result of an election, population demography, earthquakes, crop production etc. are all based on such models and their predictions.

"The assumption of equally likely outcomes" (which is valid in many experiments, as in two of the examples seen, of a coin and of a die) is one of the assumption that leads us to the following definition of probability of an event.

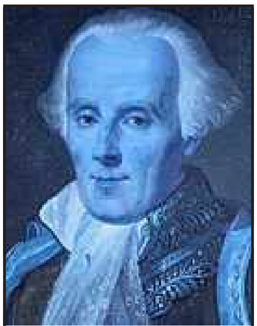
The theoretical probability (also called classical probability) of an event T, written as P(T), is defined as

$$P(T) = \frac{\text{Number of outcomes favourable to T}}{\text{Number of all possible outcomes of the experiment}}$$

where we assume that the outcomes of the experiment are equally likely. We usually simply refer to theoretical probability as Probability.

The definition of probability was given by Pierre Simon Laplace in 1795.

Probability theory had its origin in the 16^{th} century when an Italian physician and mathematician J. Cardan wrote the first book on the subject, The Book on Games of Chance. James Bernoulli (1654 -1705), A. De Moivre (1667-1754), and Pierre Simon Laplace are among those who made significant contributions to this field. In recent years, probability has been used extensively in many areas such as biology, economics, genetics, physics, sociology etc.



Pierre Simon Laplace
(1749 – 1827)

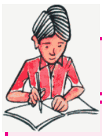
13.3 Mutually Exclusive Events

If a coin is tossed, we get head or tail, but not both. Similarly, if we select a student of a high school that student may belong to one of either 6, 7, 8, 9 or 10 class, but not to any two or more classes. In both these examples, occurrence of an event prevents the occurrence of other events. Such events are called mutually exclusive events.

Two or more events of an experiment, where occurance of an event prevents occurances of all other events, are called **Mutually Exclusive Events.** We will discuss this in more detail later in the chapter.

13.4.1 Finding Probability

How do we find the probability of events that are equally likely? We consider the tossing of a coin as an event associated with experiments where the equally likely assumption holds. In order to proceed, we recall that there are two possible outcomes each time. This set of outcomes is called the sample space. We can say that the sample space of one toss is {H, T}. For the experiement of drawing out a ball from a bag containing red, blues, yellow and white ball, the sample space is {R, B, Y, W}. What is the sample space for the throw of a die?



Do This

Think of 5 situations with equally likely events and find the sample space.

Let us now try to find the probability of equally likely events that are mutually exclusive.

Example-1. Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

Solution : In the experiment of tossing a coin once, the number of possible outcomes is two - Head (H) and Tail (T). Let E be the event 'getting a head'. The number of outcomes favourable to E, (i.e., of getting a head) is 1. Therefore,

$$P(E) = P\text{ (head)} = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes}} = \frac{1}{2}$$

Similarly, if F is the event 'getting a tail', then

$$P(F) = P(\text{tail}) = \frac{1}{2} \text{ (Guess why?)}$$

Example-2. A bag contains a red ball, a blue ball and an yellow ball, all the balls being of the same size. Manasa takes out a ball from the bag without looking into it. What is the probability that she takes a (i) yellow ball? (ii) red ball? (iii) blue ball?

Solution : Manasa takes out a ball from the bag without looking into it. So, it is equally likely that she takes out any one of them.

Let Y be the event 'the ball taken out is yellow', B be the event 'the ball taken out is blue', and R be the event 'the ball taken out is red'.

Now, the number of possible outcomes = 3.

(i) The number of outcomes favourable to the event Y = 1.

So, $P(Y) = \frac{1}{3}$. Similarly, $P(R) = \frac{1}{3}$ and $P(B) = \frac{1}{3}$

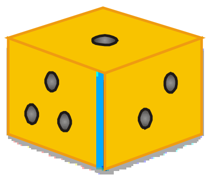
Remarks

1. An event having only one outcome in an experiment is called an elementary event. In Example 1, both the events E and F are elementary events. Similarly, in Example 2, all the three events, Y, B and R are elementary events.
2. In Example 1, we note that : P(E) + P(F) = 1
   In Example 2, we note that : P(Y) + P(R) + P(B) = 1.

   If we find the probability of all the elementary events and add them, we would get the total as 1.
3. In events like a throw of dice, probability of getting less than 3 and of getting a 3 or more than three are not elementary events of the possible outcomes. In tossing two coins {HH}, {HT}, {TH} and {TT} are elementary events.

Example-3. Suppose we throw a die once. (i) What is the probability of getting a number greater than 4? (ii) What is the probability of getting a number less than or equal to 4?

Solution : (i) In rolling an unbaised dice



Sample space	S	= {1, 2, 3, 4, 5, 6}
No. of outcomes	n(S)	= 6
Favourable outcomes for number greater than 4	E	= {5, 6}
No. of favourable outcomes	n(E)	= 2
Probability	P(E)	$= \frac{2}{6} = \frac{1}{3}$

(ii) Let F be the event 'getting a number less than or equal to 4'.

Sample space	S	= {1, 2, 3, 4, 5, 6}
No. of outcomes	n(S)	= 6
Favourable outcomes for number less or equal to 4	F	= {1, 2, 3, 4}
No. of favourable outcomes	n(F)	= 4
Probability	P(F)	$= \frac{4}{6} = \frac{2}{3}$

Note : Are the events E and F in the above example elementary events?

No, they are not elementary events. The event E has 2 outcomes and the event F has 4 outcomes.

13.4.2 Complementary Events and Probability

In the previous section we read about elementary events. Then in example-3, we calculated probability of events which are not elementary. We saw,

$$P(E) + P(F) = \frac{1}{3} + \frac{2}{3} = 1$$

Here F is the same as 'not E' because there are only two events.

We denote the event 'not E' by $\overline{E}$. This is called the **complement** event of event E.

So, P(E) + P(not E) = 1

i.e., $P(E) + P(\overline{E}) = 1$, which gives us $P(\overline{E}) = 1 - P(E)$.

In general, it is true that for an event E, $P(\overline{E}) = 1 - P(E)$

Do This

(i) Is getting a head complementary to getting a tail? Give reasons.

(ii) In case of a die is getting a 1 complementary to events getting 2, 3, 4, 5, 6? Give reasons for your answer.

(iii) Write of five new pair of events that are complementary.

13.4.3 Impossible and Certain Events

Consider the following about the throws of a die with sides marked as 1, 2, 3, 4, 5, 6.

(i) What is the probability of getting a number 7 in a single throw of a die?

We know that there are only six possible outcomes in a single throw of this die. These outcomes are 1, 2, 3, 4, 5 and 6. Since no face of the die is marked 7, there is no outcome favourable to 7, i.e., the number of such outcomes is zero. In other words, getting 7 in a single throw of a die, is impossible.

$$\text{So P(getting 7)} = \frac{0}{6} = 0$$

That is, the probability of an event which is impossible to occur is 0. Such an event is called an **impossible event**.

(ii) What is the probability of getting 6 or a number less than 6 in a single throw of a die?

Since every face of a die is marked with 6 or a number less than 6, it is sure that we will always get one of these when the dice is thrown once. So, the number of favourable outcomes is the same as the number of all possible outcomes, which is 6.

Therefore, P(E) = P(getting 6 or a number less than 6) = $\frac{6}{6} = 1$

So, the probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a **certain event**.

Note : From the definition of probability P(E), we see that the numerator (number of outcomes favourable to the event E) is always less than or equal to the denominator (the number of all possible outcomes). Therefore, $\mathbf{0 \leq P(E) \leq 1}$.

Try This

1. A child has a die whose six faces show the letters A, B, C, D, E and F. The die is thrown once. What is the probability of getting (i) A? (ii) D?
2. Which of the following cannot be the probability of an event?

   (a) 2.3 (b) -1.5 (c) 15% (D) 0.7

Think - Discuss

1. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of any game?
2. Can $\frac{7}{2}$ be the probability of an event? Explain.
3. Which of the following arguments are correct and which are not correct? Give reasons.

   i) If two coins are tossed simultaneously there are three possible outcomes - two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

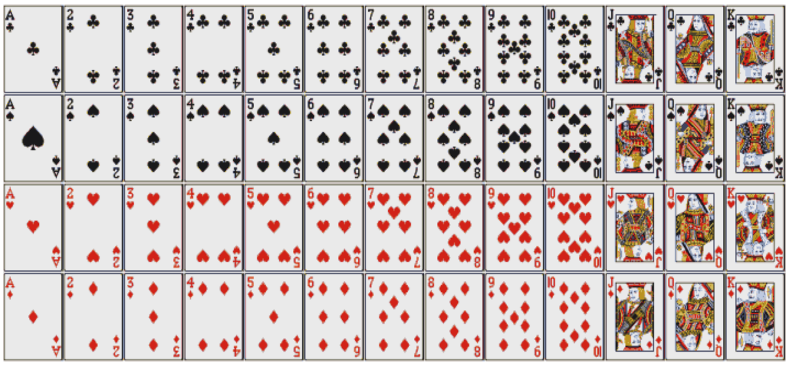
   ii) If a die is thrown, there are two possible outcomes - an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

13.5 Deck of Cards and Probability

Have you seen a deck of playing cards?

A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. They are black spades (♠), red hearts (♥), red diamonds (♦) and black clubs (♣).

The cards in each suit are Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, Queens and Jacks are called face cards. Many games are played with this deck of cards, some games are played with part of the deck and some with two decks even. The study of probability has a lot to do with card and dice games as it helps players to estimate possibilities and predict how the cards could be distributed among players.



Example-4. One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will (i) be an ace, (ii) not be an ace.

Solution : Well-shuffling ensures equally likely outcomes.

(i) There are 4 aces in a deck.

Let E be the event 'the card is an ace'.

The number of outcomes favourable to E = 4

The number of possible outcomes = 52 (Why ?)

Therefore, $P(E) = \frac{4}{52} = \frac{1}{13}$

(ii) Let F be the event 'card drawn is not an ace'.

The number of outcomes favourable to the event F = 52 - 4 = 48 (Why?)

The number of possible outcomes = 52

Therefore, $P(F) = \frac{48}{52} = \frac{12}{13}$

Alternate Method : Note that F is nothing but $\overline{E}$.

Therefore, we can also calculate P(F) as follows:

$P(F) = P(\overline{E}) = 1 - P(E) = 1 - \frac{1}{13} = \frac{12}{13}$

TRY THIS

You have a single deck of well shuffled cards. Then,

1. What is the probability that the card drawn will be a queen?

2. What is the probability that it is a face card?
3. What is the probability it is a spade?
4. What is the probability that is the face card of spades?
5. What is the probability it is not a face card?

13.6 Use of Probability

Let us look at some more occasions where probability may be useful. We know that in sports some countries are strong and others are not so strong. We also know that when two players are playing it is not that they win equal times. The probability of winning of the player or team that wins more often is more than the probability of the other player or team. We also discuss and keep track of birthdays. Sometimes happens it that people we know have the same birthdays. Can we find out whether this is a common event or would it only happen occassionally. Classical probability helps us do this.

Example-5. Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match?

Solution : Let S and R denote the events that Sangeeta wins the match and Reshma wins the match, respectively.

The probability of Sangeeta's winning chances = P(S) = 0.62 (given)

The probability of Reshma's winning chances = P(R) = 1 - P(S)

= 1 -0.62 = 0.38 [R and S are complementary]

Example-6. Sarada and Hamida are friends. What is the probability that both will have (i) different birthdays? (ii) the same birthday? (ignoring a leap year).

Solution : Out of the two friends, one girl, say, Sarada's birthday can be any day of the year. Now, Hamida's birthday can also be any day of 365 days in the year. We assume that these 365 outcomes are equally likely.

(i) If Hamida's birthday is different from Sarada's, the number of favourable outcomes for her birthday is 365 - 1 = 364

$$\text{So, P (Hamida's birthday is different from Sarada's birthday)} = \frac{364}{365}$$

(ii) P(Sarada and Hamida have the same birthday) = 1 - P (both have different birthdays)

$$= 1 - \frac{364}{365} \text{ [Using } P(\overline{E}) = 1 - P(E)] = \frac{1}{365}$$

Example-7. There are 40 students in Class X of a school of whom 25 are girls and 15 are boys. The class teacher has to select one student as a class representative. She writes the name of each student on a separate cards, the cards being identical. Then she puts cards in a box and stirs them thoroughly. She then draws one card from the box. What is the probability that the name written on the card is the name of (i) a girl? (ii) a boy?

Solution : There are 40 students, and only one name card has to be chosen.

The number of all possible outcomes is 40

(i) The number of outcomes favourable for a card with the name of a girl = 25 (Why?)

$$\therefore \text{ P (card with name of a girl)} = \text{P(Girl)} = \frac{25}{40} = \frac{5}{8}$$

(ii) The number of outcomes favourable for a card with the name of a boy = 15 (Why?)

$$\text{Therefore, P(card with name of a boy)} = \text{P(Boy)} = \frac{15}{40} = \frac{3}{8}$$

or $$\text{P(Boy)} = 1 - \text{P(not Boy)} = 1 - \text{P(Girl)} = 1 - \frac{5}{8} = \frac{3}{8}$$

Exercise - 13.1

1. Complete the following statements:

   (i) Probability of an event E + Probability of the event 'not E' = ______________

   (ii) The probability of an event that cannot happen is__________.

   Such an event is called __________

   (iii) The probability of an event that is certain to happen is __________.

   Such an event is called______

   (iv) The sum of the probabilities of all the elementary events of an experiment is_________

   (v) The probability of an event is greater than or equal to __________ and less than or equal to _______

2. Which of the following experiments have equally likely outcomes? Explain.

   (i) A driver attempts to start a car. The car starts or does not start.

   (ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

   (iii) A trial is made to answer a true-false question. The answer is right or wrong.

   (iv) A baby is born. It is a boy or a girl.

3. If P(E) = 0.05, what is the probability of 'not E'?

4. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

   (i) an orange flavoured candy? (ii) a lemon flavoured candy?

5. Rahim takes out all the hearts from the cards. What is the probability of

   i. Picking out an ace from the remaining pack.

   ii. Picking out a diamonds.

   iii. Picking out a card that is not a heart.

   iv. Picking out the Ace of hearts.

6. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

7. A die is thrown once. Find the probability of getting

   (i) a prime number; (ii) a number lying between 2 and 6; (iii) an odd number.

8. What is the probability of drawing out a red king from a deck of cards?

9. Make 5 more problem of this kind using dice, cards or birthdays and discuss with friends and teacher about their solutions.

13.7 More Applications of Probability

We have seen some example of use of probability. Think about the contents and ways probability has been used in these. We have seen again that probability of complementary events add to 1. Can you identify in the examples and exercises given above, and those that follow, complementary events and elementary events? Discuss with teachers and friends. Let us see more uses.

Example-8. A box contains 3 blue, 2 white, and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will be

(i) white? (ii) blue? (iii) red?

Solution : Saying that a marble is drawn at random means all the marbles are equally likely to be drawn.

$\therefore$ The number of possible outcomes = 3 +2 + 4 = 9 (Why?)

Let W denote the event 'the marble is white', B denote the event 'the marble is blue' and R denote the event 'marble is red'.

(i) The number of outcomes favourable to the event W = 2

So, $P(W) = \frac{2}{9}$

Similarly, (ii) $P(B) = \frac{3}{9} = \frac{1}{3}$ and (iii) $P(R) = \frac{4}{9}$

Note that $P(W) + P(B) + P(R) = 1$.

Example-9. Harpreet tosses two different coins simultaneously (say, one is of ₹1 and other of ₹2). What is the probability that she gets at least one head?

Solution : We write H for 'head' and T for 'tail'. When two coins are tossed simultaneously, the possible outcomes are (H, H), (H, T), (T, H), (T, T), which are all equally likely. Here (H, H) means heads on the first coin (say on ₹1) and also heads on the second coin (₹2). Similarly (H, T) means heads up on the first coin and tail up on the second coin and so on.

The outcomes favourable to the event E, 'at least one head' are (H, H), (H, T) and (T, H).

So, the number of outcomes favourable to E is 3.

$\therefore P(E) = \frac{3}{4}$ [Since the total possible outcomes = 4]

i.e., the probability that Harpreet gets at least one head is $\frac{3}{4}$

Check This

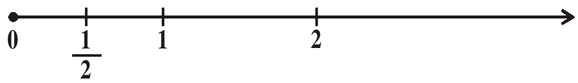
Did you observe that in all the examples discussed so far, the number of possible outcomes in each experiment was finite? If not, check it now.

There are many experiments in which the outcome is number between two given numbers, or in which the outcome is every point within a circle or rectangle, etc. Can you count the number of all possible outcomes in such cases? As you know, this is not possible since there are infinitely many numbers between two given numbers, or there are infinitely many points within a circle. So, the definition of theoretical probability which you have learnt so far cannot be applied in the present form.

What is the way out? To answer this, let us consider the following example:

Example-10. (Not for examination) In a musical chair game, the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the first half-minute after starting?

Solution : Here the possible outcomes are all the numbers between 0 and 2. This is the portion of the number line from 0 to 2



Let E be the event that 'the music is stopped within the first half-minute'.

The outcomes favourable to E are points on the number line from 0 to $\frac{1}{2}$

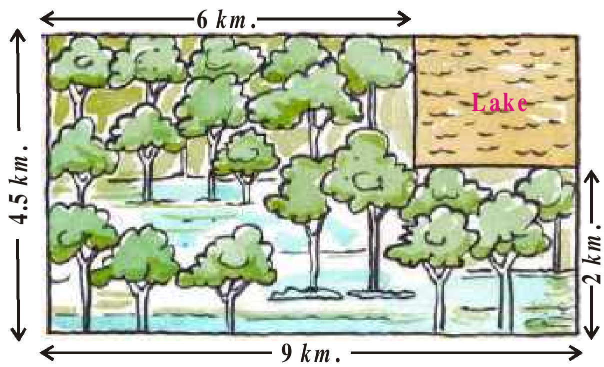
The distance from 0 to 2 is 2, while the distance from 0 to $\frac{1}{2}$ is $\frac{1}{2}$

Since all the outcomes are equally likely, we can argue that, of the total distance is 2 and the distance favourable to the event E is $\frac{1}{2}$

$$\text{So, } P(E) = \frac{\text{Distance favourable to the event E}}{\text{Total distance in which outcomes can lie}} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

We now try to extend this idea of for finding the probability as the ratio of the favourable area to the total area.

Example-11. A missing helicopter is reported to have crashed somewhere in the rectangular region as shown in the figure. What is the probability that it crashed inside the lake shown in the figure?



Solution : The helicopter is equally likely to crash anywhere in the region. Area of the entire region where the helicopter can crash = $(4.5 \times 9)\ km^2 = 40.5\ \text{km}^2$

Area of the lake = $(2 \times 3)\ km^2 = 6\ km^2$

Therefore, P (helicopter crashed in the lake) = $\frac{6}{40.5} = \frac{4}{27}$

Example-12. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jhony, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that

(i) it is acceptable to Jhony? (ii) it is acceptable to Sujatha?

Solution : One shirt is drawn at random from the carton of 100 shirts. Therefore, there are 100 equally likely outcomes.

(i) The number of outcomes favourable (i.e., acceptable) to Jhony = 88 (Why?)

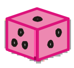
Therefore, P (shirt is acceptable to Jhony) = $\frac{88}{100} = 0.88$

(ii) The number of outcomes favourable to Sujatha = 88 + 8 = 96 (Why?)

So, P (shirt is acceptable to Sujatha) = $\frac{96}{100} = 0.96$

Example-13. Two dice, one red and one white, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 8 (ii) 13 (iii) less than or equal to 12?

Solution : When the red dice shows '1', the white dice could show any one of the numbers 1, 2, 3, 4, 5, 6. The same is true when the red dice shows '2', '3', '4', '5' or '6'. The possible outcomes of the experiment are shown in the figure; the first number in each ordered pair is the number appearing on the red dice and the second number is that on the white dice.



	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Note that the pair (1, 4) is different from (4, 1). (Why?)

So, the number of possible outcomes $n(S) = 6 \times 6 = 36$.

(i) The outcomes favourable to the event 'the sum of the two numbers is 8' denoted by E, are: (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) (See figure)

i.e., the number of outcomes favourable to E is n(E) = 5.

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$

(ii) As there is no outcome favourable to the event F, 'the sum of two numbers is 13',

So, $P(F) = \frac{0}{36} = 0$

(iii) As all the outcomes are favourable to the event G, 'sum of two numbers is 12',

So, $P(G) = \frac{36}{36} = 1$

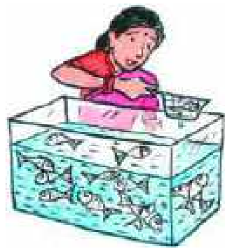
Exercise - 13.2

1. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red ? (ii) not red?

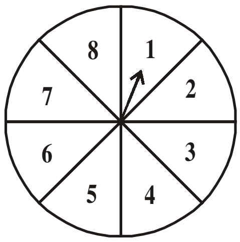
2. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) white ? (iii) not green?

3. A Kiddy bank contains hundred 50p coins, fifty ₹1 coins, twenty ₹2 coins and ten ₹5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin? (ii) will not be a ₹5 coin?

4. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (See figure). What is the probability that the fish taken out is a male fish?



5. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (See figure), and these are equally likely outcomes. What is the probability that it will point at



(i) 8 ? (ii) an odd number?

(iii) a number greater than 2? (iv) a number less than 9?

6. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red colour (ii) a face card (iii) a red face card

(iv) the jack of hearts (v) a spade (vi) the queen of diamonds

7. Five cards-the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

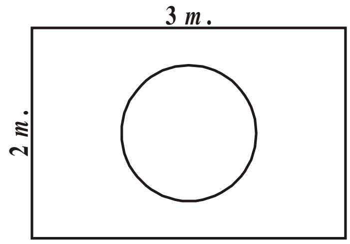
(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

8. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

9. A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective? Suppose the bulb drawn in previous case is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

10. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

11. Suppose you drop a die at random on the rectangular region shown in figure. What is the probability that it will land inside the circle with diameter 1m?



12. A lot consists of 144 ball pens of which 20 are defective and the others are good. The shopkeeper draws one pen at random and gives it to Sudha. What is the probability that (i) She will buy it? (ii) She will not buy it ?

13. Two dice are rolled simultaneously and counts are added (i) complete the table given below:

Event : 'Sum on 2 dice'	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{12}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

14. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

15. A die is thrown twice. What is the probability that (i) 5 will not come up either time? (ii) 5 will come up at least once? [**Hint** : Throwing a die twice and throwing two dice simultaneously are treated as the same experiment].

Optional Exercise

[This exercise is not meant for examination]

1. Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) consecutive days? (iii) different days?

2. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

3. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

4. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

What We Have Discussed

In this chapter, you have studied the following points:

1. We have looked at experimental probability and theoretical probability.
2. The theoretical (classical) probability of an event E, written as P(E), is defined as

$$P\,(E) = \frac{\text{Number of trails in which the event happened}}{\text{Total number of trails}}$$ where we assume that the outcomes of the experiment are equally likely.

3. The probability of a sure event (or certain event) is 1.
4. The probability of an impossible event is 0.
5. The probability of an event E is a number P(E) such that $0 \leq P\,(E) \leq 1$
6. An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
7. For any event E, $P\,(E) + P\,(\overline{E}) = 1$, where $\overline{E}$ stands for 'not E'. E and $\overline{E}$ are called complementary events.
8. Some more terms used in the chapter are given below:

Equally likely events : Two or more events are said to be equally likely if each one of them has an equal chance of occurrance.

Mutually Exclusive events : Two or more events are mutually exclusive if the occurrence of each event prevents the every other event.

Complementary events : Consider an event has few outcomes. Event of all other outcomes in the sample survey which are not in the favourable event is called Complementary event.

Exhaustive events : All the events are exhaustive events if their union is the sample space.

Sure events : The sample space of a random experiment is called sure or certain event as any one of its elements will surely occur in any trail of the experiment.

Impossible event : An event which will occur on any account is called an impossible event.

CHAPTER 14 Statistics

14.1 Introduction

Ganesh had recorded the marks of 26 children in his class in the mathematics Summative Assessment - I in the register as follows:

Arjun	76	Narayana	12
Kamini	82	Suresh	24
Shafik	64	Durga	39
Keshav	53	Shiva	41
Lata	90	Raheem	69
Rajender	27	Radha	73
Ramu	34	Kartik	94
Sudha	74	Joseph	89
Krishna	76	Ikram	64
Somu	65	Laxmi	46
Gouri	47	Sita	19
Upendra	54	Rehana	53
Ramaiah	36	Anitha	69

Is the data given in the list above organized? Why or why not?

His teacher asked him to report on how his class students had performed in mathematics in their Summative Assessment - I .

Ganesh made the following table to understand the performance of his class:

Marks	Number of children
0 - 33	4
34 - 50	6
51 - 75	10
76 - 100	6

Is the data given in the above table grouped or ungrouped?

He showed this table to his teacher and the teacher appreciated him for organising the data to be understood easily. We can see that most children have got marks between 51-75. Do you think Ganesh should have used smaller range? Why or why not?

In the previous class, you had learnt about the difference between grouped and ungrouped data as well as how to present this data in the form of tables. You had also learnt to calculate the mean value for ungrouped data. Let us recall this learning and then learn to calculate the mean, median and mode for grouped data.

14.2 Mean of Ungrouped Data

As we know the mean (or average) of observations is the sum of the values of all the observations divided by the total number of observations. Let $x_1, x_2, \ldots, x_n$ be observations with respective frequencies $f_1, f_2, \ldots, f_n$. This means that observation x_1 occurs f_1 times, x_2 occurs f_2 times, and so on.

Now, the sum of the values of all the observations $= f_1x_1 + f_2x_2 + \ldots + f_nx_n$, and the number of observations $= f_1 + f_2 + \ldots + f_n$.

So, the mean $\bar{x}$ of the data is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots\ldots\ldots + f_nx_n}{f_1 + f_2 + \ldots\ldots\ldots + f_n}$$

Recall that we can write this in short, using the Greek letter $\sum$ which means summation

i.e., $\bar{x} = \dfrac{\sum f_ix_i}{\sum f_i}$

Example-1. The marks obtained in mathematics by 30 students of Class X of a certain school are given in table below. Find the mean of the marks obtained by the students.

Marks obtained (x_i)	10	20	36	40	50	56	60	70	72	80	88	92	95
Number of student (f_i)	1	1	3	4	3	2	4	4	1	1	2	3	1

Solution : Let us re-organize this data and find the sum of all observations.

Marks obtained (x_i)	Number of students (fi)	$f_i x_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
Total	$\sum f_i = 30$	$\sum f_i x_i = 1779$

So, $$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1779}{30} = 59.3$$

Therefore, the mean marks are 59.3.

In most of our real life situations, data is usually so large that to make a meaningful study, it needs to be condensed as grouped data. So, we need to convert ungrouped data into grouped data and devise some method to find its mean.

Let us convert the ungrouped data of Example 1 into grouped data by forming class-intervals of width, say 15. Remember that while allocating frequencies to each class-interval, students whose score is equal to in any **upper class-boundary** would be considered in the next class, e.g., 4 students who have obtained 40 marks would be considered in the class-interval 40-55 and not in 25-40. With this convention in our mind, let us form a grouped frequency distribution table.

Class interval	10-25	25-40	40-55	55-70	70-85	85-100
Number of students	2	3	7	6	6	6

Now, for each class-interval, we require a point which would serve as the representative of the whole class. ***It is assumed that the frequency of each class-interval is centred around its mid-point.*** So, the *mid-point* of each class can be chosen to represent the observations falling in that class and is called the class mark. Recall that we find the class mark by finding the average of the upper and lower limit of the class.

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

For the class 10 -25, the class mark is $\frac{10+25}{2}$ =17.5. Similarly, we can find the class marks of the remaining class intervals. We put them in the table. These class marks serve as our x_i's. We can now proceed to compute the mean in the same manner as in the previous example.

Class interval	Number of students (f_i)	Class Marks (x_i)	$f_i x_i$
10-25	2	17.5	35.0
25-40	3	32.5	97.5
40-55	7	47.5	332.5
55-70	6	62.5	375.0
70-85	6	77.5	465.0
85-100	6	92.5	555.0
Total	$\sum f_i$ =30		$\sum f_i x_i$ =1860.0

The sum of the values in the last column gives us $\sum f_i x_i$. So, the mean $\bar{x}$ of the given data is given by

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1860}{30} = 62$$

This new method of finding the mean is known as the **Direct Method**.

We observe that in the above cases we are using the same data and employing the same formula for calculating the mean but the results obtained are different. In example (1), 59.3 is the exact mean and 62 is the approximate mean. Can you think why this is so?

THINK - DISCUSS

1. The mean value can be calculated from both ungrouped and grouped data. Which one do you think is more accurate? Why?
2. When it is more convenient to use grouped data for analysis?

Sometimes when the numerical values of x_1 and f_1 are large, finding the product of x_1 and f_1 becomes tedious and time consuming. So, for such situations, let us think of a method of reducing these calculations.

We can do nothing with the f_i's, but we can change each x_i to a smaller number so that our calculations become easy. How do we do this? What is about subtracting a fixed number from each of these x_i's? Let us try this method for the data in example 1.

The first step is to choose one among the x_i's as the *assumed mean*, and denote it by 'a'. Also, to further reduce our calculation work, we may take 'a' to be that x_i which lies in the centre of $x_1, x_2, ..., x_n$. So, we can choose $a = 47.5$ or $a = 62.5$. Let us choose $a = 47.5$.

The second step is to find the **deviation** of 'a' from each of the x_i's, which we denote as d_i

i.e., $d_i = x_i - a = x_i - 47.5$

The third step is to find the product of d_i with the corresponding f_i, and take the sum of all the $f_i d_i$'s. These calculations are shown in table given below-

Class interval	Number of students (f_i)	Class Marks (x_i)	$d_i = x_i - 47.5$ $x_i = a$	$f_i d_i$
10-25	2	17.5	-30	-60
25-40	3	32.5	-15	-45
40-55	7	47.5 (a)	0	0
55-70	6	62.5	15	90
70-85	6	77.5	30	180
85-100	6	92.5	45	270
Total	$\sum f_i = 30$			$\sum f_i d_i = 435$

So, from the above table, the mean of the deviations, $\bar{d} = \dfrac{\sum f_i d_i}{\sum f_i}$

Now, let us find the relation between $\bar{d}$ and $\bar{x}$.

Since, in obtaining d_i we subtracted 'a' from each x_i so, in order to get the mean $\bar{x}$ we need to add 'a' to $\bar{d}$. This can be explained mathematically as:

Mean of deviations, $\bar{d} = \dfrac{\sum f_i d_i}{\sum f_i}$

So, $\bar{d} = \dfrac{\sum f_i (x_i - a)}{\sum f_i}$

$= \dfrac{\sum f_i x_i}{\sum f_i} - \dfrac{\sum f_i a}{\sum f_i}$

$= \bar{x} - a\dfrac{\sum f_i}{\sum f_i}$

$\bar{d} = \bar{x} - a$

Therefore $\bar{x} = a + \dfrac{\sum f_i d_i}{\sum f_i}$

Substituting the values of a, $\sum f_i d_i$ and $\sum f_i$ from the table, we get

$$\bar{x} = 47.5 + \frac{435}{30} = 47.5 + 14.5 = 62$$

Therefore, the mean of the marks obtained by the students is 62.

The method discussed above is called the **Assumed Mean Method**.

Activity

Consider the data given in example 1 and calculate the arithmetic mean by deviation method by taking successive values of x_i i.e., 17.5, 32.5, ... as assumed means. Now discuss the following:

1. Are the values of arithmetic mean in all the above cases equal?
2. If we take the actual mean as the assumed mean, how much will $\sum f_i d_i$ be?
3. Reason about taking any mid-value (class mark) as assumed mean?

Observe that in the table given below the values in Column 4 are all multiples of 15. So, if we divide all the values of Column 4 by 15, we would get smaller numbers which we then multiply with f_i. (Here, 15 is the class size of each class interval.)

So, let $u_i = \dfrac{x_i - a}{h}$ where a is the assumed mean and h is the class size.

Now, we calculate u_i in this way and continue as before (i. e., find $f_i u_i$ and then $\sum f_i u_i$). Taking $h = 15$, [Generally size of the class is taken as h but it need not be size of the class always].

Let $\bar{u} = \dfrac{\sum f_i u_i}{\sum f_i}$

Class interval	Number of students (f_i)	Class Marks (x_i)	$d_i = x_i - a$	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$
10-25	2	17.5	-30	-2	-4
25-40	3	32.5	-15	-1	-3
40-55	7	47.5	0	0	0
55-70	6	62.5	15	1	6
70-85	6	77.5	30	2	12
85-100	6	92.5	45	3	18
Total	$\sum f_i$=30				$\sum f_i u_i$ =29

Here, again let us find the relation between $\bar{u}$ and $\bar{x}$.

We have $\qquad u_i = \dfrac{x_i - a}{h}$

So $\qquad \bar{u} = \dfrac{\sum f_i u_i}{\sum f_i}$

So
$$\bar{u} = \frac{\sum f_i \dfrac{(x_i - a)}{h}}{\sum f_i}$$

$$= \frac{1}{h}\left\{\frac{\sum f_i x_i}{\sum f_i} - \frac{\sum f_i a}{\sum f_i}\right\}$$

$$= \frac{1}{h}(\bar{x} - a)$$

or $\qquad h\bar{u} = \bar{x} - a$

$$\bar{x} = a + h\bar{u}$$

Therefore,
$$\bar{x} = a + h\left\{\frac{\sum f_i u_i}{\sum f_i}\right\}$$

or $$\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$$

Substituting the values of a, $\sum f_i u_i$ and $\sum f_i$ from the table, we get

$$\bar{x} = 47.5 + 15 \times \frac{29}{30}$$

$$= 47.5 + 14.5 = 62$$

So, the mean marks obtained by a student are 62.

The method discussed above is called the **Step-deviation** method.

We note that:

- The step-deviation method will be convenient to apply if all the d_i's have a common factor.
- The mean obtained by all the three methods is the same.
- The assumed mean method and step-deviation method are just simplified forms of the direct method.
- The formula $\bar{x} = a + h\bar{u}$ still holds if a and h are not as given above, but are any non-zero numbers such that $u_i = \frac{x_i - a}{h}$

Let us apply these methods in other examples.

Example-2. The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers using all the three methods.

Percentage of female teachers	15 - 25	25 - 35	35 – 45	45 - 55	55 - 65	65 - 75	75 – 85
Number of States/U.T.	6	11	7	4	4	2	1

Source : *Seventh All India School Education Survey conducted by NCERT*

Solution : Let us find the class marks x_i of each class, and put them in a table

Here we take $a = 50$, $h = 10$,

then $d_i = x_i - 50$ and $u_i = \frac{x_i - 50}{10}$

We now find d_i and u_i and put them in the table

Percentage of female teachers	Number of States/U.T.	x_i	$d_i = x_i - 50$	$u_i = \frac{x_i - 50}{10}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
15 – 25	6	20	-30	-3	120	-180	-18
25 – 35	11	30	-20	-2	330	-220	-22
35 – 45	7	40	-10	-1	280	-70	-7
45 – 55	4	50	0	0	200	0	0
55 – 65	4	60	10	1	240	40	4
65 – 75	2	70	20	2	140	40	4
75 – 85	1	80	30	3	80	30	3
Total	35				1390	-360	-36

From the table above, we obtain $\sum f_i = 35, \sum f_i x_i = 1390, \sum f_i d_i = -360, \sum f_i u_i = -36$.

Using the direct method $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1390}{35} = 39.71$

Using the assumed mean method $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i} = 50 + \frac{-360}{35} = 50 - 10.29 = 39.71$

Using the step-deviation method $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h = 50 + \frac{-36}{35} \times 10 = 39.71$

Therefore, the mean percentage of female teachers in the primary schools of rural areas is 39.71.

Think - Discuss

1. Is the result obtained by all the three methods the same?
2. If x_i and f_i are sufficiently small, then which method is an appropriate choice?
3. If x_i and f_i are numerically large numbers, then which methods are appropriate to use?

Even if the class sizes are unequal, and x_i are large numerically, we can still apply the step-deviation method by taking h to be a suitable divisor of all the d_i's.

Example-3. The distribution below shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean number of wickets by choosing a suitable method. What does the mean signify?

Number of wickets	20 - 60	60 - 100	100 - 150	150 - 250	250 - 350	350 – 450
Number of bowlers	7	5	16	12	2	3

Solution : Here, the class size varies, and the x_i's are large. Let us still apply the step deviation method with $a = 200$ and $h = 20$. Then, we obtain the data as given in the table.

Number of wickets	Number of bowlers (f_i)	x_i	$d_i = x_i - a$	$u_i = \frac{x_i - a}{h}$ ($h = 20$)	$f_i u_i$
20 – 60	7	40	-160	-8	-56
60 – 100	5	80	-120	-6	-30
100 – 150	16	125	-75	-3.75	-60
150 – 250	12	200 (a)	0	0	0
250 – 350	2	300	100	5	10
350 – 450	3	400	200	10	30
Total	45				-106

So $$\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h = 200 + \frac{-106}{45} \times 20 = 200 - 47.11 = 152.89$$

Thus, the average number of wickets taken by these 45 bowlers in one-day cricket is 152.89.

Classroom Project :

1. Collect the marks obtained by all the students of your class in Mathematics in the recent examination conducted in your school. Form a grouped frequency distribution of the data obtained. Do the same regarding other subjects and compare. Find the mean in each case using a method you find appropriate.
2. Collect the daily maximum temperatures recorded for a period of 30 days in your city. Present this data as a grouped frequency table. Find the mean of the data using an appropriate method.
3. Measure the heights of all the students of your class and form a grouped frequency distribution table of this data. Find the mean of the data using an appropriate method.

Exercise - 14.1

1. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 - 2	2 - 4	4 – 6	6 - 8	8 - 10	10 - 12	12 – 14
Number of houses	1	2	1	5	6	2	3

2. Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages in Rupees	200 - 250	250 - 300	300 - 350	350 - 400	400– 450
Number of workers	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

3. The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is ₹ 18. Find the missing frequency f.

Daily pocket allowance(in Rupees)	11 - 13	13 - 15	15 - 17	17 - 19	19 - 21	21 - 23	23 - 25
Number of children	7	6	9	13	f	5	4

4. Thirty women were examined in a hospital by a doctor and their of heart beats per minute were recorded and summarised as shown. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats/minute	65-68	68-71	71-74	74-77	77-80	80-83	83-86
Number of women	2	4	3	8	7	4	2

5. In a retail market, fruit vendors were selling oranges kept in packing baskets. These baskets contained varying number of oranges. The following was the distribution of oranges.

Number of oranges	10-14	15–19	20-24	25-29	30–34
Number of baskets	15	110	135	115	25

Find the mean number of oranges kept in each basket. Which method of finding the mean did you choose?

6. The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in Rupees)	100-150	150-200	200-250	250-300	300-350
Number of house holds	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

7. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 in ppm	0.00-0.04	0.04-0.08	0.08-0.12	0.12-0.16	0.16-0.20	0.20-0.24
Frequency	4	9	9	2	4	2

Find the mean concentration of SO_2 in the air.

8. A class teacher has the following attendance record of 40 students of a class for the whole term. Find the mean number of days a student was present out of 56 days in the term.

Number of days	35-38	38-41	41-44	44-47	47-50	50-53	53-56
Number of students	1	3	4	4	7	10	11

9. The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate in %	45–55	55-65	65-75	75-85	85-95
Number of cities	3	10	11	8	3

14.3 Mode

A mode is that value among the observations which occurs most frequently.

Before learning about calculating the mode of grouped data let us first recall how we found the mode for ungrouped data through the following example.

Example-4. The wickets taken by a bowler in 10 cricket matches are as follows: 2, 6, 4, 5, 0, 2, 1, 3, 2, 3. Find the mode of the data.

Solution : Let us arrange the observations in order i.e., 0, 1, 2, 2, 2, 3, 3, 4, 5, 6

Clearly, 2 is the number of wickets taken by the bowler in the maximum number of matches (i.e., 3 times). So, the mode of this data is 2.

Do This

1. Find the mode of the following data.
   a) 5, 6, 9, 10, 6, 12, 3, 6, 11, 10, 4, 6, 7.
   b) 20, 3, 7, 13, 3, 4, 6, 7, 19, 15, 7, 18, 3.
   c) 2, 2, 2, 3, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6.
2. Is the mode always at the centre of the data?
3. Does the mode change. If another observation is added to the data in Example? Comment.
4. If the maximum value of an observation in the data in Example 4 is changed to 8, would the mode of the data be affected? Comment.

In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. Here, we can only locate a class with the maximum frequency, called the modal class. The mode is a value inside the modal class, and is given by the formula.

Mode $$= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where, l = lower boundary of the modal class,

h = size of the modal class interval,

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

Let us consider the following examples to illustrate the use of this formula.

Example-5. A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household.

Family size	1-3	3-5	5-7	7-9	9-11
Number of families	7	8	2	2	1

Find the mode of this data.

Solution : Here the maximum class frequency is 8, and the class corresponding to this frequency is 3-5. So, the modal class is 3-5.

Now,

modal class = 3-5, boundary limit (l) of modal class = 3, class size (h) = 2

frequency of the modal class (f_1) = 8,

frequency of class preceding the modal class (f_0) = 7,

frequency of class succeeding the modal class (f_2) = 2.

Now, let us substitute these values in the formula-

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 3 + \left(\frac{8-7}{2 \times 8 - 7 - 2}\right) \times 2 = 3 + \frac{2}{7} = 3.286$$

Therefore, the mode of the data above is 3.286.

Example-6. The marks distribution of 30 students in a mathematics examination are given in the adjacent table. Find the mode of this data. Also compare and interpret the mode and the mean.

Class interval	Number of students (f_i)	Class Marks (x_i)	$f_i x_i$
10-25	2	17.5	35.0
25-40	3	32.5	97.5
40-55	7	47.5	332.5
55-70	6	62.5	375.0
70-85	6	77.5	465.0
85-100	6	92.5	555.0
Total	$\sum f_i = 30$		$\sum f_i x_i = 1860.0$

Solution : Since the maximum number of students (i.e., 7) have got marks in the interval, 40-65 the modal class is 40 - 55.

The lower boundary (l) of the modal class = 40,

The class size (h) = 15,

The frequency of modal class (f_1) = 7,

the frequency of the class preceding the modal class (f_0) = 3,

the frequency of the class succeeding the modal class (f_2) = 6.

Now, using the formula:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{7-3}{2\times 7 - 6 - 3}\right) \times 15 = 40 + 12 = 52$$

Interpretation : The mode marks is 52. Now, from Example 1, we know that the mean marks is 62. So, the maximum number of students obtained 52 marks, while on an average a student obtained 62 marks.

Think - Discuss

1. It depends upon the demand of the situation whether we are interested in finding the average marks obtained by the students or the marks obtained by most of the students.
   a. What do we find in the first situation?
   b. What do we find in the second situation?
2. Can mode be calculated for grouped data with unequal class sizes?

Exercise - 14.2

1. The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
Number of patients	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

2. The following data gives the information on the observed life times (in hours) of 225 electrical components :

Lifetimes (in hours)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100	100 - 120
Frequency	10	35	52	61	38	29

Determine the modal lifetimes of the components.

3. The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the mean monthly expenditure :

Expenditure (in rupees)	1000-1500	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of families	24	40	33	28	30	22	16	7

4. The following distribution gives the state-wise, teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret the two measures.

Number of students	15-20	20-25	25-30	30-35	35-40	40-45	45-50	50-55
Number of States	3	8	9	10	3	0	0	2

5. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000	10000-11000
Number of batsmen	4	18	9	7	6	3	1	1

Find the mode of the data.

6. A student noted the number of cars passing through a spot on a road for 100 periods, each of 3 minutes, and summarised this in the table given below.

Number of cars	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Frequency	7	14	13	12	20	11	15	8

Find the mode of the data.

14.4 Median of Grouped Data

Median is a measure of central tendency which gives the value of the middle-most observation in the data. Recall that for finding the median of ungrouped data, we first arrange the data values or the observations in ascending order.

Then, if n is odd, the median is the $\left(\frac{n+1}{2}\right)^{th}$ observation and

if n is even, then the median will be the average of the $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

Suppose, we have to find the median of the following data, which is about the marks, out of 50 obtained by 100 students in a test :

Marks obtained	20	29	28	33	42	38	43	25
Number of students	6	28	24	15	2	4	1	20

First, we arrange the marks in ascending order and prepare a frequency table 14.9 as follows :

Marks obtained	Number of students (frequency)
20	6
25	20
28	24
29	28
33	15
38	4
42	2
43	1
Total	100

Here $n = 100$, which is even. The median will be the average of the $\left(\frac{n}{2}\right)^{th}$ and the $\left(\frac{n}{2}+1\right)^{th}$ observations, i.e., the 50^{th} and 51^{st} observations. To find the position of these middle values, we construct cumulative frequency.

Marks obtained	Number of students	Cumulative frequency
20	6	6
upto 25	6 + 20 = 26	26
upto 28	26 + 24 = 50	50
upto 29	50 + 28 = 78	78
upto 33	78 + 15 = 93	93
upto 38	93 + 4 = 97	97
upto 42	97 + 2 = 99	99
upto 43	99 + 1 = 100	100

Now we add another column depicting this information to the frequency table above and name it as *cumulative frequency column*.

From the table above, we see that :

50^{th} observation is 28 (Why?)

51^{st} observation is 29

$$\text{Median} = \frac{28+29}{2} = 28.5$$

Remark : Column 1 and column 3 in the above table are known as *Cumulative Frequency Table*. The median marks 28.5 conveys the information that about 50% students obtained marks less than 28.5 and another 50% students obtained marks more than 28.5.

Consider a grouped frequency distribution of marks obtained, out of 100, by 53 students, in a certain examination, as shown in adjacent table.

Marks	Number of students
0-10	5
10-20	3
20-30	4
30-40	3
40-50	3
50-60	4
60-70	7
70-80	9
80-90	7
90-100	8

From the table, try to answer the following questions :

How many students have scored marks less than 10? The answer is clearly 5.

How many students have scored less than 20 marks? Observe that the number of students who have scored less than 20 include the number of students who have scored marks from 0-10 as well as the number of students who have scored marks from 10-20. So, the total number of students with marks less than 20 is 5 + 3, i.e., 8. We say that the cumulative frequency of the class 10-20 is 8. (As shown in table 14.11)

Marks obtained	Number of students (Cumulative frequency)
Less than 10	5
Less than 20	5 + 3 = 8
Less than 30	8 + 4 = 12
Less than 40	12 + 3 = 15
Less than 50	15 + 3 = 18
Less than 60	18 + 4 = 22
Less than 70	22 + 7 = 29
Less than 80	29 + 9 = 38
Less than 90	38 + 7 = 45
Less than 100	45 + 8 = 53

Similarly, we can compute the cumulative frequencies of the other classes, i.e., the number of students with marks less than 30, less than 40, ..., less than 100.

This distribution is called the cumulative frequency distribution of the less than type. Here 10, 20, 30, ..., 100, are the upper boundaries of the respective class intervals.

We can similarly make the table for the number of students with scores more than or equal to 0 (this number is same as sum of all the frequencies), more than above sum minus the frequency of the first class interval), more than or equal to 20 (this number is same as the sum of all frequencies minus the sum of the frequencies of the first two class intervals), and so on. We observe that all 53 students have scored marks more than or equal to 0. Since there are 5 students scoring marks in the interval 0-10, this means that there

Marks obtained	Number of students (Cumulative frequency)
More than or equal to 0	53
More than or equal to 10	53 - 5 = 48
More than or equal to 20	48 - 3 = 45
More than or equal to 30	45 - 4 = 41
More than or equal to 40	41 - 3 = 38
More than or equal to 50	38 - 3 = 35
More than or equal to 60	35 - 4 = 31
More than or equal to 70	31 - 7 = 24
More than or equal to 80	24 - 9 = 15
More than or equal to 90	15 - 7 = 8

are 53-5 = 48 students getting more than or equal to 10 marks. Continuing in the same manner, we get the number of students scoring 20 or above as 48-3 = 45, 30 or above as 45-4 = 41, and so on, as shown in the table a side.

This table above is called a cumulative frequency distribution of the more than type. Here 0, 10, 20, ..., 90 give the lower boundaries of the respective class intervals.

Now, to find the median of grouped data, we can make use of any of these cumulative frequency distributions.

Now in a grouped data, we may not be able to find the middle observation by looking at the cumulative frequencies as the middle observation will be some value in a class interval. It is, therefore, necessary to find the value inside a class that divides the whole distribution into two halves. But which class should this be?

To find this class, we find the cumulative frequencies of all the clases and $\frac{n}{2}$. We now locate the class whose cumulative frequency exceeds $\frac{n}{2}$ for the first time. This is called the median class.

Marks	Number of students (f)	Cumulative frequency (cf)
0-10	5	5
10-20	3	8
20-30	4	12
30-40	3	15
40-50	3	18
50-60	4	22
60-70	7	29
70-80	9	38
80-90	7	45
90-100	8	53

In the distribution above, $n = 53$. So $\frac{n}{2} = 26.5$. Now 60-70 is the class whose cumulative frequency 29 is greater than (and nearest to) $\frac{n}{2}$, i.e., 26.5.

Therefore, 60-70 is the median class.

After finding the median class, we use the following formula for calculating the median.

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

where l = lower boundary of median clas,

n = number of observations,

cf = cumulative frequency of class preceding the median class,

f = frequency of median class,

h = class size (size of the median class).

Substituting the values $\frac{n}{2} = 26.5$, $l = 60$, $cf = 22$, $f = 7$, $h = 10$

in the formula above, we get

$$\text{Median} = 60 + \left[\frac{26.5 - 22}{6} \right] \times 10$$

$$= 60 + \frac{45}{7}$$

$$= 66.4$$

So, about half the students have scored marks less than 66.4, and the other half have scored marks more than 66.4.

Example-7. A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and data was obtained as shown in table. Find their median.

Height (in cm)	Number of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Solution : To calculate the median height, we need to find the class intervals and their corresponding frequencies. The given distribution being of the *less than type*, 140, 145, 150, . . ., 165 give the upper limits of the corresponding class intervals. So, the classes should be below 140, 140 - 145, 145 - 150, . . ., 160 - 165.

Class intervals	Frequency	Cumulative frequency
Below 140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51

Observe that from the given distribution, we find that there are 4 girls with height less than 140, i.e., the frequency of class interval below 140 is 4 . Now, there are 11 girls with heights less than 145 an d 4 girls with height less than 140. Therefore, the number of girls with height in the interval 140 - 145 is $11 - 4 = 7$. Similarly, the frequencies can be calculated as shown in table.

Number of observations, $n = 51$

$\frac{n}{2} = \frac{51}{2} = 25.5^{\text{th}}$ observation, which lies in the class 145 - 150.

$\therefore$ 145 - 150 is median class

Then, l (the lower boundary) = 145,

cf (the cumulative frequency of the class preceding 145 - 150) = 11,

f (the frequency of the median class 145 - 150) = 18,

h (the class size) = 5.

Using the formula, Median $= l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$

$$= 145 + \frac{(25.5 - 11)}{18} \times 5$$

$$= 145 + \frac{72.5}{18} = 149.03$$

So, the median height of the girls is 149.03 cm. This means that the height of about 50% of the girls is less than this height, and that of other 50% is greater than this height.

Example-8. The median of the following data is 525. Find the values of x and y, if the total frequency is 100. Here, CI stands for class interval and Fr for frequency.

CI	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Fr	2	5	x	12	17	20	y	9	7	4

Solution :

It is given that $n = 100$

So, $76 + x + y = 100$, i.e., $x + y = 24$ (1)

The median is 525, which lies in the class 500 – 600

So, $l = 500, f = 20, cf = 36 + x, h = 100$

Using the formula

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

$$525 = 500 + \frac{50 - 36 - x}{20} \times 100$$

i.e., $525 - 500 = (14 - x) \times 5$

i.e., $25 = 70 - 5x$

i.e., $5x = 70 - 25 = 45$

So, $x = 9$

Therefore, from (1), we get $9 + y = 24$

i.e., $y = 15$

Class intervals	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	x	$7+x$
300-400	12	$19+x$
400-500	17	$36+x$
500-600	20	$56+x$
600-700	y	$56+x+y$
700-800	9	$65+x+y$
800-900	7	$72+x+y$
900-1000	4	$76+x+y$

Note :

The median of grouped data with unequal class sizes can also be calculated.

14.5 Which value of Central Tendency

Which measure would be best suited for a particular requirement.

The mean is the most frequently used measure of central tendency because it takes into account all the observations, and lies between the extremes, i.e., the largest and the smallest observations of the entire data. It also enables us to compare two or more distributions. For example, by comparing the average (mean) results of students of different schools of a particular examination, we can conclude which school has a better performance.

However, extreme values in the data affect the mean. For example, the mean of classes having frequencies more or less the same is a good representative of the data. But, if one class has frequency, say 2, and the five others have frequency 20, 25, 20, 21, 18, then the mean will certainly not reflect the way the data behaves. So, in such cases, the mean is not a good representative of the data.

In problems where individual observations are not important, especially extreem values, and we wish to find out a 'typical' observation, the median is more appropriate, e.g., finding the typical productivity rate of workers, average wage in a country, etc. These are situations where extreme values may exist. So, rather than the mean, we take the median as a better measure of central tendency.

In situations which require establishing the most frequent value or most popular item, the mode is the best choice, e.g., to find the most popular T.V. programme being watched, the consumer item in greatest demand, the colour of the vehicle used by most of the people, etc.

Exercise - 14.3

1. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

2. If the median of 60 observations, given below is 28.5, find the values of x and y.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	20	15	y	5

3. A life insurance agent found the following data about distribution of ages of 100 policy holders. Calculate the median age. [Policies are given only to persons having age 18 years onwards but less than 60 years.]

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
Number of policy holders	2	6	24	45	78	89	92	98	100

4. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table :

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
Number of leaves	3	5	9	12	5	4	2

Find the median length of the leaves. (**Hint :** The data needs to be converted to continuous classes for finding the median, since the formula assumes continuous classes. The classes then change to 117.5 - 126.5, 126.5 - 135.5, . . ., 171.5 - 180.5.)

5. The following table gives the distribution of the life-time of 400 neon lamps

Life time (in hours)	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
Number of lamps	14	56	60	86	74	62	48

Find the median life time of a lamp.

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabet in the surnames was obtained as follows

Number of letters	1-4	4-7	7-10	10-13	13-16	16-19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames? Also, find the modal size of the surnames.

7. The distribution below gives the weights of 30 students of a class. Find the median weight of the students.

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students	2	3	8	6	6	3	2

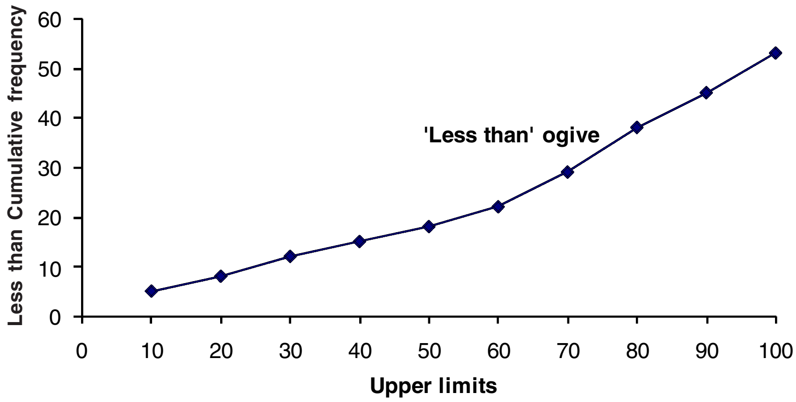
14.6 Graphical Representation of Cumulative Frequency Distribution

As we all know, pictures speak better than words. A graphical representation helps us in understanding given data at a glance. In Class IX, we have represented the data through bar graphs, histograms and frequency polygons. Let us now represent a cumulative frequency distribution graphically.

For example, let us consider the cumulative frequency distribution given in example.

For drawing ozires, it should be ensured that the class intervals are continuous, because cumulative frequencies are linked with boundaries, but not with limits.

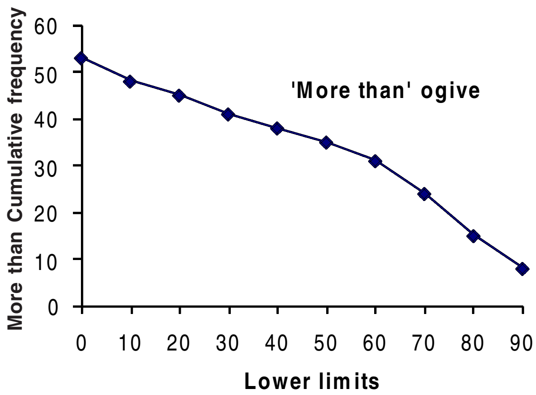
Recall that the values 10, 20, 30, ..., 100 are the upper boundaries of the respective class intervals. To represent the data graphically, we mark the upper boundaries of the class intervals on the horizontal axis (X-axis) and their corresponding cumulative frequencies on the vartical axis (Y-axis), choosing a convenient scale. Now plot the points corresponding to the ordered pairs given by (upper boundary, corresponding cumulative frequency), i.e., (10, 5), (20, 8), (30,, 12), (40, 15), (50, 18), (60, 22), (70, 29), (80, 38), (90, 45), (100, 53) on a graph paper and join them by a free hand smooth curve. The curve we get is called a cumulative frequency curve, or an ogive (of the less than type).



The term 'ogive is pronounced as 'ojeev' and is derived from the word ogee. An ogee is a shape consisting of a concave arc flowing into a convex arc, so forming an S-shaped curve with vertical ends. In architecture, the ogee shape is one of the characteristics of the 14th and 15th century Gothic styles.

Again we consider the cumulative frequency distribution and draw its ogive (of the more than type).

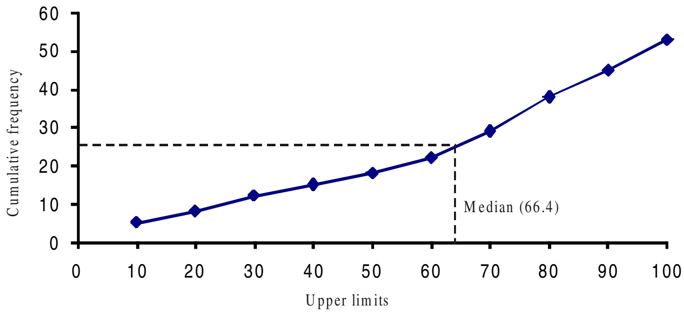
Recall that, here 0, 10, 20, ..., 90 are the lower boundaries of the respective class intervals 0-10, 10-20, ..., 90-100. To represent 'the more than type' graphically, we plot the lower boundaries on the X-axis and the corresponding cumulative frequencies on the Y-axis. Then we plot the points (lower boundaries, corresponding cumulative frequency), i.e., (0, 53), (10, 48), (20, (45), (30, 41), (40, 38), (50, 35), (60, 31), (70, 24), (80, 15), (90, 8), on a graph paper, and join them by a free hand smooth curve. The curve we get is a cumulative frequency curve, or an ogive (of the more than type).



14.6.1 Obtaining Median from give curve:

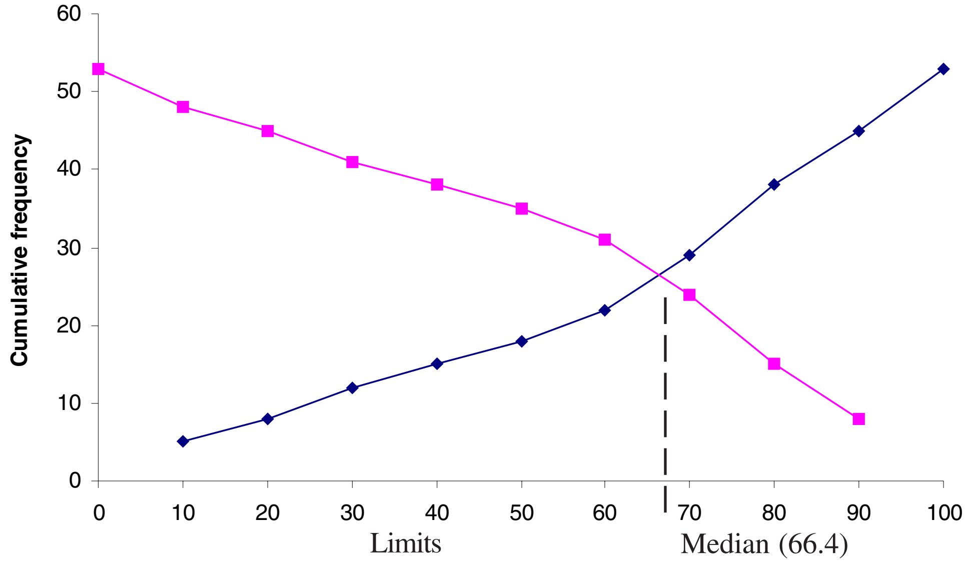
Is it possible to obtain the median from these two cumulative frequency curves . Let us see.

One obvious way is to locate on $\frac{n}{2} = \frac{53}{2} = 26.5$ on the y-axis. From this point, draw a line parallel to the x-axis cutting the curve at a point. From this point, draw a perpendicular to the x-axis. Foot of this perpendicular determines the median of the data.



Another way of obtaining the median :

Draw both ogives (i.e., of the less than type and of the more than type) on the same axis. The two ogives will intersect each other at a point. From this point, if we draw a perpendicular on the x-axis, the point at which it cuts the x-axis gives us the median.

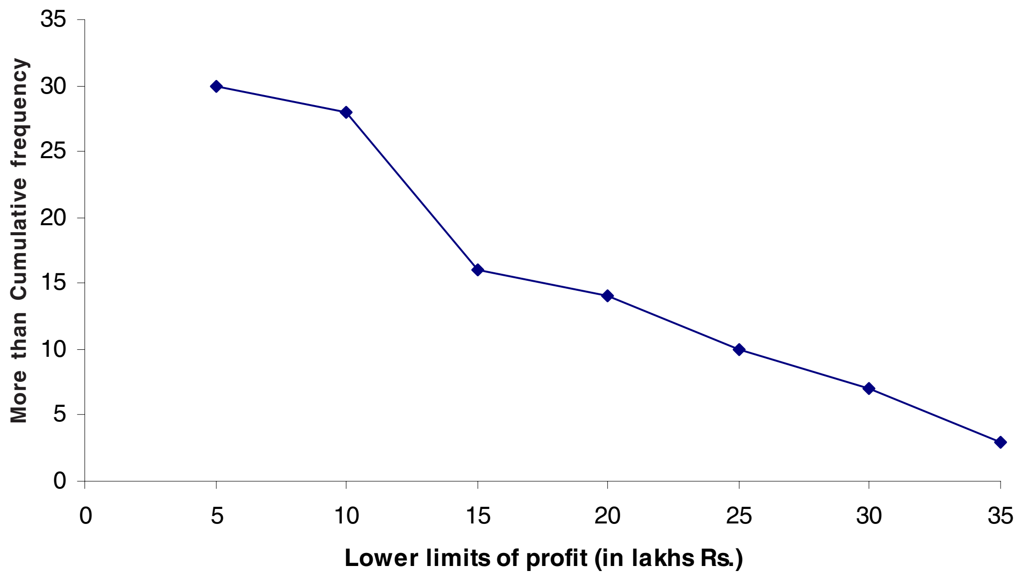


Example-9. The annual profits earned by 30 shops in a locality give rise to the following distribution :

Profit (in lakhs)	Number of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Draw both ogives for the data above. Hence obtain the median profit.

Solution : We first draw the coordinate axes, with lower limits of the profit along the horizontal axis, and the cumulative frequency along the vertical axes. Then, we plot the points (5, 30), (10, 28), (15, 16), (20, 14), (25, 10), (30, 7) and (35, 3). We join these points with a smooth curve to get the more than ovive, as shown in the figure below-

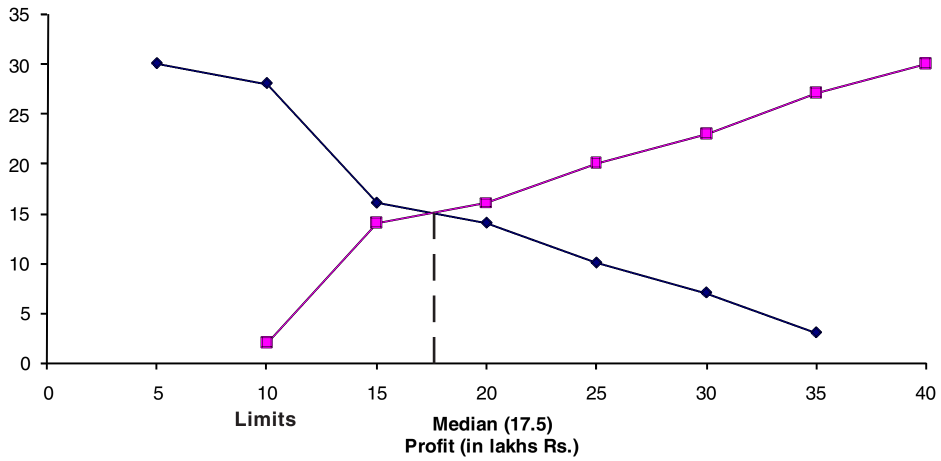


Now, let us obtain the classes, their frequencies and the cumulative frequency from the table above.

Classes	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Number of shops	2	12	2	4	3	4	3
Cumulative frequency	2	14	16	20	23	27	30

Using these values, we plot the points (10, 2), (15, 14), (20, 16), (25, 20), (30, 23), (35, 27), (40, 30) on the same axes as in last figure to get the less than ogive, as shown in figure below.

The abcissa of their point of intersection is nearly 17.5, which is the median. This can also be verified by using the formula. Hence, the median profit (in lakhs) is ₹ 17.5.



Exercise - 14.4

1. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in Rupees)	250-300	300-350	350-400	400-450	450-500
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

2. During the medical check-up of 35 students of a class, their weights were recorded as follows :

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

3. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (Qui/Hec)	50-55	55-60	60-65	65-70	70-75	75-80
Number of farmers	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

What We Have Discussed

In this chapter, you have studied the following points :

1. The mean for grouped is calculated by :

   (i) The direct method : $\overline{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

   (ii) The assumed mean method : $\overline{x} = a + \dfrac{\sum f_i d_i}{\sum f_i}$

   (iii) The step deviation method : $\overline{x} = a + \left(\dfrac{\sum f_i u_i}{\sum f_i}\right) \times h$

2. The mode for grouped data can be found by using the formula :

   $$\text{Mode } = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

   where, symbols have their usual meaning.

3. The median for grouped data is formed by using the formula :

   Median $= l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h$ Where symbols have their usual meanings.

4. In order to find median, class intervals should be continuous.

5. Representing a cumulative frequency distribution graphically as a cumulative frequency curve, or an ogive of the less than type and of the more than type.

6. While drowing ogives boundaries are taken on X-main and cumulative frequencies are taken on Y-axis.

7. Scale on both the axes mya not be equal.

8. The median of grouped data can be obtained graphically as the x-coordinate of the point of intersection of the two ogives for this data.

APPENDIX

Mathematical Modelling

A.I.1 Introduction

On 25th February 2013, the ISRO launcher PSLV C20, put the satellite SARAL into orbit. The satellite weighs 407 kg. It is at an altitude of 781 km and its orbit is inclined at an angle of 98.5º.

On reading the above information, we may wonder:

(i) How did the scientists calculate the altitude as 781km. Did they go to space and measure it?

(ii) How did they conclude that the angle of orbit is 98.5º without actually measuring?

Some more examples are there in our daily life where we wonder how the scientists and mathematicians could possibly have estimated these results. Observe these examples:

(i) The temperature at the surface of the sun is about 6,000ºC.

(ii) The human heart pumps 5 to 6 liters of blood in the body every minute.

(iii) We know that the distance between the sun and the earth is 1,49,000 km.

In the above examples, we know that no one went to the sun to measure the temperature or the distance from earth. Nor can we take the heart out of the body and measure the blood it pumps. The way we answer these and other similar questions is through mathematical modelling.

Mathematical modelling is used not only by scientists but also by us. For example, we might want to know how much money we will get after one year if we invest ₹100 at 10% simple interest. Or we might want to know how many litres of paint is needed to whitewash a room. Even these problems are solved by mathematical modelling.

Think - Discuss

Discuss with your friends some more examples in real life where we cannot directly measure and must use mathematical modelling .

A.I.2 Mathematical Models

Do you remember the formula to calculate the area of a triangle?

Area of Triangle = $\frac{1}{2} \times$ base $\times$ height.

Similarly, simple interest calculation uses the formula $I = \frac{PTR}{100}$. This formula or equation is a relation between the Interest (I); Principle (P); Time (T); and Rate of Interest (R). These formulae are examples of mathematical models.

Some more examples for mathematical models.

(i) Speed (S) $= \frac{Distance\ (d)}{time\ (t)}$

(ii) In compound interest sum (A) = $P\left(1+\frac{r}{100}\right)^n$

Where P = Principle

r = rate of interest

n = no. of times to be calculated interest.

So, Mathematical model is nothing but a mathematical description or relation that describes some real life situation.

Do This

Write some more mathematical models which you have learnt in previous classes.

A.I.3 Mathematical Modelling

We often face problems in our day to day life. To solve them, we try to write it as an equivalent mathematical problem and find its solution. Next we interpret the solution and check to what extent the solution is valid. This process of constructing a mathematical model and using it to find the answer is known as mathematical modelling.

Now we have to observe some more examples related to mathematical modelling.

Example-1. Vani wants to buy a TV that costs ₹19,000 but she has only ₹15,000. So she decides to invest her money at 8% simple interest per year. After how many years will she be able to buy the TV?

Step 1 : (Understanding the problem): In this stage, we define the real problem. Here, we are given the principal, the rate of simple interest and we want to find out the number of years after which the amount will become Rs. 19000.

Step 2 : (Mathematical description and formulation) In this step, we describe, in mathematical terms, the different aspects of the problem. We define variables, write equations or inequalities and gather data if required.

Here, we use the formula for simple interest which is

$$I = \frac{PTR}{100} \text{ (Model)}$$

where P = Principle, T = number of years, R = rate of interest, I = Interest

We need to find time $= T = \dfrac{100I}{RP}$

Step 3: (Solving the mathematical problem) In this step, we solve the problem using the formula which we have developed in step 2.

We know that Vani already has ₹15,000 which is the principal, P

The final amount is ₹19000 so she needs an extra (19000-15000) = ₹4000. This will come from the interest, I.

P = ₹15,000, Rate = 8%, then I = 4000; $T = \dfrac{100 \times 4\cancel{0}\cancel{0}\cancel{0}}{150\cancel{0}\cancel{0} \times 8} = \dfrac{40\cancel{0}}{12\cancel{0}}$

$$T = 3\frac{4}{12} = 3\frac{1}{3} \text{ years}$$

or **Step4 : (Interpreting the solution):** The solution obtained in the previous step is interpreted here.

Here T = $3\dfrac{1}{3}$. This means three and one third of a year or three years and 4 months.

So, Vani can buy a washing machine after 3 year 4 months

Step5 : (Validating the model): We can't always accept a model that gives us an answer that does not match the reality. The process of checking and modifying the mathematical model, if necessary, is validation.

In the given example, we are assuming that the rate of interest will not change. If the rate changes then our model $\frac{PTR}{100}$ will not work. We are also assuming that the price of the washing machine will remain Rs. 19,000.

Let us take another example.

Example-2. In Lokeshwaram High school, 50 children in the 10th class and their Maths teacher want to go on tour from Lokeshwaram to Hyderabad by vehicles. Each vehicle can hold six persons not including driver. How many vehicles they need to hire?

Step 1 : We want to find the number of vehicles needed to carry 51 persons, given that each jeep can seat 6 persons besides the driver.

Step 2 : Number of vehicles = (Number of persons) / (Persons that can be seated in one jeep)

Step 3 : Number of vehicles = 51/6 = 8.5

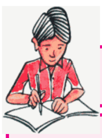
Step 4 : Interpretation

We know that it is not possible to have 8.5 vehicles. So, the number of vehicles needed has to be the nearest whole number which is 9.

$\therefore$ Number of vehicles need is 9.

Step 5 : Validation

While modelling, we have assumed that lean and fat children occupy same space.



Do This

1. Take any word problem from your textbook, make a mathematical model for the chosen problem and solve it.

2. Make a mathematical model for the problem given below and solve it.

   Suppose a car starts from a place A and travels at a speed of 40 Km/h towards another place B. At the same time another car starts from B and travels towards A at a speed of 30 Km/h. If the distance between A and B is 100 km; after how much time will that cars meet?

So far, we have made mathematical models for simple word problems. Let us take a real life example and model it.

Example-3. In the year 2000, 191 member countries of the U.N.signed a declaration to promote gender equality. One indicator for deciding whether this goal has been achieved is the ratio of girls to boys in primary, secondary education. India also signed the declaration. The data for the percentage of girls in India who are enrolled in primary schools is given in Table **A.I.1.**

Table A.I.1

Year	Enrolment (in %)
1991 – 92	41.9
1992 – 93	42.6
1993 – 94	42.7
1994 – 95	42.9
1995 – 96	43.1
1996 – 97	43.2
1997 -98	43.5
1998 – 99	43.5
1999 – 2000	43.6
2000 – 01	43.7
2001 - 02	44.1

Using this data, mathematically describe the rate at which the proportion of girls enrolled in primary schools grew. Also, estimate the year by which the enrollment of girls will reach 50%.

Solution :

Step 1 : Formulation Let us first convert the problem into a mathematical problem.

Table A.I.1 gives the enrollment for the years 1991 – 92, 1992- 93 etc. Since the students join at the begining of an academic year, we can take the years as 1991, 1992 etc. Let us assume that the percentage of girls who join primary schools will continue to grow at the same rate as the rate in Table A.I.1. So, the number of years is important, not the specific years. (To give a similar situation, when we find the simple interest for say, ₹ 15000 at the rate 8% for three years, it does not matter whether the three – year period is from 1999 to 2002 or from 2001 to 2004. What is important is the interest rate in the years being considered)

Here also, we will see how the enrollment grows after 1991 by comparing the number of years that has passed after 1991 and the enrollment. Let us take 1991 as the 0^{th} year, and write 1 for 1992 since 1 year has passed in 1992 after 1991. Similarly we will write 2 for 1993, 3 for 1994 etc. So, Table A.I.1 will now look like as Table A.I.2

Table A.I.2

Year	Enrolment (in%)
0	41.9
1	42.6
2	42.7
3	42.9
4	43.1
5	43.2
6	43.5
7	43.5
8	43.6
9	43.7
10	44.1

The increase in enrolment is given in the following table A.I.3.

Table A.I.3

Year	Enrolment (in%)	Increase
0	41.9	0
1	42.6	0.7
2	42.7	0.1
3	42.9	0.2
4	43.1	0.2
5	43.2	0.1
6	43.5	0.3
7	43.5	0
8	43.6	0.1
9	43.7	0.1
10	44.1	0.4

At the end of the first year period from 1991 to 1992, the enrollment has increased by 0.7% from 41.9% to 42.6%. At the end of the second year, this has increased by 0.1% from 42.6% to 42.7%. From the table above, we cannot find a definite relationship between the number of years and percentage. But the increase is fairly steady. Only in the first year and in the 10^{th} year there is a jump. The mean of these values is

$$\frac{0.7+0.1+0.2+0.2+0.1+0.3+0+0.1+0.1+0.4}{10} = 0.22 \qquad \text{.... (1)}$$

Let us assume that the enrolment steadily increases at the rate of 0.22 percent.

Step 2 : (Mathematical Description)

We have assumed that the enrolment increases steadily at the rate of 0.22% per year.

So, the Enrolment Percentage (EP) in the first year = 41.9 + 0.22

EP in the second year = 41.9 + 0.22 + 0.22 = 41.9 + 2 $\times$ 0.22

EP in the third year = 41.9 + 0.22 + 0.22 + 0.22 = 41.9 + 3 $\times$ 0.22

So, the enrolment percentage in the n^{th} year = 41.9 + 0.22n, for n $\geq$ 1. .... (2)

Now, we also have to find the number of years by which the enrolment will reach 50%. So, we have to find the value of *n* from this equation

$$50 = 41.9 + 0.22n$$

Step 3 : Solution : Solving (2) for n, we get

$$n = \frac{50-41.9}{0.22} = \frac{8.1}{0.22} = 36.8$$

Step 4 : (Interpretation) : Since the number of years is an integral value, we will take the next higher integer, 37. So, the enrolment percentage will reach 50% in 1991 + 37 = 2028.

Step 5 : (Validation) Since we are dealing with a real life situation, we have to see to what extent this value matches the real situation.

Let us check Formula (2) is in agreement with the reality. Let us find the values for the years we already know, using Formula (2), and compare it with the known values by finding the difference. The values are given in Table A.I.4.

Table A.I.4

Year	Enrolment (in %)	Values given by (2) (in %)	Difference (in %)
0	41.9	41.90	0
1	42.6	42.12	0.48
2	42.7	42.34	0.36
3	42.9	42.56	0.34
4	43.1	42.78	0.32
5	43.2	43.00	0.20
6	43.5	43.22	0.28
7	43.5	43.44	0.06
8	43.6	43.66	-0.06
9	43.7	43.88	-0.18
10	44.1	44.10	0.00

As you can see, some of the values given by Formula (2) are less than the actual values by about 0.3% or even by 0.5%. This can give rise to a difference of about 3 to 5 years since the increase per year is actually 1% to 2%. We may decide that this much of a difference is acceptable and stop here. In this case, (2) is our mathematical model.

Suppose we decide that this error is quite large, and we have to improve this model. Then, we have to go back to Step 2, and change the equation. Let us do so.

Step 1 : Reformulation : We still assume that the values increase steadily by 0.22%, but we will now introduce a correction factor to reduce the error, For this, we find the mean of all the errors. This is

$$\frac{0+0.48+0.36+0.34+0.32+0.2+0.28+0.06-0.06-0.18+0}{10}=0.18$$

We take the mean of the errors, and correct our formula by this value.

Revised Mathematical Description : Let us now add the mean of the errors to our formula for enrolment percentage given in (2). So, our corrected formula is :

Enrolment percentage in the nth year

$$= 41.9 + 0.22n + 0.18 = 42.08 + 0.22n, \text{ for } n \geq 1 \qquad \text{... (3)}$$

We will also modify Equation (2) appropriately. The new equation for n is :

$$50 = 42.08 + 0.22n \qquad \text{... (4)}$$

Altered Solution : Solving Equation (4) for n, we get

$$n = \frac{50 - 42.08}{0.22} = \frac{7.92}{0.22} = 36$$

Interpretation : Since n = 36, the enrolment of girls in primary schools will reach 50% in the year 1991 + 36 = 2027.

Validation : Once again, let us compare the values got by using Formula (4) with the actual values. Table A.I.5 gives the comparison.

Table A.I.5

Year	Enrolment (in %)	Values given by (2)	Difference between Values	Values given by (4)	Difference between values
0	41.9	41.90	0	41.9	0
1	42.6	42.12	0.48	42.3	0.3
2	42.7	42.34	0.36	42.52	0.18
3	42.9	42.56	0.34	42.74	0.16
4	43.1	42.78	0.32	42.96	0.14
5	43.2	43.00	0.20	43.18	0.02
6	43.5	43.22	0.28	43.4	0.1
7	43.5	43.44	0.06	43.62	-0.12
8	43.6	43.66	-0.06	43.84	-0.24
9	43.7	43.88	-0.18	44.06	-0.36
10	44.1	44.10	0.00	44.28	-0.18

As you can see, many of the values that (4) gives are closer to the actual value than the values that (2) gives. The mean of the errors is 0 in this case.

A.I.4 Advantages of Mathematics Modeling

1. The aim of mathematical modeling is to get some useful information about a real world problem by converting it into mathematical problem. This is especially useful when it is not possible or very expensive to get information by other means such as direct observation or by conducting experiments.

   For example, suppose we want to study the corrosive effect of the discharge of the Mathura refinery on the Taj Mahal. We would not like to carry out experiments on the Taj Mahal directly because that would damage a valuable monument. Here mathematical modeling can be of great use.

2. Forecasting is very important in many types of organizations, since predictions of future events have to be incorporated into the decision – making process.

For example

(i) In marketing departments, reliable forecasts of demand help in planning of the sale strategies

(ii) A school board needs to able to forecast the increase in the number of school going children in various districts so as to decide where and when to start new schools.

3. Often we need to estimate large values like trees in a forest; fishes in a lake; estimation of votes polled etc.

   Some more examples where we use mathematical modelling are:

   (i) Estimating future population for certain number of years

   (ii) Predicting the arrival of Monsoon

   (iii) Estimating the literacy rate in coming years

   (iv) Estimating number of leaves in a tree

   (v) Finding the depth of oceans

A.I.5 Limitations of Mathematical modeling

Is mathematical modeling the answer to all our problems?

Certainly not; it has it's limitations. Thus, we should keep in mind that a model is only a simplification of a real world problem, and the two are not same. It is some thing like the difference between a map that gives the physical features of a country, and the country itself. We can find the height of a place above the sea level from this map, but we cannot find the characteristics of the people from it. So, we should use a model only for the purpose it is supposed to serve, remembering all the factors we have neglected while constructing it. We should apply the model only within the limits where it is applicable.

A.I.6 To What Extent We Should Try To Improve Our Model?

To improve a model we need to take into account several additional factors. When we do this we add more variables to our mathematical equations. The equations becomes complicated and the model is difficult to use. A model must be simple enough to use yet accurate; i.e the closer it is to reality the better the model is.

Try This

A problem dating back to the early 13th century, posed by Leonardo Fibonacci, asks how many rabbits you would have in one year if you started with just two and let all of them reproduce. Assume that a pair of rabbits produces a pair of offspring each month and that each pair of rabbits produces their first offspring at the age of 2 months. Month by month, the number of pairs of rabbits is given by the sum of the rabbits in the two preceding months, except for the 0th and the 1st months. The table below shows how the rabbit population keeps increasing every month.

Month	Pairs of Rabbits
0	1
1	1
2	2
3	3
4	5
5	8
6	13
7	21
8	34
9	55
10	89
11	144
12	233
13	377
14	610
15	987
16	1597

After one year, we have 233 rabbits. After just 16 months, we have nearly 1600 pairs of rabbits.

Clearly state the problem and the different stages of mathematical modelling in this situation.

We will finish this chapter by looking at some interesting examples.

Example-4. (Rolling of a pair of dice) : Deekshitha and Ashish are playing with dice. Then Ashish said that, if she correctly guess the sum of numbers that show up on the dice, he would give a prize for every answer to her. What numbers would be the best guess for Deekshitha.

Solution :

Step 1 (Understanding the problem) : You need to know a few numbers which have higher chances of showing up.

Step 2 (Mathematical description) : In mathematical terms, the problem translates to finding out the probabilities of the various possible sums of numbers that the dice could show.

We can model the situation very simply by representing a roll of the dice as a random choice of one of the following thirty six pairs of numbers.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The first number in each pair represents the number showing on the first die, and the second number is the number showing on the second die.

Step 3 (Solving the mathematical problem) : Summing the numbers in each pair above, we find that possible sums are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. We have to find the probability for each of them, assuming all 36 pairs are equally likely.

We do this in the following table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Observe that the chance of getting a sum of a seven is $\frac{1}{6}$, which is larger than the chances of getting other numbers as sums.

Step 4 (Interpreting the solution) : Since the probability of getting the sum 7 is the highest, you should repeatedly guess the number seven.

Step 5 (Validating the model) : Toss a pair of dice a large number of times and prepare a relative frequency table. Compare the relative frequencies with the corresponding probabilities. If these are not close, then possibly the dice are biased. Then, we could obtain data to evaluate the number towards which the bias is.

Before going to the next try this exercise, we need some background information.

Not having the money you want when you need it, is a common experience for many people. Whether it is having enough money for buying essentials for daily living, or for buying comforts, we always require money. To enable the customers with limited funds to purchase goods like scooters, refrigerators, televisions, cars, etc., a scheme known as an instalment scheme (or plan) is introduced by traders.

Sometimes a trader introduces an instalment scheme as a marketing strategy to allow customers to purchase these articles. Under the instalment scheme, the customer is not required to make full payment of the article at the time of buying it. She/he is allowed to pay a part of it at the time of purchase and the rest can be paid in instalments, which could be monthly, quarterly, half-yearly, or even yearly. Of course, the buyer will have to pay more in the instalment plan, because the seller is going to charge some interest on account of the payment made at a later date (called deferred payment).

There are some frequently used terms related to this concept. You may be familiar with them. For example, the cash price of an article is the amount which a customer has to pay as full payment of the article at the time it is purchased. Cash down payment is the amount which a customer has to pay as part payment of the price of an article at the time of purchase.

Now, try to solve the problem given below by using mathematical modelling.

Try This

Ravi wants to buy a bicycle. He goes to the market and finds that the bicycle of his choice costs ₹2,400. He has only ₹1,400 with him. To help, the shopkeepr offers to help him. He says that He can make a down payment of ₹1400 and pay the rest in monthly instalments of ₹550 each. Ravi can either take the shopkeepers offer or go to a bank and take a loan at 12% per annum simple interest. From these two opportunities which is the best one to Ravi. Help him.

Answers

Exercise - 1.1

1. (i) Terminating (ii) Non-terminating (iii) Terminating
   (iv) Terminating (v) Non-terminating
2. (i) $\frac{3}{4}$ (ii) $3\frac{1}{2}$ (iii) $\frac{31}{25}$
3. (i) Rational (ii) Irrational (iii) Rational (iv) Rational
   (v) Rational (vi) Irrational (vii) Rational

Exercise - 1.2

1. (i) $2^2 \times 5 \times 7$ (ii) $2^2 \times 3 \times 13$ (iii) $3^2 \times 5^2 \times 17$
   (iv) $5 \times 7 \times 11 \times 13$ (v) $17 \times 19 \times 23$
2. (i) LCM = 420, HCF = 3 (ii) LCM = 11339, HCF = 1
   (iii) LCM = 1800, HCF = 1 (iv) LCM = 216, HCF = 36
   (v) LCM = 22338, HCF = 9

Exercise - 1.3

1. (i) 0.375 (terminating) (ii) 0.5725 (terminating) (iii) 4.2 (terminating)
   (iv) $0.\overline{18}$ (non-terminating, repeating) (v) 0.064 (terminating)
2. (i) Terminating (ii) Non-terminating,repeating
   (iii) Non-terminating,repeating (iv) Terminating
   (v) Non-terminating, repeating (vi) Terminating
   (vii) Non-terminating, repeating (viii) Terminating (ix) Terminating
   (x) Non-terminating, repeating
3. (i) 0.52 (ii) 0.9375 (iii) 0.115 (iv) 32.08 (v) 1.3
4. (i) Rational (ii) Not a rational (iii) Rational

5. m = 5, n = 3

6. m = 4, n = 2

Exercise - 1.5

1. (i) $\log_3 243 = 5$ (ii) $\log_2 1024 = 10$ (iii) $\log_{10} 1000000 = 6$

(iv) $\log_{10} 0.001 = -3$ (v) $\log_3 \frac{1}{9} = -2$ (vi) $\log_6 1 = 0$

(vii) $\log_5 \frac{1}{5} = -1$ (viii) $\log_{\sqrt{49}} 7 = 1$ (ix) $\log_{27} 9 = \frac{2}{3}$

(x) $\log_{32} \frac{1}{4} = -\frac{2}{5}$

2. (i) $18^2 = 324$ (ii) $10^4 = 10000$ (iii) $a^b = \sqrt{x}$

iv) $4x = 8$ (v) $3y = \frac{1}{27}$

3. (i) $\frac{1}{2}$ (ii) $\frac{1}{4}$ (iii) -4 (iv) 0

(v) $\frac{1}{2}$ (vi) 9 (vii) -2 (viii) 3

4. (i) log 10 (ii) log 8 (iii) log 64

(iv) $\log \frac{9}{8}$ (v) log 243 (vi) log 45

5. (i) 3(log 2 + log 5) (ii) 7log 2 – 4log 5 (iii) $2\log x + 3\log y + 4\log z$

(iv) $2\log p + 3\log q - \log r$ (v) $\frac{1}{2}(3\log x - 2\log y)$

Exercise - 2.1

1. (i) Set (ii) Not set (iii) Not set

(iv) Set (v) Set

2. (i) $\in$ (ii) $\notin$ (iii) $\notin$ (iv) $\notin$

(v) $\in$ (vi) $\in$

3. (i) $x \notin A$ (ii) $d \in B$ (iii) $1 \in N$ (iv) $8 \notin P$

4. (i) Not true (ii) Not true (iii) True (iv) Not true

5. (i) B = {1, 2, 3, 4, 5}
(ii) C = {17, 26, 35, 44, 53, 62, 71}
(iii) D = {5, 3}
(iv) E = {B, E, T, R}

6. (i) A = {x : x is multiple of 3 & less than 13}
(ii) B = {x : x is in power of 2^x & x is less than 6}
(iii) C = {x : x is in power of 5 & x less than 5)
(iv) D = {x : x in square of natural number and not greater than 10}

7. (i) A = {51, 52, 53, ..., 98, 99}
(ii) B = {+2, – 2}
(iii) D = {L, O, Y, A}

8. (i) – (c)
(ii) – (a)
(iii) (d)
(iv) (b)

Exercise - 2.2

1. (i) Not empty (ii) Empty (iii) Empty
(iv) Empty (v) Not empty

2. (i) Finite (ii) Finite (iii) Finite

3. (i) Finite (ii) Infinite (iii) Infinite (iv) Infinite

Exercise - 2.3

1. Yes, equal set

2. (i) Equal (=) (ii) Not equal (≠) (iii) Equal (=) (iv) Not equal (≠)
(v) Not equal (≠) (vi) Not equal (≠) (vii) Not equal (≠)

3. (i) A = B (ii) A ≠ B (iii) A = B (iv) A ≠ B

Exercise - 2.4

1. (i) True (ii) Not false (iii) False (iv) False

2. (i) $\{1, 2, 3, ... 10\} \neq \{2, 3, 4, ... 9\}$

(ii) $x = 2x + 1$ means x is odd

(iii) x is multiple of 15. So 5 does not exist

(iv) x is prime number but 9 is not a prime number

3. (i) $\{p\}, \{q\}, \{p, q\}, \{\phi\}$

(ii) $\{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{z, x\}, \{x, y, z\}, \phi$

(iii) $\{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}$, $\{b, \text{c}, d\}, \{a, b, d\}, \{a, c, d\}, \{a, b, c, d\}, \phi$

(iv) $\phi, \{1\}, \{4\}, \{9\}, \{16\}, \{1, 4\}, \{1, 9\}, \{1, 16\}, \{4, 9\}, \{4, 16\}, \{9, 16\}$, $\{1, 4, 9\}, \{1, 9, 16\}, \{4, 9, 16\}, \{1, 4, 16\}, \{1, 4, 9, 16\}$

(v) $\phi, \{10\}, \{100\}, \{1000\}, \{10, 100\}, \{100, 1000\}, \{10, 1000\}$, $\{10, 100, 1000\}$

Exercise - 2.5

1. Yes, $A \cap B$ & $B \cap B$ are same

2. $A \cap \phi = \phi$

$A \cap A = A$

3. $A - B = \{2, 4, 8, 10\}$

$B - A = \{3, 9, 12, 15\}$

4. $A \cup B = B$

5. $A \cap B = \{\text{even natural number}\}$

$\{2, 4, 6, ...\}$

$A \cap C = \{\text{odd natural numbers}\}$

$A \cap D = \{4, 6, 8, 9, 10, 12, ... 100\}$

$B \cap C = \phi$

$B \cap D = \{\text{even natural number}\}$

$C \cap D = \{4, 6, 8, 9, ... 99\}$

6. (i) $A - B = \{3, 6, 9, 15, 18, 21\}$

(ii) $A - C = \{3, 9, 15, 18, 21\}$

(iii) $A - D = \{3, 6, 9, 12, 18, 21\}$

(iv) $B - A = \{4, 8, 16, 20\}$

(v) $C - A = \{2, 4, 8, 10, 14, 16\}$

(vi) D – A = {5, 10, 20}

(vii) B – C = {20}

(viii) B – D = {4, 8, 12, 16}

(ix) C – B = {2, 6, 10, 14)

(x) D – B = {5, 10, 15}

7. (i) False, because they have common element '3'

(ii) False, because the two sets have a common element 'a'

(iii) True, because no common elements for the sets.

(iv) True, because no common elements for the sets.

Exercise - 3.1

1. (a) (i) – 6 (ii) 7 (iii) – 6

(b) Left to children

2. (i) False ($\sqrt{2}$ is coefficient of x^2 not a degree)

(ii) False (Coefficient of x^2 is – 4)

(iii) True (For any constant term, degree is zero)

(iv) False (It is not a polynomial at all)

(v) False (Degree of a polynomial is not related with number of terms)

3. p(1) = 0, p (– 1) = – 2, p(0) = – 1, p(2) = 7, p(– 2) = – 9

4. Yes, – 2 and – 2 are zeroes of the polynomial x^4-16

5. Yes, 3 and – 2 are zeroes of the polynomial $x^2 – x – 6$

Exercise - 3.2

1. (i) No zeroes (ii) 1 (iii) 3

(iv) 2 (v) 4 (vi) 3

2. (i) 0 (ii) – 2, – 3 (iii) – 2, – 3 (iv) – 2, 2, $\pm\sqrt{-4}$

3. (i) 4, – 3 (ii) 3, 3 (iii) No zeroes

(iv) – 4, 1 (v) – 1, 1

4. $p\left(\frac{1}{4}\right) = 0$ and p (– 1) = 0

Exercise - 3.3

1. (i) $4, -2$ (ii) $\frac{1}{2}, \frac{1}{2}$ (iii) $\frac{3}{2}, \frac{-1}{3}$

   (iv) $0, -2$ (v) $\sqrt{15}\ -\sqrt{15}$ (vi) $-1, \frac{4}{3}$

2. (i) $4x^2 - x - 4$ (ii) $3x^2 - 3\sqrt{2}x + 1$ (iii) $x^2 + \sqrt{5}$

   (iv) $x^2 - x + 1$ (v) $4x^2 + x + 1$ (vi) $x^2 - 4x + 1$

3. (i) $x^2 - x - 2$ (ii) $x^2 - 3$ (iii) $4x^2 + 3x - 1$

   (iv) $4x^2 - 8x + 3$

4. $-1, -1$ and 3 are zeros of the given polynomial .

Exercise - 3.4

1. (i) Quotient $= x - 3$ and remainder $= 7x - 9$

   (ii) Quotient $= x^2 + x - 3$ and remainder $= 8$

   (iii) Quotient $= -x^2 - 2$ and remainder $= -5x + 10$

2. (i) Yes (ii) Yes (iii) No

3. $-1, -1$

4. $g(x) = x^2 - x + 1$

5. (i) $p(x) = 2x^2 - 2x + 14$, $g(x) = 2$, $q(x) = x^2 - x + 7$, $r(x) = 0$

   (ii) $p(x) = x^3 + x^2 + x + 1$, $g(x) = x^2 - 1$, $q(x) = x + 1$, $r(x) = 2x + 2$

   (iii) $p(x) = x^3 + 2x^2 - x + 2$, $g(x) = x^2 - 1$, $q(x) = x + 2$, $r(x) = 4$

Exercise - 4.1

1. (a) Intersect at a point

   (b) Coincident

   (c) Parallel

2. (a) Consistent (b) Inconsistent (c) Consistent

   (d) Consistent (e) Consistent (f) Inconsistent

   (g) Inconsistent (h) Consistent (i) Inconsistent

3. Number of pants = 1; Number of shirts = 0

4. Number of Girls = 7; Number of boys = 4

5. Cost of pencil = 23; Cost of pen = 25

6. Length = 20 m; Width = 16 m

7. (i) $3x + 2y - 7 = 0$

(ii) $3x + 3y - 12 = 0$

(iii) $4x + 6y - 16 = 0$

8. Length = 56 units; Breadth = 100 units

9. Number of students = 16; Number of benches = 5

Exercise - 4.2

1. Income of I[st] person = ₹ 18000; Income of II[nd] person = ₹ 14000

2. 42 and 24

3. Angles are 54° and 36°

4. (i) Fixed charge = ₹ 40; Charge per km = ₹ 18 (ii) ₹ 490

5. $\frac{7}{9}$

6. 60 km/h; 40 km/h.

7. 61° and 119°

8. 659 and 723

9. 40 ml and 60 ml

10. ₹ 7200 and ₹ 4800

Exercise - 4.3

1. (i) (4, 5) (ii) $\left(\frac{-1}{2}, \frac{1}{4}\right)$ (iii) (4, 9)

(iv) (1, 2) (v) (3, 2) (vi) $\left(\frac{1}{2}, \frac{1}{3}\right)$

(vii) (3, 2) (viii) (1, 1)

2. (i) Speed of boat = 8 km/h; Speed of streem = 3 km/h

(ii) Speed of train = 60 km/h; Speed of car = 80 km/h

(iii) Number of days by man = 18; Number of days by woman = 36

Exercise - 5.1

1. (i) Yes (ii) Yes (iii) No

(iv) Yes (v) Yes (vi) No

(vii) No (viii) Yes

2. (i) $2x^2 + x - 528 = 8$ (x = Breadth)
   (ii) $x^2 + x - 306 = 0$ (x = Smaller integer)
   (iii) $x^2 + 32x - 273 = 0$ (x = Rohan's Age)
   (iv) $x^2 - 8x + 1280 = 0$ (x = Speed of the train)

Exercise - 5.2

1. (i) $-2; 5$ (ii) $-2; \frac{3}{2}$ (iii) $-\sqrt{2}; \frac{-5}{\sqrt{2}}$
   (iv) $\frac{1}{4}; \frac{1}{4}$ (v) $\frac{1}{10}; \frac{1}{10}$ (vi) $-6; 2$
   (vii) $1, \frac{2}{3}$ (viii) $-1; 3$ (ix) $7, \frac{8}{3}$
2. 13, 14
3. 17, 18; -17, -18
4. 5 cm, 12 cm
5. Number of articles = 6; Cost of each article = 15
6. 4 m; 10 m
7. Base = 12 cm; Altitide = 8 cm
8. 15 km, 20 km
9. 20 or 40
10. 9 kmph

Exercise - 5.3

1. (i) $\frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4}$ (ii) $\frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$
   (iii) $\frac{7+\sqrt{-71}}{10}, \frac{7-\sqrt{71}}{10}$ (iv) $-1, -5$
3. (i) $\frac{3-\sqrt{13}}{2}, \frac{3+\sqrt{13}}{2}$ (ii) 1, 2
4. 7 years
5. Maths = 12, Einglish = 18 (or) Maths = 13, English = 17
6. 120 m; 90 m

7. 18, 12; – 18, – 12
8. 40 kmph
9. 15 hours, 25 hours
10. Speed of the passenger train = 33 kmph
    Speed of the express train = 44 kmph
11. 18 m; 12 m
12. 6 seconds
13. 13 sides; No

Exercise - 5.4

1. (i) Real roots do not exist
   (ii) Equal roots; $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$
   (iii) Distinct roots; $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$
2. (i) $k = \pm 2\sqrt{6}$ (ii) $k = 6$
3. Yes; 40 m; 20 m
4. No
5. Yes; 20 m; 20 m
8. $\frac{3}{7}$

Exercise - 6.1

1. (i) AP (ii) Not AP (iii) AP (iv) Not AP
2. (i) 10, 20, 30, 40 (ii) – 2, – 2, – 2, – 2
   (iii) 4, 1, – 2, – 5 (iv) $-1, -\frac{1}{2}, 0, \frac{1}{2}$
   (v) – 1.25, – 1.5, – 1.75, – 2
3. (i) $a_1 = 3$; $d = -2$ (ii) $a_1 = -5$; $d = 4$
   (iii) $a_1 = \frac{1}{3}$; $d = \frac{4}{3}$ (iv) $a_1 = 0.6$; $d = 1.1$
4. (i) Not AP

(ii)	AP, next three terms = 4, $\frac{9}{2}$, 5
(iii)	AP, next three terms = – 9.2, – 11.2, – 13.2
(iv)	AP, next three terms = 6, 10, 14
(v)	AP, next three terms = $3 + 4\sqrt{2}$, $3 + 5\sqrt{2}$, $3 + 6\sqrt{2}$
(vi)	Not AP
(vii)	AP, next three terms = – 16, – 20, – 24
(viii)	AP, next three terms = $\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}$
(ix)	Not AP
(x)	AP, next three term = $5a$, $6a$, $7a$
(xi)	Not AP
(xii)	AP, next three terms = $\sqrt{50}$, $\sqrt{72}$, $\sqrt{98}$
(xiii)	Not AP

Exercise - 6.2

1.	(i)	$a_8 = 28$	(ii)	$d = 2$	(iii)	$a = 46$		
	(iv)	$n = 10$	(v)	$a_n = 3.5$				
2.	(i)	– 84	(ii)	22				
3.	(i)	$a_2 = 14$						
	(ii)	$a_1 = 18$;	$a_3 = 8$					
	(iii)	$a_2 = \frac{13}{2}$;	$a_3 = 8$					
	(iv)	$a_2 = -2$;	$a_3 = 0$;	$a_4 = 2$;	$a_5 = 4$			
	(v)	$a_1 = 53$;	$a_3 = 23$;	$a_4 = 8$;	$a_5 = -7$			
4.	16th term							
5.	(i)	34	(ii)	27				
6.	No	7.	178	8.	5	9.	1	

10.	100	11.	128	12.	60	13.	13
14.	AP = 4, 10, 16, ...			15.	158		
16.	– 13, – 8, – 3			17.	11	18.	13

Exercise - 6.3

1. (i) 245 (ii) – 180 (iii) 555 (iv) $\frac{33}{20} = 1\frac{13}{20}$
2. (i) $\frac{2093}{2} = 1046\frac{1}{2}$ (ii) 286 (iii) – 8930
3. (i) 440 (ii) $d = \frac{7}{3},\quad S_{13} = 273$

   (iii) $a = 4, S_{12} = 246$ (iv) $d = 1,\ a_{10} = 22$

   (v) $x = 5;\quad a_5 = 37$ (vi) $x = 7;\quad a = -8$

   (vii) $a = 4$
4. $x = 38;$ $S_{38} = 6973$
5. 5610
6. x^2
7. (i) 525 (ii) – 465
8. $S_1 = 3;$ $S_2 = 4;$ $a_2 = 1;$ $a_3 = -1;$ $a_{10} = -15$

   $a_n = 5 - 2x$
9. 4920
10. 160, 140, 120, 100, 80, 60, 40
11. 234
12. 143
13. 16
14. 370

Exercise - 6.4

1. (i) No (ii) No (iii) Yes
2. (i) 4, 12, 36, ... (ii) $\sqrt{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{25}, ...$

   (iii) 81, – 27, 9, ... (iv) $\frac{1}{64}, \frac{1}{32}, \frac{1}{16}, ...$
3. (i) Yes; 32, 64, 128 (ii) Yes, $\frac{-1}{24}, \frac{1}{48}, \frac{-1}{96}$

(iii) No (iv) No (v) No

(vi) Yes; $-81, 243, -729$ (vii) Yes; $\frac{1}{x^2}, \frac{1}{x^3}, \frac{1}{x^4}, \ldots\ldots$

(viii) Yes; $-16,\ 32\sqrt{2},\ -128$ (ix) Yes; 0.0004, 0.00004, 0.000004

4. -4

Exercise - 6.5

1. (i) $r_a = \frac{1}{2}$; $a_n = 3\left(\frac{1}{2}\right)^{n-1}$

   (ii) $r = -3$; $a_n = 2(-3)^{n-1}$

   (iii) r = 3; $a_n = 3(3)^{n-1}$

   (iv) $r = \frac{2}{5}$; $a_n = 5\left(\frac{2}{5}\right)^{n-1}$

2. $a_{10} = 5^{10}$; $a_n = 5^n$

3. (i) $\frac{1}{3^4}$ (ii) $\frac{-4}{3^4}$

4. (i) 5 (ii) 12 (iii) 7

5. 2^{12} 6. $\frac{9}{4}, \frac{3}{2}, 1, \ldots.$ 7. 5

Exercise - 7.1

1. (i) $2\sqrt{2}$ (ii) $4\sqrt{2}$ (iii) $5\sqrt{2}$ (iv) $2\sqrt{a^2+b^2}$
2. 39
3. Points are not collinear
9. (i) Square (ii) Trepezium (iii) Parallelogram
10. (7, 0) 11. 7 or -5
12. 3 or -9 13. $2\sqrt{5}$ units

Exercise - 7.2

1. (1, 3)
2. $\left(2, \frac{-5}{3}\right)$ and $\left(0, \frac{-7}{3}\right)$
3. 2 : 7
4. $x = 6$; $y = 3$
5. (3, – 10)
6. $\left(\frac{-2}{7}, \frac{-20}{7}\right)$
7. $\left(-3, \frac{3}{2}\right), (-2, 3), \left(-1, \frac{9}{2}\right)$
8. $\left(1, \frac{13}{2}\right)$
9. 24 sq. units
10. $\left(\frac{5a-b}{5}, \frac{5a+b}{5}\right)$
11. (i) $\left(\frac{2}{3}, 2\right)$ (ii) $\left(\frac{10}{3}, \frac{-5}{3}\right)$ (iii) $\left(\frac{-2}{3}, \frac{5}{3}\right)$

Exercise - 7.3

1. (i) $2\frac{1}{2}$ sq. units (ii) 32 sq. units (iii) 3 sq. units
2. (i) K = 4 (ii) K = 3 (iii) $K = \frac{7}{3}$
3. 1 sq. unit ; 1 : 4
4. $\frac{33}{2}$ sq. units
5. $1500\sqrt{3}$ sq. units

Exercise - 7.4

1. (i) 6 (ii) $\sqrt{3}$ (iii) $\frac{4b}{a}$ (iv) $\frac{-a}{b}$
   (v) $\frac{-25}{19}$ (vi) 0 (vii) $\frac{1}{7}$ (viii) – 1

Exercise - 8.1

4. $x = 5$ cm and $y = 2\frac{13}{16}$ cm or 2.8125 cm

Exercise - 8.2

1.	(ii)	DE = 2.8 cm			
2.	8 cm	3.	1.6 m	7.	16 m

Exercise - 8.3

3.	1:4	4.	$\frac{\sqrt{2}-1}{1}$	6.	96 cm^2	8.	3.5 cm

Exercise - 8.4

8.	$6\sqrt{7}$ m	9.	13 m	12.	1: 2

Exercise - 9.1

1.	(i)	One	(ii)	Secant of a circle	(iii)	Two
	(iv)	Point of contact	(v)	Infinite		
2.	PQ = 13 cm		4.	$\sqrt{306}$ cm		

Exercise - 9.2

1.	(i)	d	(ii)	a	(iii)	b	(iv)	a	(v)	c
2.	8 cm	4.	AB = 15 cm, AC = 13 cm							
5.	8 cm each	6.	$2\sqrt{5}$ cm			9.	Two			

Exercise - 9.3

1.	(i) 28.5 cm^2	(ii)	285.5 cm^2		
2.	88.368 cm^2	3.	1254.96 cm^2	4.	57 cm^2
5.	10.5 cm^2	6.	9.625 cm^2	7.	102.67 cm^2
8.	57 cm^2				

Exercise - 10.1

1.	5500 cm^2	2.	124800 cm^2 (12.48 m^2)	3.	264 c.c.
4.	1 : 2	5.	4772	7.	29645 cm^3
8.	188.57 m^2	9.	37 cm		

Exercise - 10.2

1. 103.71 cm^2 2. 1156.57 cm^2 3. 220 mm^2
4. 160 cm^2 5. ₹ 765.6 6. $4:4:\sqrt{5}$
7. $a^2\left(5+\frac{\pi}{2}\right)$ sq. units 8. 374 cm^2

Exercise - 10.3

1. 693 kg 2. Height of cone = 22.05 cm; Surface area of toy = 793 cm^2
3. 88.83 cm^3 4. 616 cm^3 5. 309.57 cm^3
6. 150 7. 523.9 cm^3

Exercise - 10.4

1. 2.74 cm 2. 12 cm 3. 0.714 m (71.4 cm)
4. 5 m 5. 10 6. 57
7. 100 8. 224

Exercise - 11.1

1. $\sin A = \frac{15}{17}$; $\cos A = \frac{18}{17}$; $\tan A = \frac{15}{8}$
2. $\frac{527}{168}$ 3. $\cos\ \theta = \frac{49}{25}$; $\tan\theta = \frac{24}{49}$
4. $\sin A = \frac{5}{13}$; $\tan A = \frac{5}{12}$
5. $\sin A = \frac{4}{5}$; $\cos A = \frac{3}{5}$
7. (i) $\frac{47}{62}$ (ii) $\frac{\sqrt{111}+8}{7}$
8. (i) 1 (ii) 0

Exercise - 11.2

1. (i) $\sqrt{2}$ (ii) $\frac{\sqrt{3}}{4\sqrt{2}}$ (iii) 1

	(iv)	$\frac{-1}{3}$		(v)	-1				
2.	(i)	c		(ii)	d		(iii)	b	
3.	1			4.	Yes				
5.	QR = $6\sqrt{3}$ cm;			PR = 12 cm					
6.	$\angle$YXZ = 60º;			$\angle$YXZ = 30º			7.	It is true	

Exercise - 11.3

1.	(i)	1	(ii)	0	(iii)	0	
	(iv)	1	(v)	1			
3.	A = 24º		6.	cos 15º + sin 25º			

Exercise - 11.4

1.	(i)	2	(ii)	2	(iii)	1	
6.	1		8.	1	9.	$\frac{1}{p}$	

Exercise - 12.1

1.	15 m	2.	$6\sqrt{3}$ m	3.	4 m		
4.	60º	5.	11.55 m	6.	$4\sqrt{3}$ m		
7.	4.1568 m	8.	300 m	9.	15 m	10.	12.99 cm^2

Exercise - 12.2

1. Height of the tower = $5\sqrt{3}$ m; Width of the road = 5 m

2.	32.908 m	3.	1.464 m	4.	19.124 m				
5.	7.608 m	6.	10 m	7.	51.96 feets;	30 feets		8.	6 m
9.	200 m/sec.	10.	24 m						

Exercise - 13.1

1.	(i)	1	(ii)	0, Impossible event		(iii)	1, Sure event	
	(iv)	1	(v)	0, 1				
2.	(i)	No	(ii)	No	(iii)	Yes	(iv)	Yes

3.	0.95		4.	(i)	0		(ii)	1	
5.	0.008		6.	(i)	$\frac{1}{2}$	(ii)	$\frac{1}{2}$	(iii)	$\frac{1}{2}$

Exercise - 13.2

1.	(i)	$\frac{3}{8}$	(ii)	$\frac{5}{8}$				
2.	(i)	$\frac{5}{17}$	(ii)	$\frac{4}{17}$	(iii)	$\frac{13}{17}$		
3.	(i)	$\frac{5}{9}$	(ii)	$\frac{17}{18}$				
4.	$\frac{5}{13}$		5.	0.35				
6.	(i)	$\frac{1}{8}$	(ii)	$\frac{1}{2}$	(iii)	$\frac{3}{4}$	(iv)	1
7.	(i)	$\frac{1}{26}$	(ii)	$\frac{1}{13}$	(iii)	$\frac{1}{26}$		
	(iv)	$\frac{1}{52}$	(v)	$\frac{1}{13}$	(vi)	$\frac{1}{52}$		
8.	$\frac{3}{10}$		9.	$\frac{4}{15}$				
10.	(i)	$\frac{1}{5}$	(ii)	a.	$\frac{1}{4}$		b.	$\frac{1}{4}$
11.	$\frac{11}{12}$			12.	(i)	$\frac{1}{5}$	(ii)	$\frac{15}{19}$
13.	(i)	$\frac{9}{10}$	(ii)	$\frac{1}{10}$	(iii)	$\frac{1}{5}$		
14.	$\frac{11}{21}$		15.	(i)	$\frac{31}{36}$	(ii)	$\frac{5}{36}$	

16.

Sum on 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

17. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$

Exercise - 14.1

1. 8.1 plants. We have used direct method because numerical values of x_i and f_i are small.
2. ₹ 145.20
3. $f = 20$
4. 75.9
5. 22.31
6. ₹ 211
7. 0.099 ppm
8. 49 days
9. 69.43%

Exercise - 14.2

1. Mode = 36.8 years, Mean = 35.37 years, Maximum number of patients admitted in the hospital are of the age 36.8 years (approx.), while on an average the age of a patient admitted to the hospital is 35.37 years.
2. 65.625 hours
3. Modal monthly expenditure = ₹ 1847.83, Mean monthly expenditure = ₹ 2662.5.
4. Mode : 30.6, Mean = 29.2. Most states/U.T. have a student teacher ratio of 30.6 and on an average, this ratio is 29.2.
5. Mode = 4608.7 runs.
6. Mode = 44.7 cars

Exercise - 14.3

1. Median = 137 units, Mean = 137.05 units, Mode = 135.76 units.

   The three measures are approximately the same in this case.
2. $x = 8$, $y = 7$

3. Median age = 35.76 years

4. Median length = 146.75 mm

5. Median life = 3406.98 hours

6. Median = 8.05, Mean = 8.32, Modal size = 7.88

7. Median weight = 56.67 kg

Exercise - 14.4

1.

Daily income (in ₹)	Cumulative frequency
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

Draw ogive by plotting the points : (120, 12), (140, 26), (160, 34), (180, 40) and (200, 50)

2. Draw the ogive by plotting the points : (38, 0), (40, 3), (42, 5), (44, 9), (46, 14), (48, 28), (50, 32) and (52, 35). Here $\frac{n}{2}$ =17.5. Locate the point on the ogive whose ordinate is 17.5. The x-coordinate of this point will be the median.

3.

Production yield (kg/ha)	Cumulative frequency
More than or equal to 50	100
More than or equal to 55	98
More than or equal to 60	90
More than or equal to 65	78
More than or equal to 70	54
More than or equal to 75	16

Now, draw the ogive by plotting the points : (50, 100), (55, 98), (60, 90), (65, 78), (70, 54) and (75, 16).

Note to the Teachers

Dear teachers,

The Government of Andhra Pradesh has decided to revise the curriculum of all the subjects based on Andhra Pradesh State Curriculum Framework (APSCF-2011). The framework emphasises that all children must learn and the mathematics learnt at school must be linked to the life and experience of them. The NCF-2005, the position paper on Mahtematics of the NCERT and the Govt. of Andhra Pradesh emphasise, building understanding and developing the capability, exploration and inclination to mathematize experiences. This would become more possible at the secondary level. We have consolidated the basic framework of mathematics in class-IX and now we are at level of completion of secondary level of mathematics. In previous classes, we have encouraged the students for greater abstraction and formal mathematical formulation. We made them to deal with proofs and use mathematical language. It is important to recognise that as we go forward the language- in which mathematical arguments and statements- are presented would become even more symbolic and terse. It is therefore important in this class to help children become comfortable and competent in using mathematical ideas. In class-X, we will make all such idea at level of total abstraction.

It would be important to consider all the syllabi from class-VI to X while looking at teaching class X. The nature and extent of abstraction and use of mathematical language is gradually increasing. The program here would also become axiomatic and children must be slowly empowered to deal with that. One of the major difficulties children have in moving forward and learning secondary mathematics is their inability to deal with the axiomatic nature and language of symbols. They need to have an apportunity to learn and develop these perspectives by engaging with, as a team. Peer support in overcoming the difficulties is critical and it would be important to put them in a group to think, discuss and solve problems. When children will learn such things in class-X, it will be helpful for them in future mathematical learning also.

The syllabus is based on the structural approach, laying emphasis on the discovery and understanding of basic mathematical concepts and generalisations. The approach is to encourage participation and discussion in classroom activities.

The syllabus in textbook of Class-X Mathematics has been divided broadly into six areas Number System, Arithmetic, Algebra, Geometry, Trigonometry, Statistics and Coordinate Geometry. Teaching of the topics related to these areas will develop the skills such as problem solving, logical thinking, mathematical communication, representing data in various forms, using mathematics as one of the disciplines of study and also in daily life situations.

This text book attempts to enhance this endeavor by giving higher priority and space to opportunities for contemplations. There is a scope for discussion in small groups and activities required for hands on experience in the form of 'Do this' and 'Try this'. Teacher's support is needed in setting the situations in the classroom and also for development of interest in new book.

Exercises in 'Do This' and 'Try This' are given extensively after completion of each concept. The problems which are given under 'Do This' are based on the concept taught and 'Try This' problems are intended to test the skills of generalization of facts, ensuring correctness and questioning. 'Think, Discuss and Write' has given to understand the new concept between students in their own words.

Entire syllabus in class-X Mathematics is divided into 14 chapters with an appendix, so that a child can go through the content well in bit wise to consolidate the logic and enjoy the learning of mathematics. Colourful pictures, diagrams, readable font size will certainly help the children to adopt the contents and care this book as theirs.

Chapter-1 : Real number, we are discussing about the exploration of real numbers in which the brief account of fundamental theorem of arithmetic, rational numbers their decimal expansion and non-terminating recurring rational numbers has given. Here we are giving some more about the irrational numbers. In this chapter, first time we are introducing logarithms in this we are discussing about basic laws of logarithms and their application.

Chapter-2 : Sets, this is entirely a new chapter at the level of secondary students. In old syllabus it was there but here we are introducing it in class X. This chapter is introduced with wide variety of examples which are dealing about the definition of sets, types of set, Venn diagrams, operations of sets, differences between sets. In this chapter we dealt about how to develop a common understanding of sets. How can you make set of any objects?

Chapter-3 : Polynomials, we are discussing about the fact "what are polynomials?" and degree and value of polynomials come under it. This time we look at the graphical representation of linear equations and quadratic equations. Here we are taking care of zeros and coefficients of a polynomial & their relationship. We also start with cubic polynomials and division algorithm of polynomials.

Chapter-4 : Pair of linear equations in two variables, we start the scenario with discussing about finding of unknown quantities and use of two equations together. Solution of pair of linear equations in two variables with the help of graphical and algebraic methods has done. Here we have illustrated so many examples to understand the relation between coefficients and nature of system of equations. Reduction of equation to two variable linear equation has done here.

The problem is framed in such a way to emphasis the correlation between various chapters within the mathematics and other subjects of daily life of human being. This chapter links the ability of finding unknown with every day experience.

Chapter-5 : Quadratic equations, states the meaning of quadratic equation and solution of quadratic equation with the factorizations completion of squares. Nature of roots is defined here with the use of parabola.

Chapter-6 : Progressions, we have introduced this chapter first time on secondary level. In this chapter use are taking about arithmetic progressions and geometric progressions. How the terms progressing arithmetically and geometrically in progressions discussed. The number of terms, nth terms, sum of terms are stated in this chapter.

Chapter-7 : Coordinate geometry, deals with finding the distance between two points on cartesion plane, section formula, centroid of a triangle and tsisectional points of a line. In this, we are also talking about area of the triangle on plane and finding it with the use of 'Heron's formula'. The slope on straight line is also introduced here.

We are keeping three chapters (8, 9, 10) in X mathematics book and all of them are having emphasis on learning geometry using reasoning, intuitive understanding and insightful personal experience of meanings. If helps in communicating and solving problems and obtaining new relations among various plane figures. In chapter 9 Tangents and Secants to a circle, we have introduced the new terms caked tangent and secant with their properties. We also discussed about the segment and area of that which is formed by secant. Menstruations are presented in combination of solids and finding of their volume and area.

We are keeping two new chapters (11 & 12) at second level for the first time. The applications of triangles are used with giving relation with the hypotenuse, perpendicular and base. These chapters are the introduction of trigonometry which have very big role in high level studies and also in determination of so many measurements. Applications of trigonometry are also given with brief idea of using triangle.

Chapter-13 : Probability, is little higher level chapter than the last chapter which we have introduced in class IX. Here we are taking about different terms of probability by using some daily life situations.

Chapter-14 : Statistics, deals with importance of statistics, collection of statistical grouped data, illustrative examples for finding mean, median and mode of given grouped data with different methods. The ogives are also illustrated here again. In appendix, mathematical modeling is given there with an idea about the models and their modeling methods.

The success of any course depends not only on the syllabus but also on the teachers and the teaching methods they employ. It is hoped that all teachers are concerned with the improving of mathematics education and they will extend their full co operation in this endeavour.

The production of good text books does not ensure the quality of education, unless the teachers transact the curriculum in a way as it is discussed in the text book. The involvement and participation of learner in doing the activities and problems with an understanding is ensured.

Students should be made to digest the concepts given in “What we have discussed” completely. Teachers may prepare their own problems related to the concepts besides solving the problems given in the exercises. So therefore it is hopped that the teachers will bring a paradigm shift in the classroom process from mere solving the problems in the exercises routinely to the conceptual understanding, solving of problems with ingenity.

“Good luck for happy teaching”

Syllabus

I. Number System (23 periods)

(i) **Real numbers (15 periods)**

- More about rational and irrational numbers.
- Fundamental theorem of Arithmetic - Statements.
- Proofs of results - irrationality of $\sqrt{2}, \sqrt{3}$ etc. and Decimal expansions of rational numbers in terms of terminating / non-terminating recurring decimals and vice versa.
- Properties of real numbers (after reviewing lookdone earlier and after illustrating and motivating through examples)
- Introduction of logarithms
- Conversion of a number in exponential form to a logarithmic form
- Properties of logarithms $\log_a a = 1$; $\log_a 1 = 0$
- Laws of logarithms

  $\log xy = \log x + \log y$; $\log \frac{x}{y} = \log x - \log y$; $\log x^n = n \log x$
- Standard base of logarithm and use of logarithms in daily life situations (not meant for examination)

(ii) **Sets (8 periods)**

- Sets and their representations
- Empty set, Finite and infinite sets, universal set
- Equal sets, subsets, subsets of set of real numbers (especially intervals and notations)
- Venn diagrams and cardinality of sets
- Basic set operations - union and intersection of sets
- Disjoint sets, difference of sets

II. Algebra (46 periods)

(i) **Polynomials (8 periods)**

- Zeroes of a polynomial
- Geometrical meaning of zeroes of linear, quadratic and cubic polynomials using graphs.
- Relationship between zeroes and coefficients of a polynomial.
- Simple problems on division algorithm for polynomials with integral coefficients

(ii) **Pair of Linear Equations in Two Variables (15 periods)**

- Forming a linear equation in two variables through illustrated examples.
- Graphical representation of a pair of linear equations of different possibilities of solutions / in consistency.
- Algebraic conditions for number of solutions
- Solution of pair of linear equations in two variables algebraically - by Substitution, by elimination.
- Simple and daily life problems on equations reducible to linear equations.

(iii) **Quadratic Equations (12 periods)**

- Standard form of a quadratic equation $ax^2 + bx + c = 0, a \neq 0$.
- Solutions of quadratic equations (only real roots) by factorisation and by completing the square i.e. by using quadratic formula.

- Relationship between discribmenant and nature of roots.
- Problems related to day-to-day activities.

(iv) **Progressions (11 periods)**

- Definition of Arithmetic progression (A.P)
- Finding nth term and sum of first nterms of A.P.
- Geometric progression (G.P.)
- Find nth term of G.P.

III. Geometry (33 periods)

(i) **Similar Triangles (18 periods)**

- Similarly figures difference between congruency and similarity.
- Properties of similar triangles.
- (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other twosides are divided in the same ratio.
- (Motivate) If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.
- (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar (AAA).
- (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar (SSS).
- (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar (SAS).
- (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
- (Motivate) If a perpendicular is drawn from the vertex of the right angle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.
- (Prove) In a right triangle, the square on the hypothenuse is equal to the sum of the squares on the other two sides.
- (Prove) In a triangle, if the square on one side is equal to sum of the squares on the other sides, the angles opposite to the first side is a right triangle.
- (Construction) Division of a line segment using Basic proportionality Theorem.
- (Construction) A triangle similar to a given triangle as per given scale factor.

(ii) **Tangents and secants to a circle (15 periods)**

- Difference between tangent and secant to a circle
- Tangents to a circle motivated by chords drawn from points coming closer and closer to the point
- (Prove) The tangent at any point on a circle is perpendicular to the radius through the point contact.
- (Prove) The lengths of tangents drawn from an external point to a circle are equal.
- (Construction) A tangent to a circle through a point given on it.
- (Construction) Pair of tangents to a circle from an external point.
- Segment of a circle made by the secant.
- Finding the area of minor/major segment of a circle.

IV. Coordinate Geometry

- Review the concepts of coordinate geometry by the graphs of linear equations.
- Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$

$$\overline{PQ} = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$$

- Section formula (internal division of a line segment in the ratio m : n).
- Area of triangle on coordinate plane.
- Slope of a line joining two points

V. Trigonometry (23 periods)

(i) **Trigonometry (15 periods)**

- Trigonometric ratios of an actute angle of a right angled triangle i.e. sine, cosine, tangent, cosecant, secant and cotangent.
- Values of trigonometric ratios of 30o, 45° and 60° (with proofs).
- Relationship between the ratios and trigonometric ratios for complementary angles.
- Trigonometric Identities.
  (i) $\sin^2 A + \cos^2 A = 1$, (ii) $1 + \tan^2 A = \sec^2 A$, (iii) $\cot^2 A + 1 = \text{cosec}^2 A$

(ii) **Applications of Trigonometry (8 periods)**

- Angle of elevation, angle of depression.
- Simple and daily life problems on heights and distomces
- Problems involving not more than two right triangles and angles of elevation / depression confined to 30°, 45° and 60°.

VI. Mensuration (10 periods)

(i) **Surface Areas and Volumes**

- Problems on finding surface area and volumes of combinations of any two of the following i.e. cubes, cubiods, right cicular cylinders, cones spheres and hemispheres.
- Problems involving converting one type of mettalic solid into anothers and finding volumes and other mixed problems involving not more than two different solids.

VII. Data Handling (25 periods)

(i) **Statistics**

- Revision of mean, median and mode of ungrouped (frequency distribution) data.
- Understanding the concept of Arithmetic mean, median and mode for grouped (classified) data.
- Simple problems on finding mean, median and mode for grouped/ungrouped data with different methods.
- Usage and different values of central tendencies through ogives.

(ii) **Probability (10 periods)**

- Revision of concept and definition of probability.
- Simple problems (day to day life situation) on single events using set notation.
- Concept of complimentary events.

Appendix

Mathematical Modeling (8 periods)

- Concept of Mathematical modelling
- Discussion of broad stages of modelling-real life situations (Simple interest, Fair installments payments etc. ....)

Academic Standards - High School

Academic Standards : Academic standards are clear statements about what students must know and be able to do. The following are categories on the basis of which we lay down academic standards.

	Areas of Mathematics	Content
1.	**Problem Solving**	Using concepts and procedures to solve mathematical problems like following: a. **Kinds of problems :** Problems can take various forms- puzzles, word problems, pictorial problems, procedural problems, reading data, tables, graphs etc. • Reads problems. • Identifies all pieces of information/data. • Separates relevant pieces of information. • Understanding what concept is involved. • Recalling of (synthesis of) concerned procedures, formulae etc. • Selection of procedure. • Solving the problem. • Verification of answers of raiders, problem based theorems. b. **Complexity :** The complexity of a problem is dependent on- • Making connections(as defined in the connections section). • Number of steps. • Number of operations. • Context unraveling. • Nature of procedures.
2.	**Reasoning Proof**	• Reasoning between various steps (involved invariably conjuncture). • Understanding and making mathematical generalizations and conjectures

		• Understands and justifies procedures • Examining logical arguments. • Understanding the notion of proof • Uses inductive and deductive logic • Testing mathematical conjectures
3.	**Communication**	• Writing and reading, expressing mathematical notations (verbal and symbolic forms) **Example** : $3+4=7$ $n_1+n_2= n_2+n_1$ Sum of angles in triangle = 180° • Creating mathematical expressions
4.	**Connections**	• Connecting concepts within a mathematical domain- for example relating adding to multiplication, parts of a whole to a ratio, to division. Patterns and symmetry, measurements and space. • Making connections with daily life. • Connecting mathematics to different subjects. • Connecting concepts of different mathematical domains like data handling and arithmetic or arithmetic and space. • Connecting concepts to multiple procedures.
5.	**Visualization & Representation**	• Interprets and reads data in a table, number line, pictograph, bar graph, 2-D figures, 3- D figures, pictures. • Making tables, number line, pictograph, bar graph, pictures. • Mathematical symbols and figures.