SAMPLE QUESTION PAPER

MATHEMATICS

CLASS – XII : 2015-16

TYPOLOGY

	VSA (1 M)	L A – I (4M)	L A – II (6M)	MARKS	%WEIGHTAGE
Remembering	3, 6	11, 18, 19,	25	20	20%
Understanding	1,2	9, 10	24, 26	22	22%
Applications	4	13, 15, 16, 17	22, 20	29	29%
Hots	5	7,14	23	15	15%
Evaluation	-	8, 12	21	14	14%
Total	$6\times1=6$	$13\times4=52$	$7\times6=42$	100	100%

Question- wise break up

Type of Questions	Marks per Question	Total number of Questions	Total Marks
VSA	1	6	06
L A - I	4	13	52
L A - II	6	7	42
Total		26	100

Blue Print of Sample Paper

UNIT	VSA	LA-I	LA-II	TOTAL	
Relations and functions	1(1)	*4(1)	-	5(2)	
Inverse trigonometric functions	1(1)	4(1)	-	5(2)	10
Matrices	1(1)	8(2)	-	9(3)	
Determinants	-	*4(1)	-	4(1)	13
Continuity and differentiability	1(1)	4(1) *4(1)	-	9(3)	
Applications of derivative	1(1)	-	6(1) 6(1) VBQ	13(3)	
Integrals	-	*4(1) 4(1)	-	8(2)	
Application of integrals	-		6(1)	6(1)	
Differential Equations	-	8(2)	-	8(2)	44
Vectors	-	-	6(1)	6(1)	
Three dimensional geometry	1(1)	4(1)	*6(1)	11(3)	17
Linear Programming	-	-	6(1)	6(1)	6
Probability	-	4(1)	*6(1)	10(2)	10
Total	6(6)	52(13)	42(7)	100(26)	100

Note:- * indicates the questions with internal choice. The number of questions is given in the brackets and the marks are given outside the brackets.

SAMPLE PAPER

Section A

Question numbers 1 to 6 carry 1 mark each.

Q1. Evaluate: $\sin(2\cos^{-1}(-\frac{3}{5}))$.

Q2. State the reason for the following Binary Operation *, defined on the set Z of integers, to be not commutative. $a * b = ab^3$.

Q3. Give an example of a skew symmetric matrix of order 3.

Q4. Using derivative, find the approximate percentage increase in the area of a circle if its radius is increased by 2% .

Q5. Find the derivative of $f(e^{\tan x})$ w.r. to x at x = 0. It is given that $f'(1) = 5$.

Q6. If the lines $\frac{x-1}{-2} = \frac{y-4}{3p} = \frac{z-3}{4}$ and $\frac{x-2}{4p} = \frac{y-5}{2} = \frac{z-1}{-7}$ are perpendicular to each other, then find the value of p.

Section B

Question numbers 7 to 19 carry 4 marks each.

Q7. Let $f : W \to W$ be defined as $f(n) = \begin{cases} n-1, \text{ if } n \text{ is odd} \\ n+1, \text{ if } n \text{ is even} \end{cases}$. Then show that f is invertible. Also, find the inverse of f.

OR

Show that the relation R in the set $N \times N$ defined by

$(a, b)R(c, d)$ iff $a^2 + d^2 = b^2 + c^2 \forall a, b, c, d \in N$, is an equivalence relation.

Q8. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}}\right) = \frac{\pi}{4} - \frac{x}{2}$, where $\pi < x < \frac{3\pi}{2}$

Q9. Let $A=\begin{bmatrix}-2 & 1\\ 3 & 4\end{bmatrix}$. Then verify the following: $A(\text{adj}A)=(\text{adj}A)A=|A|I$, where I is the identity matrix of order 2.

Q10. Using properties of determinants, prove that $\begin{vmatrix}1 & 1+p & 1+p+q\\ 3 & 4+3p & 2+4p+3q\\ 4 & 7+4p & 2+7p+4q\end{vmatrix}=1$.

OR

Without expanding the determinant at any stage, prove that $\begin{vmatrix}0 & 2 & -3\\ -2 & 0 & 4\\ 3 & -4 & 0\end{vmatrix}=0$.

Q11. Let $A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}, B=\begin{bmatrix}4 & -6\\ -2 & 4\end{bmatrix}$. Then compute AB. Hence, solve the following system of equations: $2x+y=4, 3x+2y=1.$

Q12. If the following function is differentiable at x = 2, then find the values of a and b.

$$f(x)=\begin{cases} x^2, \text{ if } x\le 2\\ ax+b, \text{ if } x>2\end{cases}.$$

Q13. Let $y=(\log x)^x + x^{x\cos x}$. Then find $\frac{dy}{dx}$.

OR

If $x=a\sin pt, y=b\cos pt$. Then find $\frac{d^2y}{dx^2}$ at $t=0$.

Q14. Evaluate the following indefinite integral: $\int \frac{1}{\sin x-\sin 2x}dx$.

OR

Evaluate the following indefinite integral: $\int \frac{\sin\phi}{\sqrt{\sin^2\phi+2\cos\phi+3}}d\phi$.

Q15. Evaluate the following definite integral: $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$.

Q16. Solve the following differential equation: $\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x}, x > 0$.

Q17. Solve the following differential equation: $(1+y^2)dx = (\tan^{-1} y - x)dy$.

Q18. Find the shortest distance between the following pair of skew lines:

$$\frac{x-1}{2} = \frac{2-y}{3} = \frac{z+1}{4}, \frac{x+2}{-1} = \frac{y-3}{2} = \frac{z}{3}.$$

Q19. A problem in mathematics is given to 4 students A, B, C, D. Their chances of solving the problem, respectively, are 1/3, 1/4, 1/5 and 2/3. What is the probability that (i) the problem will be solved? (ii) at most one of them will solve the problem?

Section C

Question numbers 20 to 26 carry 6 marks each.

Q20. Find the intervals in which the following function is strictly increasing or strictly decreasing. Also, find the points of local maximum and local minimum, if any.

$$f(x) = (x-1)^3(x+2)^2$$

Q21. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, then prove that $(i) \vec{a} = \pm 2(\vec{b} \times \vec{c}), (ii) \left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = \pm 1$

Q22. Using integration, find the area bounded by the tangent to the curve $4y = x^2$ at the point (2, 1) and the lines whose equations are $x = 2y$ and $x = 3y - 3$.

Q23. Find the distance of the point $3\hat{i} - 2\hat{j} + \hat{k}$ from the plane 3 x + y – z + 2 = 0 measured parallel to the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-1}{1}$. Also, find the foot of the perpendicular from the given point upon the given plane.

OR

Find the equation of the plane passing through the line of intersection of the planes

$\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})=-1$ and $\vec{r}.(\hat{i}+\hat{j}-2\hat{k})=0$ and passing through the point (3, -2, -1). Also, find the angle between the two given planes.

Q24. A Bag I contains 5 red and 4 white balls and a Bag II contains 3 red and 3 white balls. Two balls are transferred from the Bag I to the Bag II and then one ball is drawn from the Bag II. If the ball drawn from the Bag II is red, then find the probability that one red ball and one white ball are transferred from the Bag I to the Bag II.

OR

Find the mean, the variance and the standard deviation of the number of doublets in three throws of a pair of dice.

Q25. A farmer wants to construct a circular garden and a square garden in his field. He wants to keep the sum of their perimeters 600 m. Prove that the sum their areas is the least, when the side of the square garden is double the radius of the circular garden.

Do you think that a good planning can save energy, time and money?

Q26. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 hours of labour for fabricating and 1 hour for finishing. Each piece of model B requires 12 hours of labour for fabricating and 3 hours for finishing. The maximum number of labour hours, available for fabricating and for finishing, are 180 and 30 respectively. The company makes a profit of Rs 8000 and Rs 12000 on each piece of model A and model B respectively. How many pieces of each model should be manufactured to get maximum profit? Also, find the maximum profit.

Marking Scheme (Sample Paper)

Section A

Q1. $-\frac{24}{25}$ (1)

Q2. We have 1, 2 $\in$ Z such that 1 * 2 = 8 and 2 * 1 = 2. This implies that 1 * 2 $\neq$ 2 * 1. Hence, * is not commutative. (1)

Q3. $\begin{bmatrix} 0 & 1 & -3 \\ -1 & 0 & -2 \\ 3 & 2 & 0 \end{bmatrix}$ (1)

Q4. 4% (1)

Q5. 5 (1)

Q6. -14 (1)

Section B

Q7. Here, $fof : W \to W$ is such that, if n is odd, $fof(n) = f(f(n)) = f(n-1) = n-1+1 = n$

(1+1/2)

and if n is even, $fof(n) = f(f(n)) = f(n+1) = n+1-1 = n$ (1+1/2)

Hence, $fof = I$ This implies that f is invertible and $f^{-1} = f$ (1)

OR

Let $(a,b) \in N \times N$. Then $\because a^2 + b^2 = b^2 + a^2 \therefore (a,b)R(a,b)$ Hence, R is reflexive. (1)

Let $(a,b),(c,d) \in N \times N$ be such that $(a,b)R(c,d)$
$\Rightarrow a^2 + d^2 = b^2 + c^2$
$\Rightarrow c^2 + b^2 = d^2 + a^2$
$\Rightarrow (c,d)R(a,b)$

Hence, R is symmetric. (1)

Let $(a,b),(c,d),(e,f) \in N \times N$ be such that $(a,b)R(c,d),(c,d)R(e,f)$.
$\Rightarrow a^2 + d^2 = b^2 + c^2$ and $c^2 + f^2 = d^2 + e^2$
$\Rightarrow a^2 + d^2 + c^2 + f^2 = b^2 + c^2 + d^2 + e^2$
$\Rightarrow a^2 + f^2 = b^2 + e^2$
$\Rightarrow (a,b)R(e,f)$

Hence, R is transitive. (1+1/2)

Since R is reflexive, symmetric and transitive. Therefore R is an equivalence relation. (1/2)

Q8.

$$\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right) = \tan^{-1}\left(\frac{\sqrt{2\cos^2\frac{x}{2}}+\sqrt{2\sin^2\frac{x}{2}}}{\sqrt{2\cos^2\frac{x}{2}}-\sqrt{2\sin^2\frac{x}{2}}}\right) \quad (1)$$

$$= \tan^{-1}\left(\frac{-\sqrt{2}\cos\frac{x}{2}+\sqrt{2}\sin\frac{x}{2}}{-\sqrt{2}\cos\frac{x}{2}-\sqrt{2}\sin\frac{x}{2}}\right) \quad \left(\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow \cos\frac{x}{2} < 0, \sin\frac{x}{2} > 0\right) \quad (1+1/2)$$

$$= \tan^{-1}\left(\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right) \quad (1)$$

$$= \frac{\pi}{4}-\frac{x}{2} \quad \left(-\frac{\pi}{4} > \frac{\pi}{4}-\frac{x}{2} > -\frac{\pi}{2}\right) \quad (1/2)$$

Q9. $\text{adj}A = \begin{bmatrix} 4 & -3 \\ -1 & -2 \end{bmatrix}' = \begin{bmatrix} 4 & -1 \\ -3 & -2 \end{bmatrix}$ (2)

$$(\text{adj}A)A=\begin{bmatrix}-11 & 0\\ 0 & -11\end{bmatrix}=-11\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$ (1/2)

$$A(\text{adj}A)=\begin{bmatrix}-11 & 0\\ 0 & -11\end{bmatrix}=-11\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$ (1/2)

$$|A|=\begin{vmatrix}-2 & 1\\ 3 & 4\end{vmatrix}=-11$$ (1/2)

Hence, $A(\text{adj}A)=(\text{adj}A)A=|A|I$ verified. (1/2)

Q(10) $\text{LHS}=\begin{vmatrix}1 & 1+p & 1+p+q\\ 3 & 4+3p & 2+4p+3q\\ 4 & 7+4p & 2+7p+4q\end{vmatrix}=\begin{vmatrix}1 & 1+p & 1+p+q\\ 0 & 1 & -1+p\\ 0 & 3 & -2+3p\end{vmatrix}(R_2\to R_2-3R_1, R_3\to R_3-4R_1)$

(2)

$$=\begin{vmatrix}1 & 1+p & 1+p+q\\ 0 & 1 & -1+p\\ 0 & 0 & 1\end{vmatrix}(R_3\to R_3-3R_2)$$ (1)

=1 =RHS. Hence, proved. (1)

OR

Let $\Delta=\begin{vmatrix}0 & 2 & -3\\ -2 & 0 & 4\\ 3 & -4 & 0\end{vmatrix}=\begin{vmatrix}0 & -2 & 3\\ 2 & 0 & -4\\ -3 & 4 & 0\end{vmatrix}$ (interchanging rows and columns) (1 + 1/2)

$$=(-1)(-1)(-1)\begin{vmatrix}0 & 2 & -3\\ -2 & 0 & 4\\ 3 & -4 & 0\end{vmatrix}$$ (1 +1/2))

$=-\Delta$ (1/2)

$\Rightarrow 2\Delta=0 \Rightarrow \Delta=0$ (1/2)

Q11. $AB = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I$ (1/2)

$$\Rightarrow A(\frac{1}{2}B) = I \Rightarrow A^{-1} = \frac{1}{2}B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$ (1)

The given system of equations is equivalent to $A'X = C,$ where $X = \begin{bmatrix} x \\ y \end{bmatrix}, C = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ (1/2)

$$X = (A')^{-1}C = (A^{-1})'C$$ (1)

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \end{bmatrix} \Rightarrow x = 7, y = -10$$ (1)

Q12. Since, f is differentiable at x = 2, therefore, f is continuous at x = 2. (1/2)

$$\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2) \Rightarrow \lim_{x\to 2^-} x^2 = \lim_{x\to 2^+} (ax+b) = 4 \Rightarrow 4 = 2a + b$$ (1+1/2)

Since, f is differentiable at x = 2,

$$\therefore Lf'(2) = Rf'(2) \Rightarrow \lim_{h\to 0} \frac{f(2-h) - f(2)}{-h} = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \quad (h > 0)$$

$$\Rightarrow \lim_{h\to 0} \frac{(2-h)^2 - 4}{-h} = \lim_{h\to 0} \frac{a(2+h) + b - 4}{h}$$

$$\Rightarrow \lim_{h\to 0} (-h+4) = \lim_{h\to 0} \frac{4 + ah - 4}{h} \Rightarrow 4 = a$$

(1+1/2)

b = -4 (1/2)

Q13.

$$\text{Let } u = (\log x)^x. \text{Then } \log u = x\log(\log x) \Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{1}{\log x} + \log(\log x)$$

$$\Rightarrow \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right]$$

(1+1/2)

Let $v = x^{x\cos x}$. Then $\log v = x\cos x\log x \Rightarrow \frac{1}{v}\frac{dv}{dx} = \frac{x\cos x}{x} + \cos x(\log x) - x\sin x\log x$

$$\Rightarrow \frac{dv}{dx} = x^{x\cos x}\left[\cos x + \cos x(\log x) - x\sin x\log x\right]$$

(1+1/2)

$$y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] +$$
$$x^{x\cos x}\left[\cos x + \cos x(\log x) - x\sin x\log x\right]$$

(1)

OR

$$\frac{dx}{dt} = ap\cos pt, \frac{dy}{dt} = -bp\sin pt, \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\frac{b}{a}\tan pt$$ (1+1/2)

$$\frac{d^2y}{dx^2} = -\frac{b}{a}p\sec^2 pt \times \frac{dt}{dx}$$ (1+1/2)

$$\frac{d^2y}{dx^2} = -\frac{b}{a^2\cos^3 pt}$$ (1/2)

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{t=0} = -\frac{b}{a^2}$$ (1/2)

Q14. Given integral =

$$\int\frac{1}{\sin x - \sin 2x}dx = \int\frac{1}{\sin x(1-2\cos x)}dx = \int\frac{\sin x}{(1+\cos x)(1-\cos x)(1-2\cos x)}dx$$

$$= -\int\frac{dt}{(1+t)(1-t)(1-2t)} \quad (\cos x = t \Rightarrow -\sin x dx = dt)$$ (1)

$$\frac{1}{(1+t)(1-t)(1-2t)} = \frac{A}{1+t} + \frac{B}{1-t} + \frac{C}{1-2t}$$

$$\therefore 1 = A(1-t)(1-2t) + B(1+t)(1-2t) + C(1-t^2) \text{ (An identity)}$$

Putting, t= 1, ½, -1, we get A = 1/6, B= -1/2, C = 4/3 (1+1/2)

Therefore, the given integral

$$= -\frac{1}{6}\log|1+t| - \frac{1}{2}\log|1-t| + \frac{4}{6}\log|1-2t| + c$$

$$= -\frac{1}{6}\log|1+\cos x| - \frac{1}{2}\log|1-\cos x| + \frac{2}{3}\log|1-2\cos x| + c$$

(1+1/2)

OR

$$\int \frac{\sin\phi}{\sqrt{\sin^2\phi + 2\cos\phi + 3}}\,d\phi = \int \frac{\sin\phi}{\sqrt{1-\cos^2\phi + 2\cos\phi + 3}}\,d\phi \quad (1/2)$$

$$= \int \frac{\sin\phi}{\sqrt{-\cos^2\phi + 2\cos\phi + 4}}\,d\phi = \int \frac{-1}{\sqrt{-t^2 + 2t + 4}}\,dt \;\; (\cos\phi = t \Rightarrow -\sin\phi\, d\phi = dt) \quad (1)$$

$$= -\int \frac{1}{\sqrt{(\sqrt{5})^2 - (t-1)^2}}\,dt \quad (1+1/2)$$

$$= -\sin^{-1}\frac{t-1}{\sqrt{5}} + c = -\sin^{-1}\frac{\cos\phi - 1}{\sqrt{5}} + c \quad (1)$$

Q15. Let

$$I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x}\,dx + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$$

$$= 0 + 2\int_{0}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$$

(as $\frac{2x}{1+\cos^2 x}$ is odd and

$\frac{2x\sin x}{1+\cos^2 x}$ is even) (1)

$$= 4\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx\,.$$

Let

$$I_1 = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)}dx$$

$$I_1 = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}dx \qquad (1)$$

$$\text{Adding, } 2I_1 = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$$

$$= -\pi\int_1^{-1} \frac{dt}{1+t^2} \ (\cos x = t \Rightarrow -\sin x dx = dt) \qquad (1)$$

$$= \pi\left[\tan^{-1} t\right]_{-1}^{1} \qquad (1/2)$$

$$I_1 = \frac{\pi^2}{4}.\ \text{Hence, } I = \pi^2 \qquad (1/2)$$

Q16. Given differential equation is

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x}, x > 0 \text{ or}, \frac{dy}{dx} = \frac{y}{x} + \sqrt{1+(\frac{y}{x})^2} = f(\frac{y}{x}), \text{ hence, homogeneous.} \qquad (1/2)$$

Put y = v x $\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$. The differential equation becomes $v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$ (1)

$$\text{or}, \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$ (1/2).

Integrating, we get $\log\left|v+\sqrt{1+v^2}\right| = \log|x| + \log k$ (1)

$$\Rightarrow \log\left|v+\sqrt{1+v^2}\right| = \log|x|k \Rightarrow \left|v+\sqrt{1+v^2}\right| = |x|k$$
$$\Rightarrow v+\sqrt{1+v^2} = \pm kx \Rightarrow \frac{y}{x} + \sqrt{1+(\frac{y}{x})^2} = cx$$
$$\Rightarrow y + \sqrt{x^2+y^2} = cx^2,$$

which gives the general solution. (1)

Q17. We have the following differential equation: $\frac{dx}{dy} = \frac{(\tan^{-1}y - x)}{1+y^2}$ 0r, $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$,

which is linear in x (1/2)

$$\text{I.F.} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$ (1)

Multiplying both sides by I. F. and integrating we get $xe^{\tan^{-1}y} = \int e^{\tan^{-1}y} \frac{\tan^{-1}y}{1+y^2}dy$ (1/2)

$$\Rightarrow xe^{\tan^{-1}y} = \int e^t t dt \quad (\tan^{-1}y = t \Rightarrow \frac{1}{1+y^2}dy = dt)$$
$$\Rightarrow xe^{\tan^{-1}y} = te^t - e^t + c \Rightarrow xe^{\tan^{-1}y} = \tan^{-1}y e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$$

which gives the general solution of the differential equation. (2)

Q18. The vector equations of the given lines are

$$\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(2\hat{i} - 3\hat{j} + 4\hat{k}), \vec{r} = -2\hat{i} + 3\hat{j} + \mu(-\hat{i} + 2\hat{j} + 3\hat{k})$$
$$\vec{a}_1 = \hat{i} + 2\hat{j} - \hat{k}, \vec{b}_1 = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{a}_2 = -2\hat{i} + 3\hat{j}, \vec{b}_2 = -\hat{i} + 2\hat{j} + 3\hat{k}$$

(1)

$$\vec{a}_2-\vec{a}_1=-3\hat{i}+\hat{j}+\hat{k}, \vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 4\\ -1 & 2 & 3\end{vmatrix}=-17\hat{i}-10\hat{j}+\hat{k}$$ (2)

The required shortest distance $=\dfrac{\left|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)\right|}{\left|\vec{b}_1\times\vec{b}_2\right|}$ (1/2)

$=\dfrac{42}{\sqrt{390}}$ units (1/2)

Q19. Let us define the following events: E = A solves the problem, F = B solves the problem, G = C solves the problem, H = D solves the problem (1/2)

(i) The required probability = $P(E\cup F\cup G\cup H)$ (1/2)

$= 1 - P(\bar{E}\cap\bar{F}\cap\bar{G}\cap\bar{H})$

$= 1 - P(\bar{E})\times P(\bar{F})\times P(\bar{G})\times P(\bar{H})$ (1)

$= 1 - \frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}=\frac{13}{15}$ (1/2)

(ii)The required probability =

$$P(\bar{E})\times P(\bar{F})\times P(\bar{G})\times P(\bar{H})+P(E)\times P(\bar{F})\times P(\bar{G})\times P(\bar{H})+P(\bar{E})\times P(F)\times P(\bar{G})\times P(\bar{H})$$
$$+P(\bar{E})\times P(\bar{F})\times P(G)\times P(\bar{H})+P(\bar{E})\times P(\bar{F})\times P(\bar{G})\times P(H)$$ (1)

$$=\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}+\frac{1}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{1}{3}+\frac{2}{3}\times\frac{1}{4}\times\frac{4}{5}\times\frac{1}{3}$$
$$+\frac{2}{3}\times\frac{3}{4}\times\frac{1}{5}\times\frac{1}{3}+\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{2}{3}=\frac{5}{18}$$ (1/2)

Section C

Q20. $f'(x)=(x-1)^2(x+2)(5x+4)$ (1/2)

$f'(x) = 0 \Rightarrow x = 1, -2, \frac{-4}{5}$ (1/2)

In the interval	Sign of f′(x)	Nature of the function
$(-\infty, -2)$	(+ve)(-ve)(-ve)= +ve	f is strictly increasing in $(-\infty, -2]$
$(-2, -\frac{4}{5})$	(+ve)(+ve)(-ve)= -ve	f is strictly decreasing in $\left[-2, -\frac{4}{5}\right]$
$(-\frac{4}{5}, 1)$	(+ve)(+ve)(+ve)= +ve	f is strictly increasing in $\left[-\frac{4}{5}, 1\right]$
$(1, \infty)$	(+ve)(+ve)(+ve)= +ve	f is strictly increasing in $[1, \infty)$

(2+1/2)

Hence, f is strictly increasing in $(-\infty, -2]$ and $\left[-\frac{4}{5}, \infty\right)$. f is strictly decreasing in $\left[-2, -\frac{4}{5}\right]$

(1/2)

In the left nhd of -2, f′(x)>0, in the right nhd of -2, f′(x)<0 and f′(-2) = 0, therefore, by the first derivative test, -2 is a point of local maximum. (1)

In the left nhd of -4/5, f′(x)<0, in the right nhd of -4/5, f′(x)>0 and f′(-4/5) = 0, therefore, by the first derivative test, -4/5 is a point of local minimum. (1)

Q21. We have

$\vec{a}.\vec{b} = 0, \vec{a}.\vec{c} = 0 \Rightarrow \vec{a} \perp$ both $\vec{b}$ and $\vec{c}$ (as $\vec{a}, \vec{b}, \vec{c}$ are nonzero vectors)
$\Rightarrow \vec{a} \parallel \vec{b} \times \vec{c}$ (1)

Let $\vec{a} = \lambda(\vec{b} \times \vec{c})$ (1)

Then

$$|\vec{a}| = |\lambda||(\vec{b} \times \vec{c})| \Rightarrow \frac{|\vec{a}|}{|(\vec{b} \times \vec{c})|} = |\lambda| \Rightarrow |\lambda| = \frac{1}{\sin\frac{\pi}{6}} = 2 \Rightarrow \lambda = \pm 2$$

$\therefore \vec{a} = \pm 2(\vec{b} \times \vec{c})$

(2)

Now $\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right] = \left\{\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}+\vec{c}\right)\right\}.\left(\vec{c}+\vec{a}\right) = \left(\vec{a}\times\vec{b}\right).\vec{c} + \left(\vec{b}\times\vec{c}\right).\vec{a}$ (As the scalar triple product = 0 if any two vectors are equal.)

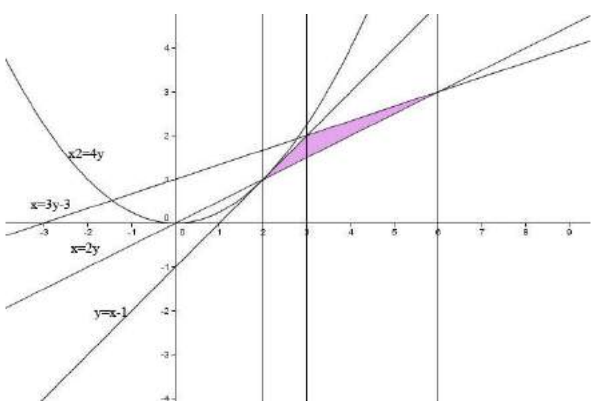
$\vec{a}.(\vec{b} \times \vec{c}) + (\vec{a} \times \vec{b}).\vec{c} = \vec{a}.(\vec{b} \times \vec{c}) + \vec{a}.(\vec{b} \times \vec{c}) = 2\vec{a}.(\vec{b} \times \vec{c})$ (1+1/2)

$= 2\vec{a}.(\pm\frac{1}{2}\vec{a}) = \pm 1$ (1/2)

Q22. We have the curve

$$4y = x^2 \Rightarrow 4\frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{2} \Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 1$$ (1)

The equation of the tangent is $y = x - 1$ (1)



Graph sketch (1)

The required area = the shaded area =

$$\int_2^3\left[(x-1)-\frac{x}{2}\right]dx+\int_3^6\left[\frac{(x+3)}{3}-\frac{x}{2}\right]dx=\int_2^3(x-1)dx+\frac{1}{3}\int_3^6(x+3)dx-\frac{1}{2}\int_2^6 xdx \qquad (1)$$

$$=\left[\frac{x^2}{2}-x\right]_2^3+\frac{1}{3}\left[\frac{x^2}{2}+3x\right]_3^6-\frac{1}{4}\left[x^2\right]_2^6 \qquad (1+1/2)$$

= 1 square units (1/2)

Q23. The equation of the line passing through the point(3, -2, 1) and parallel to the given line is

$$\frac{x-3}{2}=\frac{y+2}{-3}=\frac{z-1}{1} \qquad (1)$$

Any point on this line is $(2\lambda+3,-3\lambda-2,\lambda+1)$ (1/2)

If it lies on the plane, we have $3(2\lambda+3)-3\lambda-2-\lambda-1+2=0\Rightarrow\lambda=-4$ (1)

Hence, the point common to the plane and the line is (-5, 10, -3). (1/2)

Hence, the required distance $=\sqrt{(3+5)^2+(-2-10)^2+(1+3)^2}$ units $=4\sqrt{14}$ units (1)

The equation of the line passing through (3, -2, 1) and perpendicular to the plane is

$$\frac{x-3}{3}=\frac{y+2}{1}=\frac{z-1}{-1} \qquad (1/2)$$

Any point on it is $(3\mu+3,\mu-2,-\mu+1)$ (1/2)

If it lies on the plane, we get $3(3\mu+3)+\mu-2+\mu-1+2=0\Rightarrow\mu=\frac{-8}{7}$ (1/2)

The required foot of the perpendicular = $(\frac{-3}{7},\frac{-22}{7},\frac{15}{7})$ (1/2)

OR

Any plane through the line of intersection of the given planes is

$$\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+1+\lambda(\ \vec{r}.(\hat{i}+\hat{j}-2\hat{k}))=0$$
$$\text{or}, \vec{r}.((2+\lambda)\hat{i}+(3+\lambda)\hat{j}+(-1-2\lambda)\hat{k})=-1$$

(2)

If it contains the point (3, -2, -1), we have

$$(3)(2+\lambda)+(-2)(3+\lambda)+(-1)(-1-2\lambda)=-1 \Rightarrow \lambda=\frac{-2}{3}$$

(1)

The required equation of the plane is

$$\vec{r}.((2-\frac{2}{3})\hat{i}+(3-\frac{2}{3})\hat{j}+(-1+\frac{4}{3})\hat{k})=-1 \text{or}, \vec{r}.(4\hat{i}+7\hat{j}+\hat{k})=-3$$ (1)

If θ be the angle between the normals to the two given planes, then θ is the angle between the planes and $\cos\theta=\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}=\frac{2+3+2}{\sqrt{14}\sqrt{6}}=\frac{7}{2\sqrt{21}}$ (2)

Q24. Let us define the following events: E_1 = Two white balls are transferred, E_2 = Two red balls are transferred, E_3 = One red and one white balls are transferred, A = The ball drawn from the Bag II is red (1/2)

$$P(E_1)=\frac{^4C_2}{^9C_2}=\frac{4\times3}{9\times8}$$ (1)

$$P(E_2)=\frac{^5C_2}{^9C_2}=\frac{4\times5}{9\times8}$$ (1)

$$P(E_3)=\frac{^5C_1\times{}^4C_1}{^9C_2}=\frac{4\times5\times2}{9\times8}$$ (1)

$$P(A/E_1) = \frac{3}{8}, \quad P(A/E_2) = \frac{5}{8}, \quad P(A/E_3) = \frac{4}{8} \qquad (\frac{1}{2} + \frac{1}{2} + \frac{1}{2})$$

The required probability, $P(E_3/A)$, by Baye's Theorem,

$$= \frac{P(E_3) \times P(A/E_3)}{P(E_1) \times P(A/E_1) + P(E_2) \times P(A/E_2) + P(E_3) \times P(A/E_3)} \qquad (1/2)$$

= 20/37 (1/2)

OR

Let X represent the random variable. Then X = 0, 1, 2, 3 (1/2)

$$P(X = 0) = P(r = 0) = {}^3C_0 (\frac{1}{6})^0 (\frac{5}{6})^3 = \frac{125}{216} \qquad (1/2)$$

$$P(X = 1) = P(r = 1) = {}^3C_1 (\frac{1}{6})^1 (\frac{5}{6})^2 = \frac{75}{216} \qquad (1/2)$$

$$P(X = 2) = P(r = 2) = {}^3C_2 (\frac{1}{6})^2 (\frac{5}{6})^1 = \frac{15}{216} \qquad (1/2)$$

$$P(x = 3) = p(r = 3) = {}^3C_3 (\frac{1}{6})^3 (\frac{5}{6})^0 = \frac{1}{216} \qquad (1/2)$$

x_i	p_i	$x_i p_i$	$(x_i)^2 p_i$
0	125/216	0	0
1	75/216	75/216	75/216
2	15/216	30/216	60/216
3	1/216	3/216	9/216
Total		1/2	2/3

(2)

$$\text{Mean} = \sum x_i p_i = \frac{1}{2},\ \text{var}(X) = \sum x_i^2 p_i - (\sum x_i p_i)^2 = \frac{2}{3} - \frac{1}{4} = \frac{5}{12} \quad (1)$$

$$\text{Standard deviation } = \sqrt{\text{var}(X)} = \frac{\sqrt{15}}{6} \quad (1/2)$$

Q25. Let the radius of the circular garden be r m and the side of the square garden be x m. Then

$$600 = 2\pi r + 4x \Rightarrow x = \frac{600 - 2\pi r}{4} \quad (1)$$

$$\text{The sum of the areas} = A = \pi r^2 + x^2 \Rightarrow A = \pi r^2 + \left(\frac{600 - 2\pi r}{4}\right)^2 \quad (1)$$

$$\frac{dA}{dr} = 2\pi r + \frac{2}{16}(600 - 2\pi r)(-2\pi) = \frac{\pi}{2}(4r - 300 + \pi r), \frac{dA}{dr} = 0 \Rightarrow r = \frac{300}{\pi + 4} \quad (1)$$

$$\frac{d^2A}{dr^2} = \frac{\pi}{2}(4 + \pi), \left(\frac{d^2A}{dr^2}\right)_{r=\frac{300}{\pi+4}} > 0 \quad (1)$$

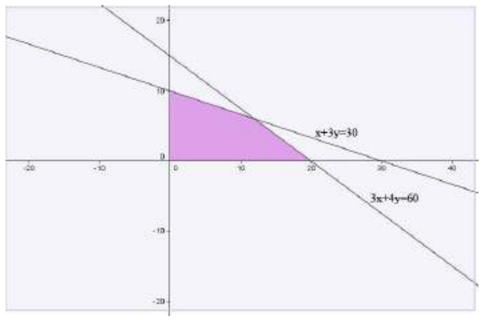
Therefore, A is minimum when $r = \frac{300}{\pi + 4}$ For this value of r, $x = 2r$ (1)

To achieve any goal, there is every possibility that energy, time and money are required to be invested. One must plan in such a manner that least energy, time and money are spent. A good planning and execution, therefore, is essentially required. (1)

Q26. Let the number of pieces of model A to be manufactured be = x and the number of pieces of model B to be manufactured be = y. (1/2)

Then to maximize the profit, P = Rs (8000x+12000y) (1/2)

subject to the constraints $9x + 12y \le 180$, or, $3x + 4y \le 60, x + 3y \le 30, x \ge 0, y \ge 0$ (2)



Graph work (on the actual graph paper) (1+1/2)

At	Profit
(0,0)	Rs 0
(20,0)	Rs 160000
(12,6)	Rs 168000 (maximum)
(0.10)	Rs 120000

(1)

The number of pieces of model A =12, the number of pieces of model B =6 and the maximum profit = Rs 168000. (1/2)