AP Intermediate 2nd Year Question Paper Maths IIB

COORDINATE GEOMETRY AND CALCULUS.

TIME: 3hrs **Max. Marks.**75

Note: This question paper consists of three sections A, B and C.

SECTION -A

Very Short Answer Type Questions. 10 X 2 =20

1. If $x^2 + y^2 + 2gx + 2fy = 0$ represents a circle with centre (-4, -3) then find g, f and the radius of the circle.

2. Find the external centre of similitude of the circles $x^2+y^2 - 2x - 6y + 9 = 0$ and $x^2+y^2 = 4$

3. Find the equation of the parabola whose focus is s(1, –7) and vertex is A(1, –2).

4. If the lines $3x - 4y = 12$ and $3x + 4y = 12$ meets on a hyperbola S = 0 then find the eccentricity of the hyperbola S = 0.

5. If the length of the tangent from (5, 4) to the circle $X^2 + y^2 + 2ky = 0$ is 1 then find k.

6. Evaluate $\int \frac{e^x}{e^{x/2}+1}dx$ on R.

7. Evaluate $\int (\tan x + \log \sec x)e^x dx$ on $\left(\left(2n-\frac{1}{2}\right)\pi, \left(2n+\frac{1}{2}\right)\pi\right) n \in Z$

8. Evaluate $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}dx$

9. Evaluate $\int_0^{\pi/2} \cos^7 x \cdot \sin^2 x dx$

10. Find the general solution of $\frac{dy}{dx} = \frac{2y}{x}$.

SECTION- B

Short Answer Type Questions.

Answer Any Five Of The Following $5 \times 4 = 20$

11. Show that the circles $x^2+y^2 - 6x - 2y + 1 = 0$; $x^2+y^2 + 2x - 8y + 13 = 0$ Touch each other. Find the point of contact and the equation of the common tangent at their point of contact.

12. Show that the common chord of the circles $x^2 + y^2 - 6x - 4y + 9 = 0$ and $x^2 + y^2 - 8x - 6y + 23 = 0$ is the diameter of the second circle also and find its length.

13. Find the length of major axis, minor axis, latus rectum, eccentricity, coordinates of the centre, foci and equations of directrices of the ellipse $4x^2 + y^2 - 8x + 2y + 1 = 0$

14. If the normal at one end of a latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through one end of the minor axis, then show that $e^4 + e^2 = 1$.

15. The point of intersection of two perpendicular tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ lies on the circle $x^2 + y^2 = a^2 - b^2$.

16. $\int_3^7 \sqrt{\frac{7-x}{x-3}}dx$

17. $(2x + y + 1)dx + (4x + 2y - 1)dy = 0$

SECTION - C

Long Answer Type Questions.

Answer Any Five of the Following 5 X 7= 35

18. Find the equation of circle passing through each of the following three points.

19. Find the equation of circle which cuts the following circles orthogonally.

$x^2 + y^2 + 2x + 4y + 1 = 0$, $2x^2 + 2y^2 + 6x + 8y - 3 = 0$, $x^2 + y^2 - 2x + 6y - 3 = 0$.

20. Prove that the area of the triangle inscribed in the parabola $y^2 = 4ax$ is

$\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ sq. units where y_1, y_2, y_3 are the ordinates of its vertices.

21. Evaluate $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$ on $I \subset R \setminus (-1, 1)$.

22. $\int \frac{dx}{3\cos x + 4\sin x + 6}$

23. $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

24. Find the equation of a curve whose gradient is $\frac{dy}{dx} = \frac{y}{x} - \cos^2\frac{y}{x}$, where $x > 0$, $y > 0$ and which passes through the point $(1, \pi/4)$.

IIB Maths paper1 Solutions

1. **If $x^2 + y^2 + 2gx + 2fy = 0$ represents a circle with centre (-4, -3) then find g, f and the radius of the circle.**

Sol. C = (-g, -f)

C = (-4, -3)

$\therefore$ g = 4, f = 3

Radius = $\sqrt{g2 + f2 - c}$

= $\sqrt{16 - 9}$

= 5 units

2. Find the external centre of similitude of the circles $x^2+y^2 - 2x - 6y + 9 = 0$ and $x^2+y^2 = 4$

Sol. S= $x^2+y^2 - 2x - 6y + 9 = 0$ Centre C_1 (1,3) , $r_1 = \sqrt{1 + 9 - 9}$ =1 and

S' = $x^2+y^2 = 4$ centreC_2 (0,0), $r_2 = 2$

The external centre of imilitude divides the line of centres C_1C_2 externally in the ratio $r_1 : r_2 = 1 :2$

Co-ordinates of E are $\left(\frac{2.1-10}{2-1}, \frac{2.3-10}{2-1}\right)$

= $\left(\frac{2}{1}, \frac{6}{1}\right)$ = (2,6)

3. Find the equation of the parabola whose focus is s(1, –7) and vertex is A(1, –2).

Sol.

Focus s = (1, -7), vertex A(1, –2)

h = 1, k = –2, a = –2 + 7 = 5

since x coordinates of S and A are equal, axis of the parabola is parallel to y-axis.

And the y coordinate of S is less than that of A, therefore the parabola is a down ward parabola.

Let equation of the parabola be

$(x - h)^2 = -4a(y - k)$

$(x - 1)^2 = -20(y + 2)$

$x^2 - 2x + 1 = -20y - 40$

$\Rightarrow x^2 - 2x + 20y + 41 = 0$

4. If the lines 3x – 4y = 12 and 3x + 4y = 12 meets on a hyperbola S = 0 then find the eccentricity of the hyperbola S = 0.

Sol. Given lines $3x - 4y = 12$, $3x + 4y = 12$

The combined equation of the lines is

$(3x - 4y)(3x + 4y) = 144$

$9x^2 - 16y^2 = 144$

$$\frac{x^2}{\frac{144}{9}} - \frac{y^2}{\frac{144}{16}} = 1 \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

$a^2 = 16, b^2 = 9$

Eccentricity $e = \sqrt{\frac{a^2 + b^2}{a^2}}$

$$= \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

5. If the length of the tangent from (5,4)to the circle $X^2 + y^{2+}2ky = 0$ is 1 then find k.

Sol. Length of tangent $= \sqrt{S_{11}} = \sqrt{(5)^2 + (4)^2} - 8k$

But length of tangent =1

$\therefore 1 = \sqrt{25 + 16 + 8k}$

Squaring both sides we get 1 =41 + 8K

K = -5 units.

6. $\int \frac{e^x}{e^{x/2}+1}dx$ **on R.**

Sol. $t=1+e^{x/2} \Rightarrow dt=\frac{1}{2}e^{x/2}dx$

$$\int \frac{e^x}{e^{x/2}+1}dx = 2\int \frac{e^{x/2}\left(\frac{1}{2}e^{x/2}dx\right)}{e^{x/2}+1}$$

$$=2\int \frac{(t-1)dt}{t} = 2\int\left(1-\frac{1}{t}\right)dt = 2(t-\log t)+C$$

$$=2(1+e^{x/2}-\log(1+e^{x/2}))+C$$

7. $\int (\tan x+\log\sec x)e^x dx$ **on** $\left(\left(2n-\frac{1}{2}\right)\pi, \left(2n+\frac{1}{2}\right)\pi\right) n\in Z$

Sol. let $f=\log|\sec x| \Rightarrow f' = \frac{1}{\sec x}\cdot \sec x\cdot \tan x\cdot$

$$=\tan x$$

$$\int (\tan x+\log\sec x)e^x dx = e^x\cdot \log|\sec x|+C$$

8. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}dx$

Sol. Let $I=\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}dx \quad \ldots(i)$

$$I=\int_{-\pi/2}^{\pi/2} \frac{\cos(\pi/2-\pi/2-x)dx}{1+e^{-x}} \left(\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$$

$$=\int_{-\pi/2}^{\pi/2} \frac{e^x\cos x dx}{1+e^x}----(2)$$

Adding (1) and (2) ,

$$2I=\int_{-\pi/2}^{\pi/2} \frac{\cos x(1+e^x)}{1+e^x}dx = \int_{-\pi/2}^{\pi/2}\cos x dx$$

$$2I = 2\int_0^{\pi/2} \cos x\, dx \left(\because \cos x \text{ is even function}\right)$$

$$\Rightarrow I = [\sin x]_0^{\pi/2} \Rightarrow I = 1$$

9. $\int_0^{\pi/2} \cos^7 x \cdot \sin^2 x dx$

Sol. $I = \int_0^{\pi/2} \cos^7 x \cdot \sin^2 x dx$,

m=2,n=7

$\int_0^{\pi/2} \sin^m x \cdot \cos^n x dx$ here m even,

n odd

$$= \frac{n-1}{m+n} \cdot \frac{n-3}{m+n-2} \cdots \frac{2}{m+3} \cdot \frac{1}{m+1}$$

$$= \frac{7-1}{9} \times \frac{7-3}{7} \times \frac{7-5}{5} \times \frac{1}{2+1}$$

$$= \frac{6}{9} \cdot \frac{4}{7} \cdot \frac{2}{5} \cdot \frac{1}{3} = \frac{16}{315}$$

10. Find the general solution of $\frac{dy}{dx} = \frac{2y}{x}$**.**

Sol. $\frac{dy}{dx} = \frac{2y}{x} \Rightarrow \int \frac{dy}{y} = 2\frac{dx}{x}$

Integrating both sides

$$\log c + \log y = 2\log x$$

$$\log cy = \log x^2$$

Solution is $cy = x^2$ where c is a constant.

SECTION B

11. Show that the circles $x^2+y^2-6x-2y+1=0$; $x^2+y^2+2x-8y+13=0$ Touch each other. Find the point of contact and the equation of the common tangent at their point of contact.

Sol. Equations of the circles are

$S \equiv x^2+y^2-6x-2y+1=0$

Centers A (3, 1), radius $r_1 = \sqrt{9+1-1} = 3$

$S' \equiv x^2+y^2+2x-8y+13=0$

Centers B(-1,4) , radius $r_2 = \sqrt{1+16-13} = 2$

$AB = \sqrt{(3+1)^2+(1-4)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$AB = 5 = 3+2 = r_1 + r_2$

∴ The circles touch each other externally. The point of contact P divides AB internally in the ratio $r_1 : r_2 = 3:2$

Co – ordinates of P are

$\left(\frac{3(-1)+2.3}{5}, \frac{3.4+2.1}{5}\right)$ i.e.,p$\left(\frac{3}{5}, \frac{14}{5}\right)$

Equation of the common tangent is

$S_1 = 0$

$\Rightarrow -8x + 6y - 12 = 0 \Rightarrow 4x - 3y + 6 = 0$

12. Show that the common chord of the circles $x^2+y^2-6x-4y+9=0$ and $x^2+y^2-8x-6y+23=0$ is the diameter of the second circle also and find its length.

Sol. $S = x^2+y^2-6x-4y+9=0$, $S' = x^2+y^2-8x-6y+23=0$

Common chord is S-S" =0

$\Rightarrow (x^2+y^2-6x-4y+9)-(x^2+y^2-8x-6y+23)=0$

$\Rightarrow 2x + 2y - 14 = 0$

$\Rightarrow x + y - 7 = 0$ …(i)

Centre of circle (4, 3)

Substituting (4, 3) in x + y – 7 =0 , we get

4+3-7 =0 $\Rightarrow$ 0=0.

(i) is a diameter of S' =0.

Radius is $\sqrt{4^2 + 3^2 - 23} = \sqrt{2} \Rightarrow$ diameter $=2\sqrt{2}$

13. Find the length of major axis, minor axis, latus rectum, eccentricity, coordinates of the centre, foci and equations of directrices of the ellipse $4x^2 + y^2 – 8x + 2y + 1 = 0$

Sol. Given equation is $4x^2 + y^2 – 8x + 2y + 1=0$

$$4(x^2 - 2x) + (y^2 + 2y) = -1$$
$$4\left((x-1)^2 - 1\right) + \left((y+1)^2 - 1\right) = -1$$
$$4(x-1)^2 + (y+1)^2 = 4+1-1 = 4$$
$$\frac{(x-1)^2}{1} + \frac{(y+1)^2}{4} = 1$$

a = 1, b = 2 where a < b $\Rightarrow$ y-axis is major axis

Length of major axis = 2b = 4

Length of minor axis = 2a = 2

Length of latus rectum = $\frac{2a^2}{b} = \frac{2}{2} = 1$

Eccentricity = $\sqrt{\frac{b^2 - a^2}{b^2}} = \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$

Centre is c(–1, 1)

be = $2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

Foci are $(-1, 1 \pm \sqrt{3})$

Equations of the directrices are

$$y+1=\pm\frac{b}{e}=\pm\frac{4}{\sqrt{3}}$$

$$\sqrt{3}y+\sqrt{3}=\pm 4$$

$$\sqrt{3}y+\sqrt{3}\pm 4=0$$

14. If the normal at one end of a latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through one end of the minor axis, then show that $e^4 + e^2 = 1$.

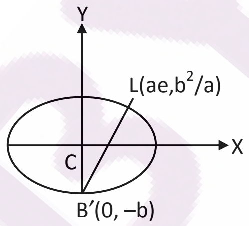
Sol. Equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

One end of the latusrectum is $L(ae, b^2/a)$

Equation of the normal at $L(ae, b^2/a)$ is

$$\frac{a^2x}{ae}-\frac{b^2y}{(b^2/a)}=a^2-b^2\left(\because \frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2\right)$$

$$\frac{ax}{e}-ay=a^2e^2$$



This normal passes through B′(0, –b)

$ab = a^2e^2$

$\Rightarrow$ $b = ae^2$

$b^2 = a^2e^4$

$a^2(1 – e^2) = a^2e^4$

$e^4 + e^2 = 1.$

15. The point of intersection of two perpendicular tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ lies on the circle $x^2 + y^2 = a^2 - b^2$.

Proof:

Equation of any tangent to the hyperbola is :

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

Suppose $P(x_1, y_1)$ is the point of intersection of tangents.

P lies on the tangent $\Rightarrow$

$$y_1 = mx_1 \pm \sqrt{a^2m^2 - b^2} \Rightarrow y_1 - mx_1 = \pm\sqrt{a^2m^2 - b^2}$$

$$\Rightarrow (y_1 - mx_1)^2 = a^2m^2 - b^2$$

$$\Rightarrow y_1^2 + m^2x_1^2 - 2mx_1y_1 - a^2m^2 + b^2 = 0$$

$$\Rightarrow m^2(x_1^2 - a^2) - 2mx_1y_1 + (y_1^2 + b^2) = 0$$

This is a quadratic in m giving the values for m say m_1 and m_2.

The tangents are perpendicular :

$$\Rightarrow m_1m_2 = -1 \Rightarrow \frac{y_1^2 + b^2}{x_1^2 - a^2} = -1$$

$$\Rightarrow y_1^2 + b^2 = -x_1^2 + a^2 \Rightarrow x_1^2 + y_1^2 = a^2 - b^2$$

$P(x_1, y_1)$ lies on the circle $x^2 + y^2 = a^2 - b^2$.

16. $\int_3^7 \sqrt{\frac{7-x}{x-3}}dx$

Sol: Let $x = 3\cos^2\theta + 7\sin^2\theta$

Then $dx = -6\cos\theta\sin\theta + 14\sin\theta\cos\theta$

$= 8\cos\theta\sin\theta$

Upper limit when x = 7 is

$7 = 3\cos^2\theta + 7\sin^2\theta$

$\Rightarrow 7(1 - \sin^2\theta) = 3\cos^2\theta$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

Lower limit when x = 3 is

$3 = 3\cos^2\theta + 7\sin^2\theta$

$\Rightarrow 3\sin^2\theta = 7\sin^2\theta$

$\Rightarrow 4\sin^2\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

Also $\sqrt{\frac{7-x}{x-3}} = \sqrt{\frac{7-3\cos^2\theta-7\sin^2\theta}{3\cos^2\theta+7\sin^2\theta-3}} = \sqrt{\frac{4\cos^2\theta}{4\sin^2\theta}} = \cot\theta$

$\therefore \int_3^7 \sqrt{\frac{7-x}{x-3}}dx = \int_0^{\pi/2} \cot\theta(8\cos\theta\sin\theta)d\theta$

$= 8\int_0^{\pi/2} \cos^2\theta\, d\theta$

$= 8\left(\frac{2-1}{2}\right)\frac{\pi}{2} = 8\left(\frac{1}{2}\right)\frac{\pi}{2} = 2\pi$

$\left(\because \int_0^{\pi/2} \cos^n x\, dx = \frac{n-1}{2} \cdot \frac{n-3}{n-2} \dots \frac{\pi}{2}\right)$

17. (2x + y + 1)dx + (4x + 2y – 1)dy = 0

Sol. $\frac{dy}{dx} = -\frac{2x+y+1}{4x+2y-1}$

$\Rightarrow a_1 = 2, b_1 = 1, a_2 = 4, b_2 = 2$

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2} = \frac{b_1}{b_2}$

Let 2x + y=v so that $\frac{dv}{dx} = 2 + \frac{dy}{dx}$

$$\frac{dv}{dx}=2-\frac{v+1}{2v-1}=\frac{4v-2-v-1}{2v-1}=\frac{3(v-1)}{2v-1}$$

$$\frac{2v-1}{3(v-1)}dv=dx \Rightarrow \frac{2v-1}{v-1}dv=3dx$$

$$\int\left(2+\frac{1}{v-1}\right)dv=3\int dx$$

$$2v+\log(v-1)=3x+c$$
$$2v-3x+\log(v-1)=c$$
$$2(2x+y)-3x+\log(2x+y-1)=c$$
$$4x+2y-3x+\log(2x+y-1)=c$$

SECTION C

18. Find the equation of circle passing through each of the following three points.

i) (3, 4); (3,2); (1,4)

Let the equation of circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

it is passing through (3, 4); (3,2); (1,4)

$\therefore$Given points satisfy above equation then

9 + 16 + 6g + 8f + c = 0

25+6g + 8f + c = 0 _____(i)

9 + 4 + 6g + 4f + c = 0

13 + 6g + 4f + c = 0 _____(ii)

1+16+2g +8f +c = 0

17+2g +8f +c = 0________(iii)

(ii) – (i) we get

-12 -4f = 0 (or) f = - 3

(ii) – (iii) we get -4 +4g -4f = 0

g –f = 1 => g = - 2

Now substituting g, f in equation (i) we get

25 + 6 (-2) +8 (-3) + c = 0

We get c = 11

Required equation of circle be

$X^2 + y^2 - 4x - 6y + 11 = 0$

19. Find the equation of circle which cuts the following circles orthogonally.

$x^2 + y^2 + 2x + 4y + 1 = 0$, $2x^2 + 2y^2 + 6x + 8y - 3 = 0$, $x^2 + y^2 - 2x + 6y - 3 = 0$.

Sol.

$S \equiv x^2 + y^2 + 2x + 4y + 1 = 0$

$S^1 \equiv 2x^2 + 2y^2 + 6x + 8y - 3 = 0$

$S^{11} \equiv x^2 + y^2 - 2x + 6y - 3 = 0$

Radical axis of $S = 0$, $S^1 = 0$ is $S - S^1 = 0$

$$-x + \frac{5}{2} = 0 \Rightarrow x = \frac{5}{2} \text{ ----(1)}$$

Radical axis of $S = 0$, $S^{11} = 0$ is $S - S^{11} = 0$

$4x - 2y + 4 = 0 \Rightarrow 2x - y + 2 = 0$

$$x = \frac{5}{2} \Rightarrow 5 - y + 2 = 0 \Rightarrow y = 7 \text{ ----(2)}$$

Solving (1) and (2),

Radical centre is P(5/2, 7)

PT = Length of the tangent from P to S = 0

$$= \sqrt{\frac{25}{4} + 49 + 5 + 28 + 1} = \sqrt{\frac{25}{4} + 83} = \sqrt{\frac{25 + 332}{4}} = \frac{\sqrt{357}}{2}$$

Equation of the circles cutting the given circles orthogonally is

$$\left(x - \frac{5}{2}\right)^2 + (y - 7)^2 = \frac{357}{4}$$

$$\Rightarrow x^2 - 5x + \frac{25}{4} + y^2 - 14y + 49 = \frac{357}{4}$$

$$\Rightarrow x^2 + y^2 - 5x - 14y + \frac{25}{4} + 49 - \frac{357}{4} = 0$$

$$\Rightarrow x^2 + y^2 - 5x - 14y + \frac{25 + 196 - 357}{4} = 0$$

$$\Rightarrow x^2 + y^2 - 5x - 14y + \frac{136}{4} = 0$$

$$\Rightarrow x^2 + y^2 - 5x - 14y - 34 = 0$$

20. Prove that the area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ sq.units where y_1, y_2, y_3 are the ordinates of its vertices.

Sol.

Given parabola $y^2 = 4ax$

let $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$, $R(at_3^2, 2at_3)$ be the vertices of ΔPQR.

Area of ΔPQR = $= \frac{1}{2}\begin{vmatrix} at_1^2 - at_2^2 & at_2^2 - at_3^2 \\ 2at_1 - 2at_2 & 2at_2 - 2at_3 \end{vmatrix} = \frac{1}{2}\left|2a^2\left(t_1^2 - t_2^2\right)(t_2 - t_3) - 2a^2\left(t_2^2 - t_3^2\right)(t_1 - t_2)\right|$

$= a^2\left|(t_1 - t_2)(t_2 - t_3)\left(t_1 + t_2 - t_2 - t_3\right)\right|$

$$= a^2\left|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)\right|$$

$$= \frac{a^3}{a}\left|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)\right|$$

$$= \frac{1}{8a}|(2at_1 - 2at_2)(2at_2 - 2at_3)(2at_3 - 2at_1)|$$

$$= \frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$$

Where $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ are the vertices of ΔPQR.

21. Evaluate $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$ on $I \subset R \setminus (-1, 1)$.

Sol. Let $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\frac{2x}{1-x^2} = \frac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}(\tan 2\theta) = 2\theta + n\pi$$

Where $n = 0$ if $|x| < 1$

$= -1$ if $x > 1$

$= 1$ if $x < -1$

We have $d\theta = \frac{1}{1+x^2}dx$ and

$1+x^2 = 1 + \tan^2\theta = \sec^2\theta$

$$\therefore \int \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$$

$$= \int \left(\tan^{-1}\left(\frac{2x}{1-x^2}\right)\right)(1+x^2)\frac{1}{1+x^2}dx$$

$$= \int (2\theta + n\pi) \int \sec^2\theta d\theta$$

$$= 2\int \theta\sec^2\theta d\theta + n\pi\int \sec^2\theta d\theta + c$$

$$= 2\left(\theta\tan\theta - \int \tan\theta d\theta\right) + n\pi\tan\theta + c$$

$$= 2(\theta\tan\theta + \log|\cos\theta| + n\pi\tan\theta + c$$

$$= (2\theta + n\pi)\tan\theta + 2\log\cos\theta + c$$

$$= (2\theta + n\pi)\tan\theta + \log\cos^2\theta + c$$

$$= (2\theta + n\pi)\tan\theta + \log\sec^2\theta + c$$

$$= x\tan^{-1}\left(\frac{2x}{1-x^2}\right) - \log(1+x^2) + c$$

22. $\int \frac{dx}{3\cos x + 4\sin x + 6}$

Sol: Let $\tan\frac{x}{2} = t$ then $dx = \frac{2\,dt}{1+t^2}$

$\sin x = \frac{2t}{1+t^2}$ and $\cos x = \frac{1-t^2}{1+t^2}$

$$\therefore \int \frac{dx}{3\cos x + 4\sin x + 6}$$

$$= \int \frac{\frac{2dt}{1+t^2}}{3\left(\frac{1-t^2}{1+t^2}\right) + 4\left(\frac{2t}{1+t^2}\right) + 6}$$

$$= \int \frac{2\,dt}{3 - 3t^2 + 8t + 6 + 6t^2}$$

$$= \int \frac{2dt}{3t^2 + 8t + 9}$$

$$= \frac{2}{3}\int \frac{dt}{t^2 + \frac{8}{3}t + 3}$$

$$= \frac{2}{3}\int \frac{dt}{\left(t + \frac{4}{3}\right)^2 + 3 - \frac{16}{9}}$$

$$= \frac{2}{3}\int \frac{dt}{\left(t + \frac{4}{3}\right)^2 + \frac{11}{9}}$$

$$= \frac{2}{3}\int \frac{dt}{\left(t + \frac{4}{3}\right)^2 + \left(\frac{\sqrt{11}}{3}\right)^2}$$

$$= \frac{2}{3}\frac{3}{\sqrt{11}}\tan^{-1}\left(\frac{t + \frac{4}{3}}{\frac{\sqrt{11}}{3}}\right)$$

$$=\frac{2}{\sqrt{11}}\text{Tan}^{-1}\left(\frac{3t+4}{\sqrt{11}}\right)+c.$$

23. $\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}dx$

Sol. $\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}dx=\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16[1-(\sin x-\cos x)^2]}dx$

put $\sin x-\cos x=t\Rightarrow(\cos x+\sin x)dx=dt$

$x=0\Rightarrow t=-1$ and $x=\frac{\pi}{4}\Rightarrow t=0$

$$=\int_{-1}^{0}\frac{dt}{25-16t^2}=\frac{1}{16}\int_{-1}^{0}\frac{dt}{\frac{25}{16}-t^2}$$

$$=\frac{1}{16}\times\frac{1}{2\times\frac{5}{4}}\left[\ln\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|\right]_{-1}^{0}$$

$$=-\frac{1}{40}\ln\left[\frac{1/4}{9/4}\right]=\frac{1}{40}\cdot 2\ln\cdot 3=\frac{1}{20}\ln 3$$

24. Find the equation of a curve whose gradient is $\frac{dy}{dx}=\frac{y}{x}-\cos^2\frac{y}{x}$, where x > 0, y > 0 and which passes through the point (1, π/4).

Sol. $\frac{dy}{dx}=\frac{y}{x}-\cos^2\frac{y}{x}$ which is homogeneous d.eq.

Put y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$v+x\frac{dv}{dx}=v-\cos^2 v\Rightarrow\int\frac{dv}{\cos^2 v}=-\int\frac{dx}{x}$$

$$\int\sec^2 v=-\int\frac{dx}{x}\Rightarrow\tan v=-\log|x|+c$$

This curve passes through (1, π/4)

$$\tan\left(\frac{\pi}{4}\right) = c - \log 1 \Rightarrow c = 1$$

Equation of the curve is :

$$\tan v = 1 - \log|x| \Rightarrow \tan\left(\frac{y}{x}\right) 1 - \log|x|$$