MAXIMUM MARKS 100

Section A – Each question carries 1 mark

1. Write a statement that is a negation of the statement, “Every class has a fan.”

2. Write the following set in set-builder notation: \{\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}\}

3. For the following statement, write the contra-positive: If a quadrilateral is a rhombus, then it is a square.

4. Given the inequality: \(-5 \leq 8 - 5x \leq 15\) where \(x\) belongs to the set of real numbers \(R\), find the solution set in interval notation and represent it on a number line.

5. Two lines are perpendicular. The first one passes through the points (-2,6) and (4,8) the second line passes through the points (8,12) and (x,24). What could be the possible value of the \(x\)?

6. For the following statement, write a statement that could be a possible inverse of it: If you get good marks then you have studied hard.

Section B - Each question carries 4 marks

7. Prove the following trigonometric identity: \(\frac{\cos(2\theta)}{1+\sin(2\theta)} = \cot\left(\frac{\pi}{4} + \theta\right)\)

8. Given that for two sets \(P\) and \(Q\), \(P \cap Q' = \phi\). Using this relationship show that \(P = P \cap Q\). And using that relationship show that \(P \subseteq Q\).

9. \(A = \{-8, -2, 1, 5, 9, 15\}\). The given set \(A\) forms the domain of the function \(f(x) = x^2 - 3x - 6\). Another set \(B = \{80, 4, -8, 4, 44, 172\}\). Is set for \(f(A)\) equal to set \(B\)? If not find \(f(A)\).

10. Given that \(z_1 = 4 - 3i\) and that \(z_2 = 12 + 5i\), prove the following:
    a. \(|z_2 - z_1| > |z_2| - |z_1|\)
    b. \(\frac{|z_1|}{|z_2|} = \frac{|z_1|}{|z_2|}\)

   OR

   For the following complex number, find its conjugate and absolute value:
   \(\frac{(2 + 1)^3}{3 + 2i}\)
11. Find the number of ways in which the alphabets of the word PROBABILITY can be arranged?

12. The numbers, \( x, y \) and \( z \) are in geometric progression. Their sum is 21. The sum of their squares is 189. Find the possible values of \( x, y \) and \( z \).

13. A system of linear inequalities consist of the following inequalities:

\[
\begin{align*}
2x + y &\leq 10 \\
x - y &\leq 8 \\
x &\leq 5 \\
x &\geq -1 \\
y &\geq 0
\end{align*}
\]

Find the solution region for the above system graphically.

14. If \( n \) is a natural number, then using the principle of mathematical induction prove that:

\[
1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \cdots + n(n + 2) = \frac{1}{6}n(n + 1)(2n + 7)
\]

15. The linear equations \( 3x - 2y = -6 \) and \( 4x + 5y = 20 \) represent two sides of a triangle in the xOY co-ordinate plane. The orthocentre of this triangle is at the point (1,1). Find the linear equation that would represent the third side of the triangle.

16. Solve any one of the following trigonometric equations in suitable domain:
   a. \( 3 \tan^2 x + 4 \tan x + 2 = 0 \)
   b. \( 5 \cos x + 12 \sin x = 13 \)

17. The equation of an ellipse is \( 9x^2 + 4y^2 = 144 \). Find the co-ordinates of the vertices of this ellipse. Also find the foci, the eccentricity and the length of the latus rectum of this ellipse. What would be the length of the major axis and the minor axis of this ellipse?

OR

A circle has the equation \( x^2 + y^2 - 8x = 0 \). A chord of the circle lies on the line \( y = x \). Consider another circle whose diameter is this chord of this first circle. Find the equation of this new circle.

18. There are two points in space \( A(1,0,3) \) and \( B(1, -1, 2) \). Find the locus of a point that is equidistant from both these points.

19. Given that the ratio of the \( m^{th} \) and \( n^{th} \) terms of an arithmetic progression is \( (2m - 1):(2n - 1) \) prove that the ratio of the sum of \( m \) terms to that of the sum of \( n \) terms would be \( m^2:n^2 \).

OR

120 degree is the smallest angle of a polygon. If the angles of this polygon form an arithmetic progression such that the common difference between consecutive angles is 5 degrees. How many sides does this polygon have?
Section C – Each question carries 6 marks

20. The ratio 1:3:5 represents the ratio of the coefficients of three consecutive terms of the binomial expansion of the expression $(1 + x)^n$. Find the value of $n$ and the order of the three terms.

21. Use the limit definition to find the first derivative of any one of the following functions:
   a. $f(x) = xe^{2x}$
   b. $f(x) = \frac{x}{\tan(x)}$

22. Consider a triangle ABC, whose sides measure $a$, $b$ and $c$. Prove the following identity:
   
   $$a \sin(B - C) + b \sin(C - A) + c \sin(A - B) = 0$$

23. The arithmetic mean of two positive number is A and their geometric mean is G. A exceeds G by 2. The harmonic mean of the two number is H. This harmonic mean is smaller than the geometric mean by 8/5. What would the two numbers be?

24. The following table gives the data of the amount contributed by 40 students for a class project.

<table>
<thead>
<tr>
<th>Amount in Rs.</th>
<th>Number of students</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 – 30</td>
<td>5</td>
</tr>
<tr>
<td>30 – 40</td>
<td>8</td>
</tr>
<tr>
<td>40 – 50</td>
<td>9</td>
</tr>
<tr>
<td>50 – 60</td>
<td>10</td>
</tr>
<tr>
<td>60 – 70</td>
<td>6</td>
</tr>
<tr>
<td>70 – 80</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
</tr>
</tbody>
</table>

Use the above table to calculate the mean and standard deviation of the data.

25. We are given that $\tan(y) = \frac{5}{12}$ and that $\pi < y < \frac{3\pi}{2}$. Use this information to find the values of $\sin\left(\frac{y}{2}\right)$, $\cos\left(\frac{y}{2}\right)$ and $\tan\left(\frac{y}{2}\right)$.

26. A standard deck of 52 cards is shuffled properly. A card it then drawn randomly from this deck. First describe the sample space of this experiment. Then use that to find the following probabilities:
   a. The card drawn is a heart.
   b. The card drawn is of red colour.
   c. The card drawn is an ace.
   d. The card drawn is a black face card.

OR
100 students are divided into two groups: The tenting group and the cooking group. The tenting group would have 40 students and the cooking group is to have 60 students. Kane and Abel are both among the 100 students. Find the probability that:

1. They are both in the same group, that is, they are both either in tenting group or are both in the cooking group.

2. They are both in different groups. That means, if Kane is in tenting group then Able would be in cooking group or vice versa. But they both are not in the same group.