

1. There are 5 girls and 7 boys. A team of 3 boys and 2 girls is to be formed such that no two specific boys are in the same team. Number of way to do so

- (1) 400                      (2) 250                      (3) 200                      (4) 300

**Answer:** (4)

**Solution:**

$$\text{Total number of teams} = {}^7C_3 \times {}^5C_2 = 35 \times 10 = 350$$

Let A, B be the specific boys.

$$\text{Number of teams with these two boys in the same team} = {}^5C_1 \times {}^5C_2 = 5 \times 10 = 50$$

$$\therefore \text{Required number of ways} = 350 - 50 = 300$$

2. The equation  $x^2 + 2x + 2 = 0$  has roots  $\alpha$  and  $\beta$ . Then value of  $\alpha^{15} + \beta^{15}$  is

- (1) 512                      (2) 256                      (3) -512                      (4) -256

**Answer:** (4)

**Solution:**

$$(x+1)^2 = -1 \Rightarrow x+1 = \pm i$$

$$x = -1+i, -1-i$$

$$\alpha = -1+i, \beta = -1-i$$

$$\alpha = \sqrt{2}e^{i\left(\frac{3\pi}{4}\right)}, \beta = \sqrt{2}e^{i\left(\frac{-3\pi}{4}\right)}$$

$$\alpha^{15} + \beta^{15} = (\sqrt{2})^{15} \left( e^{i\frac{45\pi}{4}} + e^{i\left(\frac{-45\pi}{4}\right)} \right)$$

$$= 2^{\frac{15}{2}} \left( 2 \cos\left(\frac{45\pi}{4}\right) \right)$$

$$= 2^{\frac{15}{2}} \left( 2 \left( \frac{-1}{\sqrt{2}} \right) \right)$$

$$= -2^8 = -256$$

3.  $\int_0^\pi |\cos x|^3 dx$  is equal to

(1)  $\frac{4}{3}$

(2)  $\frac{2}{3}$

(3) 0

(4)  $\frac{-8}{3}$

**Answer:** (1)

**Solution:**

$$\int_0^\pi |\cos x|^3 dx = 2 \int_0^{\frac{\pi}{2}} \cos^3 x \cdot dx$$

$$= 2 \int_0^{\frac{\pi}{2}} \frac{3 \cos x + \cos 3x}{4}$$

$$= \frac{1}{2} \left( 3 \sin x + \frac{\sin 3x}{3} \right)_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2} \left( 3 - \frac{1}{3} \right) = 2 \times \frac{8}{3} = \frac{4}{3}$$

4. If  $x^2 \neq n\pi + 1, n \in N$  then  $\int x \sqrt{\frac{2 \sin(x^2 - 1) - \sin 2(x^2 - 1)}{2 \sin(x^2 - 1) + \sin 2(x^2 - 1)}} dx$  is equal to

(1)  $\ln \cos \left( \frac{x^2 - 1}{2} \right) + c$

(2)  $\frac{1}{2} \ln \cos \left( \frac{x^2 - 1}{2} \right) + c$

(3)  $\ln \sec \left( \frac{x^2 - 1}{2} \right) + c$

(4)  $\frac{1}{2} \ln \sec \left( \frac{x^2 - 1}{2} \right) + c$

**Answer:** (3)

**Solution:**

$$\text{Let } x^2 - 1 = t \Rightarrow x dx = \frac{dt}{2}$$

$$\begin{aligned} \int \sqrt{\frac{2 \sin t - \sin 2t}{2 \sin t + \sin 2t}} \cdot \frac{dt}{2} &= \frac{1}{2} \int \sqrt{\frac{2 \sin t(1 - \cos t)}{2 \sin t(1 + \cos t)}} \cdot dt \\ &= \frac{1}{2} \int \tan \frac{t}{2} \cdot dt \\ &= \log \left| \sec \frac{t}{2} \right| + c \\ &= \log \left| \sec \left( \frac{x^2 - 1}{2} \right) \right| + c \end{aligned}$$

5. If  $\vec{a} = i - j, \vec{b} = i + j + k$  are two vectors and  $\vec{c}$  is another vector such that  $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$  and  $\vec{a} \cdot \vec{c} = 4$

then  $|\vec{c}|^2 =$

- (1) 9                      (2) 8                      (3)  $\frac{19}{2}$                       (4)  $\frac{17}{2}$

**Answer:** (3)

**Solution:**

$$\vec{a} \times \vec{c} + \vec{b} = \vec{0}$$

$$\vec{a} \times \vec{c} = -\vec{b}$$

$$\vec{a} \times (\vec{a} \times \vec{c}) = \vec{b} \times \vec{a}$$

$$(\vec{a} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{a})\vec{c} = \vec{b} \times \vec{a} \quad \dots (1)$$

$$\vec{a} \cdot \vec{c} = 4; \vec{a} \cdot \vec{a} = 2 \quad \dots (2)$$

$$\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \hat{i} + \hat{j} - 2\hat{k} \quad \dots (3)$$

From (1), (2), (3)

$$4\vec{a} - 2\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$$

$$-2\vec{c} = \hat{i} + \hat{j} - 2\hat{k} - 4(\hat{i} - \hat{j})$$

$$-2\vec{c} = -3\hat{i} + 5\hat{j} - 2\hat{k}$$

$$4|\vec{c}|^2 = 38 \Rightarrow |\vec{c}|^2 = \frac{19}{2}$$

6.  $f(x) = \begin{cases} 5 & ; \quad x \leq 1 \\ a+bx & ; \quad 1 < x < 3 \\ b+5x & ; \quad 3 \leq x < 5 \\ 30 & ; \quad x \geq 5 \end{cases}$  then

(1)  $f(x)$  is discontinuous  $\forall a \in R, b \in R$

(2)  $f(x)$  is continuous if  $a = 0$  &  $b = 5$

(3)  $f(x)$  is continuous if  $a = 5$  &  $b = 0$

(4)  $f(x)$  is continuous if  $a = -5$  &  $b = 10$

**Answer:** (1)

**Solution:**

$$f(1^-) = f(1^+)$$

$$a + b = 5 \quad \dots (1)$$

$$f(3^+) = f(3^-)$$

$$b + 15 = a + 3b \quad \dots (2)$$

$$f(5^+) = f(5^-)$$

$$b + 25 = 30 \quad \dots (3)$$

From (3),  $b = 5$

From (1),  $a = 0$  & from (2),  $a = 5$

$\therefore f(x)$  is discontinuous  $\forall a \in R, b \in R$

7. Average height & variance of 5 students in a class is 150 and 18 respectively. A new student whose height is 156 cm is added to the group. Find new variance.

- (1) 20                      (2) 22                      (3) 16                      (4) 14

**Answer:** (4)

**Solution:**

Let 5 students are  $x_1, x_2, x_3, x_4, x_5$

$$\text{Given } \bar{x} = \frac{\sum x_i}{5} = 150 \quad \Rightarrow \quad \sum_{i=1}^5 x_i = 750 \quad \dots (1)$$

$$\frac{\sum x_i^2}{5} - (\bar{x})^2 = 18 \Rightarrow \frac{\sum x_i^2}{5} - (150)^2 = 18$$

$$\Rightarrow \sum x_i^2 = (22500 + 18) \times 5 \Rightarrow \sum_{i=1}^5 x_i^2 = 112590 \quad \dots (2)$$

Height of new student = 156 (Let  $X_6$ )

Now  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 750 + 156$

$$\bar{x}_{new} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \frac{906}{6} = 151 \quad \dots (3)$$

$$\text{Variance (new)} = \frac{\sum x_i^2}{6} - (\bar{X})^2$$

$$= \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2}{6} - (151)^2$$

From equation (2) and (3)

$$\text{Var (new)} = \frac{112590 + (156)^2}{6} - (151)^2 = 22821 - 22801 = 20$$

8. a, b, c are in G.P,  $a + b + c = bx$ , then  $x$  can not be

- (1) 2                      (2) -2                      (3) 3                      (4) 4

**Answer:** (1)

**Solution:**

Let the terms of G.P be  $\frac{a}{r}, a, ar$

$$\therefore \frac{a}{r} + a + ar = ax$$

$$\Rightarrow x = r + \frac{1}{r} + 1$$

But  $r + \frac{1}{r} \geq 2$  or  $r + \frac{1}{r} \leq -2$  ( using A.M,G.M inequality )

$$\therefore x - 1 \geq 2 \text{ or } x - 1 \leq -2$$

$$\Rightarrow x \geq 3 \text{ or } x \leq -1$$

9.  $\left\{ \frac{2^{403}}{15} \right\} = \frac{k}{15}$  then find k. (where  $\{.\}$  denotes fractional part function)

- (1) 2                      (2) 8                      (3) 1                      (4) 4

**Answer:** (2)

**Solution:**

$$2^4 \equiv 1 \pmod{15}$$

$$2^{400} \equiv 1 \pmod{15}$$

$$2^{403} \equiv 8 \pmod{15}$$

$$\therefore \left\{ \frac{2^{403}}{15} \right\} = \frac{8}{15} \Rightarrow K = 15$$

10.  $\lim_{y \rightarrow 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4} =$

- (1)  $\frac{1}{4\sqrt{2}}$       (2)  $\frac{1}{2\sqrt{2}}$       (3)  $\frac{1}{2\sqrt{2}(1+\sqrt{2})}$       (4) does not exist

**Answer:** (1)

**Solution:** Rationalising numerator,

$$\begin{aligned} & \lim_{y \rightarrow 0} \frac{\sqrt{1+\sqrt{1+y^4}} - \sqrt{2}}{y^4} \times \frac{\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}}{\sqrt{1+\sqrt{1+y^4}} + \sqrt{2}} \\ &= \lim_{y \rightarrow 0} \frac{\sqrt{1+y^4} - 1}{y^4 (\sqrt{1+\sqrt{1+y^4}} + \sqrt{2})} \times \frac{\sqrt{1+y^4} + 1}{\sqrt{1+y^4} + 1} = \lim_{y \rightarrow 0} \frac{y^4}{y^4 (\sqrt{1+\sqrt{1+y^4}} + \sqrt{2})(\sqrt{1+y^4} + 1)} \\ &= \frac{1}{4\sqrt{2}} \end{aligned}$$

11. There is a parabola having axis as  $x$ -axis, vertex is at a distance of 2 unit from origin & focus is at  $(4, 0)$ .

Which of the following point does not lie on the parabola.

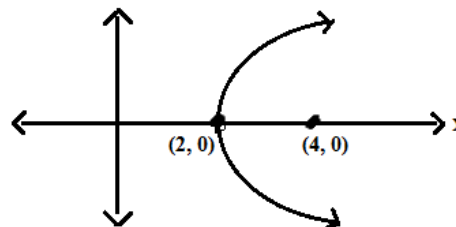
- (1)  $(6, 8)$       (2)  $(5, 2\sqrt{6})$       (3)  $(8, 4\sqrt{3})$       (4)  $(4, -4)$

**Answer:** (1)

**Solution:**

The equation of parabola is  $y^2 = 8(x - 2)$

$\therefore (6, 8)$  does not lie on this curve



12. Find sum of all possible values of  $\theta$  in the interval  $\left(-\frac{\pi}{2}, \pi\right)$  for which  $\frac{3+2i \sin \theta}{1-2i \sin \theta}$  is purely imaginary

(1)  $\frac{\pi}{3}$

(2)  $\pi$

(3)  $\frac{2\pi}{3}$

(4)  $\frac{\pi}{2}$

**Answer:** (3)

**Solution:**

$$z = \frac{3+2i \sin \theta}{1-2i \sin \theta} \times \frac{1+2i \sin \theta}{1+2i \sin \theta}$$

$$= \frac{(3-4 \sin^2 \theta) + i(8 \sin \theta)}{1+4 \sin^2 \theta}$$

For z to be purely imaginary,  $\text{Re}(z) = 0$

i.e.,  $\frac{3-4 \sin^2 \theta}{1+4 \sin^2 \theta} = 0$

$$\Rightarrow \sin^2 \theta = \frac{3}{4}$$

As  $\theta \in \left(-\frac{\pi}{2}, \pi\right) \Rightarrow \theta = \pm \frac{\pi}{3}, \frac{2\pi}{3}$

$\therefore$  sum of all values of  $\theta = \frac{2\pi}{3}$

13. Let  $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$  Find the value of  $A^{-50}$  at  $\theta = \frac{\pi}{12}$ .

(1)  $\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -1 & \frac{\sqrt{3}}{2} \end{bmatrix}$

(2)  $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -1 & \frac{\sqrt{3}}{2} \end{bmatrix}$

(3)  $\begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(4)  $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -1 \end{bmatrix}$

**Answer:** (2)



**Solution:**

$$A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

A is a rotation matrix

$$\therefore A^n = \begin{pmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{pmatrix} \Rightarrow A^{-50} = \begin{pmatrix} \cos 50\theta & \sin 50\theta \\ -\sin 50\theta & \cos 50\theta \end{pmatrix}$$

$$\therefore A^{-50} \text{ at } \theta = \frac{\pi}{12} \text{ is } \begin{pmatrix} \cos \frac{25\pi}{6} & \sin \frac{25\pi}{6} \\ -\sin \frac{25\pi}{6} & \cos \frac{25\pi}{6} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

14. If  $(A \oplus B) \wedge (\sim A \ominus B) = A \wedge B$  what should be proper symbol in place of  $\oplus$  and  $\ominus$  to hold the equation

- (1)  $\wedge$  and  $\vee$                       (2)  $\wedge$  and  $\wedge$                       (3)  $\vee$  and  $\vee$                       (4)  $\vee$  and  $\wedge$

**Answer:** (1)

**Solution:**

By inspection  $\oplus$  represents  $\wedge$  and  $\ominus$  represents  $\vee$

A	B	$A \wedge B$	$\sim A$	$\sim A \vee B$	$(A \wedge B) \wedge (\sim A \vee B)$
T	T	T	F	T	T
T	F	F	F	F	F
F	T	F	T	T	F
F	F	F	T	T	F



$$\therefore (A \wedge B) \wedge (\sim A \vee B) \equiv A \wedge B$$

15. If  $y(x)$  is solution of  $x \frac{dy}{dx} + 2y = x^2$ ,  $y(1) = 1$  then value of  $y\left(\frac{1}{2}\right) =$

(1)  $-\frac{49}{16}$

(2)  $\frac{49}{16}$

(3)  $\frac{45}{8}$

(4)  $-\frac{45}{8}$

**Answer:** (2)

**Solution:**

$$x \cdot \frac{dy}{dx} + 2y = x^2$$

$$\frac{dy}{dx} + \frac{2}{x}y = x$$

This is linear differential equation

$$I. F = e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$$

$$\therefore \text{solution is } x^2 y = \int x^3 dx$$

$$x^2 y = \frac{x^4}{4} + c$$

$$y(1) = 1 \Rightarrow c = \frac{3}{4}$$

$$\text{at } x = \frac{1}{2} \Rightarrow \frac{1}{4} y = \frac{1}{64} + \frac{3}{4}$$

$$\Rightarrow y = \frac{49}{16}$$

16. From a well shuffled deck of cards, 2 cards are drawn with replacement. If  $x$  represent numbers of times ace coming, then value of  $P(x=1) + P(x=2)$  is

(1)  $\frac{25}{169}$

(2)  $\frac{24}{169}$

(3)  $\frac{49}{169}$

(4)  $\frac{23}{169}$

**Answer:** (1)

**Solution:**

$$P(x=1) = {}^2C_1 \times \frac{4}{52} \times \frac{48}{52} = \frac{24}{169}$$

$$P(x=2) = {}^2C_2 \times \left(\frac{4}{52}\right)^2 = \frac{1}{169}$$

$$P(x=1) + P(x=2) = \frac{25}{169}$$

17. If eccentricity of the hyperbola  $\frac{x^2}{\cos^2 \theta} - \frac{y^2}{\sin^2 \theta} = 1$  is more than 2 when  $\theta \in \left(0, \frac{\pi}{2}\right)$  then values of length of latus rectum lies in the interval

- (1)  $(3, \infty)$                       (2)  $(1, 3/2)$                       (3)  $(2, 3)$                       (4)  $(-3, -2)$

**Answer:** (1)

**Solution:**

$$\text{For hyperbola, } e^2 = 1 + \frac{b^2}{a^2}$$

$$= 1 + \tan^2 \theta$$

$$= \sec^2 \theta$$

$$e > 2 \Rightarrow \sec \theta > 2$$

$$\Rightarrow \theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = 2 \tan \theta \sin \theta$$

$$= 2(> \sqrt{3}) \left(> \frac{\sqrt{3}}{2}\right)$$

> 3

18. If slant height of a right circular cone is 3 cm then the maximum volume of cone is

- (1)  $2\sqrt{3}\pi cm^3$       (2)  $4\sqrt{3}\pi cm^3$       (3)  $(2+\sqrt{3})\pi cm^3$       (4)  $(2-\sqrt{3})\pi cm^3$

**Answer:** (1)

**Solution:**

$$l = 3$$

$$\Rightarrow r^2 + h^2 = 9$$

$$\Rightarrow r^2 = 9 - h^2$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi h(9 - h^2)$$

$$V = 3\pi h - \frac{1}{3}\pi h^3$$

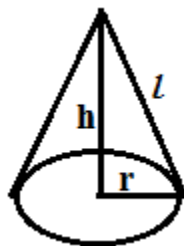
For maximum volume,  $\frac{dv}{dh} = 0$

$$\Rightarrow 3\pi - \pi h^2 = 0$$

$$\Rightarrow h = \sqrt{3}$$

$$\Rightarrow r^2 = 6$$

$$\therefore \text{Volume} = \frac{1}{3}\pi(6)(\sqrt{3}) = 2\sqrt{3}\pi cm^3$$



19. If  $\cos^{-1}\left(\frac{2}{3x}\right) + \cos^{-1}\left(\frac{3}{4x}\right) = \frac{\pi}{2}$ ,  $x > \frac{3}{4}$  then  $x =$

(1)  $\frac{\sqrt{145}}{11}$

(2)  $\frac{\sqrt{145}}{12}$

(3)  $\frac{\sqrt{146}}{10}$

(4)  $\frac{\sqrt{146}}{11}$

**Answer:** (2)

**Solution:**

$$\cos^{-1}\left(\frac{2}{3x}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{3}{4x}\right)$$

$$\cos^{-1}\left(\frac{2}{3x}\right) = \cos^{-1}\left(\frac{\sqrt{16x^2 - 9}}{4x}\right)$$

$$\frac{2}{3x} = \frac{\sqrt{16x^2 - 9}}{4x}$$

$$\sqrt{16x^2 - 9} = \frac{8}{3} \Rightarrow 16x^2 - 9 = \frac{64}{9}$$

$$\Rightarrow x^2 = \frac{145}{9 \times 16}$$

$$\Rightarrow x = \frac{\sqrt{145}}{12}$$

20. If  $px + qy + r = 0$  represent a family of straight lines such that  $3p + 2q + 4r = 0$  then

(1) All lines are parallel

(2) All lines are inconsistent

(3) All lines are concurrent at  $\left(\frac{3}{4}, \frac{1}{2}\right)$

(4) All lines are concurrent at  $(3, 2)$

**Answer:** (3)

**Solution:**

$$px + qy + r = 0 \quad \dots (1)$$

$$3p + 2q + 4r = 0$$

$$\frac{3p}{4} + \frac{q}{2} + r = 0 \quad \dots (2)$$

(1) & (2) are identical

$$\frac{x}{\frac{3}{4}} = \frac{y}{\frac{1}{2}} = 1$$

$$(x, y) = \left( \frac{3}{4}, \frac{1}{2} \right)$$

21. Consider the system of equations  $x + y + z = 1, 2x + 3y + 2z = 1, 2x + 3y + (a^2 - 1)z = a + 1$  then

(1) system has a unique solution for  $|a| = \sqrt{3}$       (2) system is inconsistent for  $|a| = \sqrt{3}$

(3) system is inconsistent for  $a = 4$       (4) system is inconsistent for  $a = 3$

**Answer:** (2)

**Solution:**

$$x + y + z = 1 \quad \dots (1)$$

$$2x + 3y + 2z = 1 \quad \dots (2)$$

$$2x + 3y + (a^2 - 1)z = a + 1 \quad \dots (3)$$

By observation, when  $a^2 - 1 = 2$

LHS of (2) & (3) are same but RHS different

$$\text{Hence } a^2 = 3 \Rightarrow |a| = \sqrt{3}$$

$\therefore$  For  $|a| = \sqrt{3}$ , the system is inconsistent.

22. The value of  $3(\cos \theta - \sin \theta)^4 + 6(\sin \theta + \cos \theta)^2 + 4\sin^6 \theta$  is, where  $\theta \in \left( \frac{\pi}{4}, \frac{\pi}{2} \right)$

(1)  $13 - 4\cos^4 \theta$

(2)  $13 - 4\cos^6 \theta$

(3)  $13 - 4\cos^6\theta + 2\sin^4\theta\cos^2\theta$

(4)  $13 - 4\cos^4\theta + 2\sin^4\theta\cos^2\theta$

Answer: (2)

Solution:

$$\begin{aligned} & 3(\cos^2\theta + \sin^2\theta - \sin 2\theta)^2 + 6(\sin^2\theta + \cos^2\theta + \sin 2\theta) + 4\sin^6\theta \\ &= 3(1 + \sin^2 2\theta - 2\sin 2\theta) + 6 + 6\sin 2\theta + 4\sin^6\theta \\ &= 9 + 3\sin^2 2\theta + 4\sin^6\theta \\ &= 9 + 3(4\sin^2\theta\cos^2\theta) + 4(1 - \cos^2\theta)^3 \\ &= 9 + 12\cos^2\theta\sin^2\theta + 4(1 - \cos^6\theta - 3\cos^2\theta\sin^2\theta) \\ &= 9 + 12\sin^2\theta\cos^2\theta + 4 - 4\cos^6\theta - 12\sin^2\theta\cos^2\theta \\ &= 13 - 4\cos^6\theta \end{aligned}$$

23. 3 circles of radii a, b, c ( $a < b < c$ ) touch each other externally and have x-axis as a common tangent then

(1) a, b, c are in A.P.

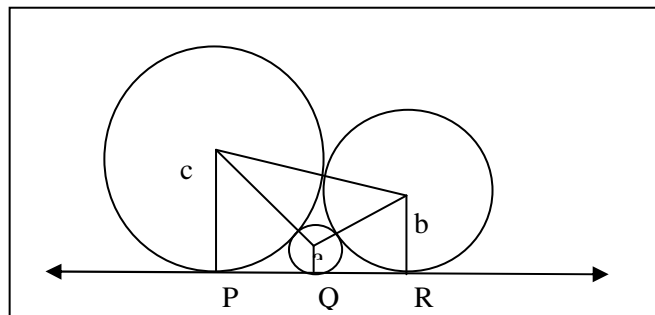
(2)  $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$

(3)  $\sqrt{a}, \sqrt{b}, \sqrt{c}$  are in A.P.

(4)  $\frac{1}{\sqrt{c}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}}$

Answer: (4)

Solution:



$$PQ + QR = PR$$

$$\sqrt{(c+a)^2 - (c-a)^2} + \sqrt{(b+a)^2 - (b-a)^2} = \sqrt{(b+c)^2 - (c-b)^2}$$

$$\sqrt{4ac} + \sqrt{4ab} = \sqrt{4bc}$$

Dividing with  $\sqrt{4abc}$ ,

$$\frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}}$$

24. If  $f(x) = \frac{1}{x}$ ,  $f_2(x) = 1-x$ ,  $f_3(x) = \frac{1}{1-x}$  then find  $J(x)$  such that  $f_2 \circ J \circ f_1(x) = f_3(x)$

(1)  $f_1(x)$

(2)  $\frac{1}{x} f_3(x)$

(3)  $f_3(x)$

(4)  $f_2(x)$

**Answer:** (3)

**Solution:**

$$f_2(J(f_1(x))) = f_3(x)$$

$$f_2\left(J\left(\frac{1}{x}\right)\right) = \frac{1}{1-x}$$

$$1 - J\left(\frac{1}{x}\right) = \frac{1}{1-x}$$

$$J\left(\frac{1}{x}\right) = 1 - \frac{1}{1-x} = \frac{x}{x-1}$$

$$J(x) = \frac{1}{1-x} = f_3(x)$$



25. Find the equation of line through  $(-4, 1, 3)$  & parallel to the plane  $x + y + z = 3$  while the line intersects another line whose equation is  $x + y - z = 0 = x + 2y - 3z + 5$

$$(1) \frac{x+4}{-3} = \frac{y-1}{-2} = \frac{z-3}{1}$$

$$(2) \frac{x+4}{1} = \frac{y-1}{2} = \frac{z-3}{1}$$

$$(3) \frac{x+4}{-3} = \frac{y-1}{2} = \frac{z-3}{1}$$

$$(4) \frac{x+4}{-1} = \frac{y-1}{2} = \frac{z-3}{-3}$$

**Answer:** (3)

**Solution:**

Family of planes containing the line of intersection of planes is  $\pi_1 + \lambda\pi_2 = 0$

$$\text{i.e., } (x + y - z) + \lambda(x + 2y - 3z + 5) = 0$$

This is passing through  $(-4, 1, 3)$

$$\Rightarrow \lambda = -1$$

Hence the equation of plane is  $y - 2z + 5 = 0$

Required line is lie in this plane & is parallel to  $x + y + z = 5$

$$\therefore \text{direction of required line} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -2 \end{vmatrix} = -3\hat{i} + 2\hat{j} + \hat{k}$$

$$\therefore \text{Required line is } \frac{x+4}{-3} = \frac{y-1}{2} = \frac{z-3}{1}$$

26. Consider the curves  $y = x^2 + 2$  and  $y = 10 - x^2$ . Let  $\theta$  be the angle between both the curves at point of intersection, then find  $|\tan \theta|$

$$(1) \frac{8}{15}$$

$$(2) \frac{5}{17}$$

$$(3) \frac{3}{17}$$

$$(4) \frac{8}{17}$$

**Answer:** (1)

**Solution:**

$$x^2 + 2 = 10 - x^2$$

$$\Rightarrow x = \pm 2 \text{ \& } y = 4$$

$\therefore$  point of intersection of curves =  $(\pm 2, 4)$

$$y = x^2 + 2; \quad y = 10 - x^2$$

$$\frac{dy}{dx} = 2x; \quad \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} \text{ at } (\pm 2, 4) = \pm 4 = m_1; \quad \frac{dy}{dx} \text{ at } (\pm 2, 4) = \mp 4 = m_2$$

$$\therefore |\tan \theta| = \left| \frac{8}{1-16} \right| = \frac{8}{15}$$

27. A plane parallel to y-axis passing through line of intersection of planes  $x + y + z = 1$  &  $2x + 3y - z - 4 = 0$  which of the point lie on the plane.

- (1)  $(3, 2, 1)$                       (2)  $(-3, 0, 1)$                       (3)  $(-3, 1, 1)$                       (4)  $(3, 1, -1)$

**Answer:** (4)

**Solution:**

Required plane is  $\pi_1 + \lambda \pi_2 = 0$

$$(x + y + z - 1) + \lambda(2x + 3y - z - 4) = 0$$

$$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z - (1 + 4\lambda) = 0$$

This is parallel to y-axis  $\Rightarrow \lambda = \frac{-1}{3}$

$\therefore$  Required plane is  $x + 4z + 1 = 0$

By inspection,  $(3, 1, -1)$  lie in this plane.

28. Find common tangent of the two curves  $y^2 = 4x$  and  $x^2 + y^2 - 6x = 0$

(1)  $y = \frac{x}{3} + 3$       (2)  $y = \left(\frac{x}{\sqrt{3}} - \sqrt{3}\right)$       (3)  $y = \frac{x}{3} - 3$       (4)  $y = \left(\frac{x}{\sqrt{3}} + \sqrt{3}\right)$

**Answer:** (4)

**Solution:**

Equation of tangent to the parabola  $y^2 = 4x$  is  $y = mx + \frac{1}{m}$

$$\Rightarrow m^2x - my + 1 = 0$$

This is also tangent to  $x^2 + y^2 - 6x = 0$

i.e.,  $\left| \frac{3m^2 + 1}{\sqrt{m^4 + m^2}} \right| = 3$

$$9m^4 + 1 + 6m^2 = 9m^4 + 9m^2$$

$$3m^2 = 1 \Rightarrow m = \pm \frac{1}{\sqrt{3}}$$

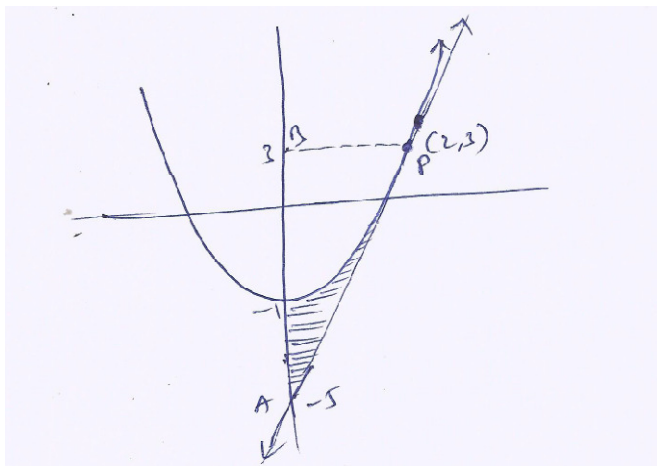
$\therefore$  common tangent is  $y = \pm \left(\frac{x}{\sqrt{3}} + \sqrt{3}\right)$

29. If the area bounded by the curve  $y = x^2 - 1$ , tangent to it at  $(2, 3)$  and y-axis is

(1)  $\frac{2}{3}$       (2)  $\frac{4}{3}$       (3)  $\frac{8}{3}$       (4) 1

**Answer:** (3)

**Solution:**



Equation of tangent at (2, 3) is

$$\frac{y+3}{2} = 2x-1$$

$$y+3 = 4x-2$$

$$4x - y = 5$$

$$\text{Required Area} = \text{Ar}(\Delta PAB) - \int_{-1}^3 x_{\text{parabola}} dy$$

$$= \frac{1}{2} \times 2 \times 8 - \int_{-1}^3 \sqrt{y+1} dy$$

$$= 8 - \frac{2}{3} \left( (y+1)^{\frac{3}{2}} \right)_{-1}^3$$

$$= 8 - \frac{16}{3} = \frac{8}{3}$$