SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following:

i. The principal solution of the equation $\cot x = -\sqrt{3}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{5\pi}{6}$ (D) $-\frac{5\pi}{6}$

ii. If the vectors $-3\hat{i}+4\hat{j}-2\hat{k}, \hat{i}+2\hat{k}, \hat{i}-p\hat{j}$ are coplanar, then the value of p is

(A) – 2 (B) 1

(C) –1 (D) 2

*iii. If the line $y = x + k$ touches the hyperbola $9x^2 - 16y^2 = 144$, then k = _____

(A) 7 (B) – 7

(C) $\pm\sqrt{7}$ (D) $\pm\sqrt{19}$

(B) Attempt any THREE of the following:

i. Write down the following statements in symbolic form:

a. A triangle is equilateral if and only if it is equiangular.

b. Price increases and demand falls.

ii. If $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$, then find A^{-1} by adjoint method.

iii. Find the separate equations of the lines represented by the equation $3x^2 - 10xy - 8y^2 = 0$.

*iv. Find the equation of the director circle of a circle $x^2 + y^2 = 100$.

v. Find the general solution of the equation $4\cos^2 x = 1$.

Q.2. (A) Attempt any TWO of the following:

i. Without using truth table show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

ii. If θ is the measure of acute angle between the pair of lines given by $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right|$, $a + b \neq 0$.

*iii. Show that the line $x + 2y + 8 = 0$ is tangent to the parabola $y^2 = 8x$. Hence, find the point of contact.

(B) Attempt any TWO of the following:

i. The sum of three numbers is 9. If we multiply third number by 3 and add to the second number, we get 16. By adding the first and the third number and then subtracting twice the second number from this sum, we get 6. Use this information and find the system of linear equations. Hence, find the three numbers using matrices.

ii. Find the general solution of $\cos x + \sin x = 1$.

iii. If $\bar{a}$ and $\bar{b}$ are any two non-zero and non-collinear vectors, then prove that any vector $\bar{r}$ coplanar with $\bar{a}$ and $\bar{b}$ can be uniquely expressed as $\bar{r} = t_1\bar{a} + t_2\bar{b}$, where t_1 and t_2 are scalars.

Q.3. (A) Attempt any TWO of the following:

i. Using truth table examine whether the following statement pattern is tautology, contradiction or contingency.
$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$

ii. Find k, if the length of the tangent segment from (8, – 3) to the circle $x^2 + y^2 - 2x + ky - 23 = 0$ is $\sqrt{10}$ units.

iii. Show that the lines given by
$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1}$ and $\frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4}$ intersect.
Also find the co-ordinates of the point of intersection.

(B) Attempt any TWO of the following:

*i. Find the equation of the locus of the point of intersection of two tangents drawn to the hyperbola $\frac{x^2}{7} - \frac{y^2}{5} = 1$ such that the sum of the cubes of their slopes is 8.

ii. Solve the following L.P.P. graphically:
Maximize : $Z = 10x + 25y$
Subject to: $x \le 3, y \le 3, x + y \le 5, x \ge 0, y \ge 0$

iii. Find the equations of the planes parallel to the plane $x + 2y + 2z + 8 = 0$ which are at the distance of 2 units from the point (1, 1, 2).

Section – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following:

i. Function $f(x) = x^2 - 3x + 4$ has minimum value at $x =$ ______

(A) 0 (B) $-\frac{3}{2}$

(C) 1 (D) $\frac{3}{2}$

ii. $\int \frac{1}{x} \cdot \log x \, dx =$ _____

(A) $\log(\log x) + c$ (B) $\frac{1}{2}(\log x)^2 + c$

(C) $2 \log x + c$ (D) $\log x + c$

iii. Order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7\frac{d^2y}{dx^2} \text{ are respectively –}$$

(A) 2, 3 (B) 3, 2
(C) 7, 2 (D) 3, 7

(B) Attempt any THREE of the following:

i. If $x = at^2$, $y = 2at$, then find $\frac{dy}{dx}$.

ii. Find the approximate value of $\sqrt{8.95}$.

iii. Find the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 3$.

*iv. For the bivariate data r = 0.3, cov(X, Y) = 18, $\sigma_x = 3$, find σ_y.

*v. A triangle bounded by the lines $y = 0$, $y = x$ and $x = 4$ is revolved about the X-axis. Find the volume of the solid of revolution.

Q.5. (A) Attempt any TWO of the following:

i. A function f(x) is defined as

$$f(x) = x + a \quad , \quad x < 0$$
$$= x \quad , \quad 0 \le x < 1$$
$$= b - x \quad , \quad x \ge 1$$

is continuous in its domain. Find a + b.

ii. If $x = a\left(t - \frac{1}{t}\right), y = a\left(t + \frac{1}{t}\right)$, then show that $\frac{dy}{dx} = \frac{x}{y}$.

iii. Evaluate : $\int \frac{1}{3+5\cos x} dx$

(B) Attempt any TWO of the following:

i. An insurance agent insures lives of 5 men, all of the same age and in good health. The probability that a man of this age will survive the next 30 years is known to be $\frac{2}{3}$. Find the probability that in the next 30 years at most 3 men will survive.

ii. The surface area of a spherical balloon is increasing at the rate of 2 cm^2/sec. At what rate is the volume of the balloon is increasing when the radius of the balloon is 6 cm?

iii. The slope of the tangent to the curve at any point is equal to $y + 2x$. Find the equation of the curve passing through the origin.

Q.6. (A) Attempt any TWO of the following:

i. If u and v are two functions of x, then prove that

$$\int uv\, dx = u\int v\, dx - \int\left[\frac{du}{dx}\int v\, dx\right] dx$$

ii. The time (in minutes) for a lab assistant to prepare the equipment for a certain experiment is a random variable X taking values between 25 and 35 minutes with p. d. f.

$f(x) = \frac{1}{10}, 25 \le x \le 35 \;\; = 0$, otherwise.

What is the probability that preparation time exceeds 33 minutes? Also find the c. d. f. of X.

iii. The probability that a certain kind of component will survive a check test is 0.6. Find the probability that exactly 2 of the next 4 tested components survive.

(B) Attempt any TWO of the following:

i. If $ax^2 + 2hxy + by^2 = 0$, show that $\frac{d^2y}{dx^2} = 0$.

ii. Find the area of the region common to the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$.

*iii. For 10 pairs of observations on two variables X and Y, the following data are available:

$\sum(x-2) = 30$, $\sum(y-5) = 40$, $\sum(x-2)^2 = 900$,

$\sum(y-5)^2 = 800$, $\sum(x-2)(y-5) = 480$.

Find the correlation coefficient between X and Y.

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following: (6)[12]

i. If A = {2, 3, 4, 5, 6}, then which of the following is not true?

(A) $\exists\, x \in A$ such that $x + 3 = 8$ (B) $\exists\, x \in A$ such that $x + 2 < 5$
(C) $\exists\, x \in A$ such that $x + 2 < 9$ (D) $\forall\, x \in A$ such that $x + 6 \geq 9$

ii. If $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$, then the value of k is

(A) $\frac{1}{2}$ (B) $\frac{11}{2}$
(C) $\frac{5}{2}$ (D) $\frac{-11}{2}$

iii. If a line is inclined at 60° and 30° with the X and Y-axes respectively, then the angle which it makes with Z-axis is

(A) 0 (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

(B) Attempt any THREE of the following:

i. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and AX = I, then find X by using elementary transformations.

ii. With usual notations, in ΔABC, prove that $a(b \cos C - c \cos B) = b^2 - c^2$.

*iii. Show that the equation of a tangent to the circle $x^2 + y^2 = a^2$ at the point $P(x_1, y_1)$ on it is $xx_1 + yy_1 = a^2$.

*iv. Find k, if the line $2x - 3y + k = 0$ touches the ellipse $5x^2 + 9y^2 = 45$.

v. Find the co-ordinates of the point, which divides the line segment joining the points A(2, – 6, 8) and B(– 1, 3, – 4) externally in the ratio 1 : 3.

Q.2. (A) Attempt any TWO of the following:

i. Using truth table, prove that $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

ii. Find the values of p and q, if the following equation represents a pair of perpendicular lines:
$px^2 - 8xy + 3y^2 + 14x + 2y + q = 0$.

*iii. Find the equations of tangents to the parabola $y^2 = 12x$ from the point (2, 5).

(B) Attempt any TWO of the following:

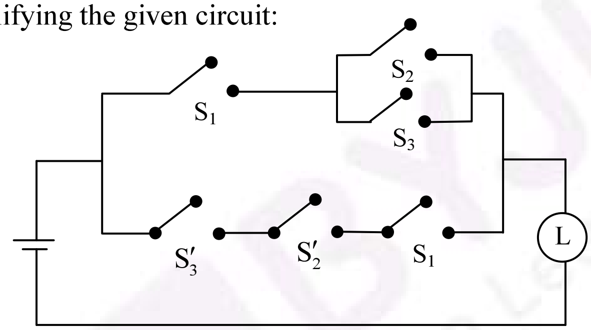
i. The cost of 2 books, 6 notebooks and 3 pens is ` 40. The cost of 3 books, 4 notebooks and 2 pens is ` 35, while the cost of 5 books, 7 notebooks and 4 pens is ` 61. Using this information and matrix method, find the cost of 1 book, 1 notebook and 1 pen separately.

ii. Prove that $\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)=\cos^{-1}\left(-\frac{1}{2}\right)$.

*iii. Show that the product of lengths of perpendicular segments drawn from the foci to any tangent to the hyperbola $\frac{x^2}{25}-\frac{y^2}{16}=1$ is equal to 16.

Q.3. (A) Attempt any TWO of the following:

i. Construct the new switching circuit for the following circuit with only one switch by simplifying the given circuit:



*ii. Find the locus of a point, the tangents from which to the circle $x^2 + y^2 = a^2$ are mutually perpendicular.

iii. Find the shortest distance between the lines

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \text{ and } \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}.$$

(B) Attempt any TWO of the following:

i. Find the angle between the line $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z+2}{4}$ and the plane $2x + y - 3z + 4 = 0$.

ii. Solve the following L. P. P. graphically:

Minimize $Z = 6x + 2y$

Subject to

$5x + 9y \leq 90$

$x + y \geq 4$

$y \leq 8$

$x \geq 0, y \geq 0$

iii. Find the volume of a tetrahedron whose vertices are

A(–1, 2, 3), B(3, –2, 1), C(2, 1, 3) and D(–1, –2, 4).

SECTION - II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following:]

i. If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ = ______

(A) $\frac{1+x}{1+\log x}$ (B) $\frac{\log x}{(1+\log x)^2}$ (C) $\frac{1-\log x}{1+\log x}$ (D) $\frac{1-x}{1+\log x}$

ii. $\int \frac{1}{1+\cos x}dx$ = _______

(A) $\tan\left(\frac{x}{2}\right)$ + c (B) $2\tan\left(\frac{x}{2}\right)$ + c

(C) $-\cot\left(\frac{x}{2}\right)$ + c (D) $-2\cot\left(\frac{x}{2}\right)$ + c

iii. If X ~ B (n, p) and E(X) = 12, Var(X) = 4, then the value of n is _______

(A) 3 (B) 48 (C) 18 (D) 36

(B) Attempt any THREE of the following:

i. Find the equation of tangent to the curve $y = 3x^2 - x + 1$ at P(1, 3).

ii. Evaluate: $\int \frac{1}{x(x-1)}dx$

iii. Solve the differential equation $y - x\frac{dy}{dx} = 0$.

*iv. In a bivariate data, n = 10, $\bar{x} = 25$, $\bar{y} = 30$ and $\sum xy = 7900$.
Find cov(X, Y).

*v. A random variable X ~ N (0, 1). Find P(X > 0) and P(X < 0).

Q.5. (A) Attempt any TWO of the following:

i. Examine the function for maximum and minimum $f(x) = x^3 - 9x^2 + 24x$.

ii. If $y = f(x)$ is a differentiable function of x such that inverse function $x = f^{-1}(y)$ exists, then prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$, where $\frac{dy}{dx} \neq 0$.

iii. The probability distribution of X, the number of defects per 10 metres of a fabric is given by

x	0	1	2	3	4
P(X = x)	0.45	0.35	0.15	0.03	0.02

Find the variance of X.

(B) Attempt any TWO of the following:

i. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, show that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

ii. Solve the differential equation $\cos^2 x\frac{dy}{dx} + y = \tan x$.

iii. Find the area of the region bounded by the curves $y^2 = 4x$ and $4x^2 + 4y^2 = 9$ with $x \geq 0$.

Q.6. (A) Attempt any TWO of the following:

i. Find the approximate value of $\tan^{-1}(1.001)$.

ii. Examine continuity of the function f(x) at $x = 0$, where

$$f(x) = \frac{10^x + 7^x - 14^x - 5^x}{1 - \cos 4x}, \quad \text{for } x \neq 0$$

$$= \frac{10}{7}, \quad \text{for } x = 0$$

iii. The probability that a person who undergoes a kidney operation will be recovered is 0.5. Find the probability that of the 6 patients who undergo similar operations:

(a) none will recover

(b) half of them will recover.

(B) Attempt any TWO of the following:

i. Prove that:

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log\left| x + \sqrt{x^2 + a^2} \right| + c$$

*ii. Find the volume of the solid generated, when the area between ellipse $4x^2 + 9y^2 = 36$ and the chord AB, with A ≡ (3, 0), B ≡ (0, 2), is revolved about X-axis.

*iii. Find Karl Pearson's coefficient of correlation between the variables X and Y for the following data:

X	11	7	9	5	8	6	10
Y	10	8	6	5	9	7	11

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following:

i. Which of the following represents direction cosines of the line?

(A) $0, \frac{1}{\sqrt{2}}, \frac{1}{2}$ (B) $0, \frac{-\sqrt{3}}{2}, \frac{1}{\sqrt{2}}$

(C) $0, \frac{\sqrt{3}}{2}, \frac{1}{2}$ (D) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$

ii. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and A (adj A) = KI, then the value of 'K' is _______.

(A) 2 (B) – 2
(C) 10 (D) – 10

iii. The general solution of the trigonometric equation $\tan^2\theta = 1$ is _______.

(A) $\theta = n\pi \pm \frac{\pi}{3}, n \in Z$ (B) $\theta = n\pi \pm \frac{\pi}{6}, n \in Z$

(C) $\theta = n\pi \pm \frac{\pi}{4}, n \in Z$ (D) $\theta = n\pi, n \in Z$

(B) Attempt any THREE of the following:

i. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$, then find the ratio in which the point C divides the line segment AB.

ii. The cartesian equation of a line is $\frac{x-6}{2} = \frac{y+4}{7} = \frac{z-5}{3}$, find its vector equation.

iii. Equation of a plane is $\bar{r} \cdot (3\hat{i} - 4\hat{j} + 12\hat{k}) = 8$. Find the length of the perpendicular from the origin to the plane.

iv. Find the acute angle between the lines whose direction ratios are 5, 12, – 13 and 3, – 4, 5.

v. Write the dual of the following statements:

a. $(p \vee q) \wedge T$

b. Madhuri has curly hair and brown eyes.

Q.2. (A) Attempt any TWO of the following:

i. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k.

ii. Prove that three vectors $\bar{a}$, $\bar{b}$ and $\bar{c}$ are coplanar, if and only if, there exists a non-zero linear combination $x\bar{a}+y\bar{b}+z\bar{c}=\bar{0}$.

iii. Using truth table, prove that
$\sim p \wedge q \equiv (p \vee q) \wedge \sim p$.

(B) Attempt any TWO of the following:

i. In any ΔABC, with usual notations, prove that
$b^2 = c^2 + a^2 - 2ca\cos B$.

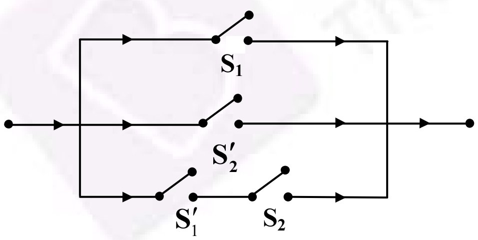
ii. Show that the equation $x^2 - 6xy + 5y^2 + 10x - 14y + 9 = 0$ represents a pair of lines. Find the acute angle between them. Also find the point of intersection of the lines.

iii. Express the following equations in the matrix form and solve them by the method of reduction:
$2x - y + z = 1,\ x + 2y + 3z = 8,\ 3x + y - 4z = 1$.

Q.3. (A) Attempt any TWO of the following:

i. Prove that a homogeneous equation of degree two in x and y (i.e., $ax^2 + 2hxy + by^2 = 0$), represents a pair of lines through the origin if $h^2 - ab \geq 0$.

ii. Find the symbolic form of the following switching circuit, construct its switching table and interpret it.



iii. If A, B, C, D are (1, 1, 1), (2, 1, 3), (3, 2, 2), (3, 3, 4) respectively, then find the volume of the parallelopiped with AB, AC and AD as the concurrent edges.

(B) Attempt any TWO of the following:

i. Find the equation of the plane passing through the line of intersection of the planes
$2x - y + z = 3,\ 4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z-3}{5}$.

ii. Minimize: $Z = 6x + 4y$
Subject to: $3x + 2y \geq 12$,
$x + y \geq 5$,
$0 \leq x \leq 4$,
$0 \leq y \leq 4$.

iii. Show that: $\cos^{-1}\left(\frac{4}{5}\right)+\cos^{-1}\left(\frac{12}{13}\right)=\cos^{-1}\left(\frac{33}{65}\right)$.

SECTION – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following:

i. If $y = 1 - \cos\theta$, $x = 1 - \sin\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is

(A) -1 (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

ii. The integrating factor of linear differential equation

$\frac{dy}{dx} + y\sec x = \tan x$ is

(A) $\sec x - \tan x$ (B) $\sec x \cdot \tan x$

(C) $\sec x + \tan x$ (D) $\sec x \cdot \cot x$

iii. The equation of tangent to the curve $y = 3x^2 - x + 1$ at the point (1, 3) is

(A) $y = 5x + 2$ (B) $y = 5x - 2$

(C) $y = \frac{1}{5}x + 2$ (D) $y = \frac{1}{5}x - 2$

(B) Attempt any THREE of the following:

i. Examine the continuity of the function

$$f(x) = \sin x - \cos x, \quad \text{for } x \neq 0$$
$$= -1 \quad , \quad \text{for } x = 0$$

at the point $x = 0$.

ii. Verify Rolle's theorem for the function

$f(x) = x^2 - 5x + 9$ on [1, 4].

iii. Evaluate: $\int \sec^n x \cdot \tan x \, dx$

iv. The probability mass function (p.m.f.) of X is given below:

$X = x$	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Find $E(X^2)$.

v. Given that X ~ B (n = 10, p). If E (X) = 8, find the value of p.

Q.5. (A) Attempt any TWO of the following:

i. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then prove that $y = f[g(x)]$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

ii. Obtain the differential equation by eliminating the arbitrary constants A, B from the equation:

$y = A\cos(\log x) + B\sin(\log x)$

iii. Evaluate: $\int \frac{x^2}{(x^2+2)(2x^2+1)} dx$

(B) Attempt any TWO of the following:

i. An open box is to be made out of a piece of a square cardboard of sides 18 cms by cutting off equal squares from the corners and turning up the sides. Find the maximum volume of the box.

ii. Prove that: $\int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(2a-x)dx$

iii. If the function f(x) is continuous in the interval [–2, 2], find the values of a and b, where

$$f(x) = \frac{\sin ax}{x} - 2, \quad \text{for } -2 \le x < 0$$
$$= 2x + 1, \quad \text{for } 0 \le x \le 1$$
$$= 2b\sqrt{x^2+3} - 1, \quad \text{for } 1 < x \le 2$$

Q.6. (A) Attempt any TWO of the following:

i. Solve the differential equation: $\frac{dy}{dx} = \frac{y + \sqrt{x^2+y^2}}{x}$.

ii. A fair coin is tossed 8 times. Find the probability that it shows heads at least once.

iii. If $x^p y^q = (x+y)^{p+q}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

(B) Attempt any TWO of the following:

i. Find the area of the sector of a circle bounded by the circle $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant.

ii. Prove that:

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + c$$

iii. A random variable X has the following probability distribution:

$X = x$	0	1	2	3	4	5	6
$P[X = x]$	k	3k	5k	7k	9k	11k	13k

a. Find k.
b. Find $P(0 < X < 4)$.
c. Obtain cumulative distribution function (c.d.f.) of X.

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following sub-questions:

i. If $\left[\bar{a} \quad \bar{b} \quad \bar{c}\right] \neq 0$ and $\bar{p} = \dfrac{\bar{b} \times \bar{c}}{\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]}$, $\bar{q} = \dfrac{\bar{c} \times \bar{a}}{\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]}$, $\bar{r} = \dfrac{\bar{a} \times \bar{b}}{\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]}$, then $\bar{a}.\bar{p} + \bar{b}.\bar{q} + \bar{c}.\bar{r}$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3

ii. The inverse of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$ (D) $-\frac{1}{2}\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iii. Direction cosines of the line passing through the points A(–4, 2, 3) and B(1, 3, –2) are

(A) $\pm\frac{1}{\sqrt{51}}, \pm\frac{5}{\sqrt{51}}, \pm\frac{1}{\sqrt{51}}$ (B) $\pm\frac{5}{\sqrt{51}}, \pm\frac{1}{\sqrt{51}}, \pm\frac{-5}{\sqrt{51}}$

(C) ± 5, ± 1, ± 5 (D) $\pm\sqrt{51}, \pm\sqrt{51}, \pm\sqrt{51}$

(B) Attempt any THREE of the following:

i. Write truth values of the following statements:

a. $\sqrt{5}$ is an irrational number but 3 + $\sqrt{5}$ is a complex number

b. $\exists n \in N$ such that n + 5 > 10

ii. If $\bar{c} = 3\bar{a} - 2\bar{b}$, then prove that $\left[\bar{a} \quad \bar{b} \quad \bar{c}\right] = 0$

iii. Find the vector equation of the plane which is at a distance of 5 units from the origin and which is normal to the vector $2\hat{i}+\hat{j}+2\hat{k}$.

iv. The Cartesian equations of the line are:
$3x + 1 = 6y - 2 = 1 - z$.
Find its equation in vector form.

v. Find the direction ratios of a vector perpendicular to the two lines whose direction ratios are $-2, 1, -1$ and $-3, -4, 1$.

Q.2. (A) Attempt any TWO of the following:

i. Using truth table, prove the following logical equivalence $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$.

ii. Find the joint equation of the pair of lines through the origin each of which is making an angle of 30° with the line $3x + 2y - 11 = 0$.

iii. Show that $2\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{24}{7}\right)$.

(B) Attempt any TWO of the following:

i. Solve the following equations by the method of reduction:
$2x - y + z = 1$, $x + 2y + 3z = 8$, $3x + y - 4z = 1$.

ii. Show that volume of parallelopiped with coterminous edges as $\bar{a}, \bar{b}$ and $\bar{c}$ is $[\bar{a} \ \bar{b} \ \bar{c}]$, hence find the volume of the parallelopiped whose coterminous edges are $\hat{i}+\hat{j}, \hat{j}+\hat{k}, \hat{k}+\hat{i}$.

iii. Solve the following LPP by using graphical method.
Maximize: $Z = 6x + 4y$,
Subject to $x \le 2$, $x + y \le 3$, $-2x + y \le 1$, $x \ge 0$, $y \ge 0$.
Also find maximum value of Z.

Q.3. (A) Attempt any TWO of the following:

i. In ΔABC with usual notations, prove that
$2\left\{a\sin^2\frac{C}{2} + c\sin^2\frac{A}{2}\right\} = (a + c - b)$

ii. If p : It is a day time, q : It is warm, write the compound statements in verbal form denoted by–
a. $p \wedge \sim q$
b. $\sim p \rightarrow q$
c. $q \leftrightarrow p$

iii. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other, then find the value of k.

(B) Attempt any TWO of the following:

i. Parametric form of the equation of the plane is
$\bar{r} = (2\hat{i}+\hat{k}) + \lambda\hat{i} + \mu(\hat{i}+2\hat{j}-3\hat{k})$.
λ and μ are parameters. Find normal to the plane and hence equation of the plane in normal form. Write its cartesian form.

ii. If the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines $2x^2 - 5xy + 3y^2 = 0$, then show that $100(h^2 - ab) = (a + b)^2$.

iii. Find the general solution of $\sin x \tan x = \tan x - \sin x + 1$.

SECTION – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following sub-questions:

i. The differential equation of the family of curves $y = c_1e^x + c_2e^{-x}$ is

(A) $\frac{d^2y}{dx^2} + y = 0$ (B) $\frac{d^2y}{dx^2} - y = 0$

(C) $\frac{d^2y}{dx^2} + 1 = 0$ (D) $\frac{d^2y}{dx^2} + 1 = 0$

ii. If X is a random variable with probability mass function

$P(x) = kx$, for $x = 1, 2, 3$

$= 0$, otherwise

then k = ……

(A) $\frac{1}{5}$ (B) $\frac{1^\circ}{4}$

(C) $\frac{1}{6}$ (D) $\frac{2}{3}$

iii. If $\sec\left(\frac{x+y}{x-y}\right) = a^2$, then $\frac{d^2y}{dx^2} = ….$

(A) y (B) x

(C) $\frac{y}{x}$ (D) 0

(B) Attempt any THREE of the following:

i. If $y = \sin^{-1}(3x) + \sec^{-1}\left(\frac{1}{3x}\right)$, find $\frac{dy}{dx}$.

ii. Evaluate: $\int x \log x \, dx$.

iii. If $\int_0^h \frac{1}{2+8x^2} dx = \frac{\pi}{16}$, then find the value of h.

iv. The probability that a certain kind of component will survive a check test is 0.5. Find the probability that exactly two of the next four components tested will survive.

v. Find the area of the region bounded by the curve $y = \sin x$, the lines $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$ and X-axis.

Q.5. (A) Attempt any TWO of the following:

i. Examine the continuity of the following function at given point:

$$f(x) = \frac{\log x - \log 8}{x - 8}, \text{ for } x \neq 8$$
$$= 8 \qquad , \text{ for } x = 8 \text{ at } x = 8$$

ii. If $x = \phi(t)$ is a differentiable function of 't', then prove that

$\int f(x)dx = \int f[\phi(t)]\phi'(t)dt$.

iii. Solve $3e^x \tan y \, dx + (1 + e^x) \sec^2 y \, dy = 0$. Also, find the particular solution when $x = 0$ and $y = \pi$.

(B) Attempt any TWO of the following:

i. A point source of light is hung 30 feet directly above a straight horizontal path on which a man of 6 feet in height is walking. How fast will the man's shadow lengthen and how fast will the tip of shadow move when he is walking away from the light at the rate of 100 ft/min.

ii. Evaluate: $\int \frac{\log x}{(1+\log x)^2} dx$

iii. If x = f(t), y = g(t) are differentiable functions of parameter 't' then prove that y is a differentiable function of 'x' and

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \frac{dx}{dt} \neq 0$$

Hence find $\frac{dy}{dx}$ if $x = a \cos t$, $y = a \sin t$.

Q.6. (A) Attempt any TWO of the following:

i. Show that the function defined by $f(x) = |\cos x|$ is continuous function.

ii. Solve the differential equation $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$.

iii. Given X ~ B(n, p). If n = 20, E(X) = 10,
find p, Var. (X) and S.D.(X).

(B) Attempt any TWO of the following:

i. A bakerman sells 5 types of cakes. Profits due to the sale of each type of cake is respectively ` 3, ` 2.5, ` 2, ` 1.5, ` 1. The demands for these cakes are 10%, 5%, 25%, 45% and 15% respectively. What is the expected profit per cake?

ii. Verify Lagrange's mean value theorem for the function

$f(x) = x + \frac{1}{x}, x \in [1, 3]$

iii. Prove that: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Hence evaluate: $\int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx$.

SECTION – I

Q.1. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions:

i. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^6 =$ _______.

(A) 6A (B) 12A (C) 16A (D) 32A

ii. The principal solution of $\cos^{-1}\left(-\frac{1}{2}\right)$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{2}$

iii. If an equation $hxy + gx + fy + c = 0$ represents a pair of lines, then

(A) fg = ch (B) gh = cf (C) fh = cg (D) hf = – cg

(B) Attempt any THREE of the following:

i. Write the converse and contrapositive of the statement-
"If two triangles are congruent then their areas are equal."

ii. Find 'k', if the sum of slopes of lines represented by equation $x^2 + kxy - 3y^2 = 0$ is twice their product.

iii. Find the angle between the planes $\bar{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) = 3$ and $\bar{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = 1$.

iv. The cartesian equations of line are $3x - 1 = 6y + 2 = 1 - z$. Find the vector equation of line.

v. If $\bar{a} = \hat{i} + 2\hat{j}$, $\bar{b} = -2\hat{i} + \hat{j}$, $\bar{c} = 4\hat{i} + 3\hat{j}$, find x and y such that $\bar{c} = x\bar{a} + y\bar{b}$.

Q.2. (A) Attempt any TWO of the following:

i. If A, B, C, D are (1, 1, 1), (2, 1, 3), (3, 2, 2), (3, 3, 4) respectively, then find the volume of the parallelopiped with AB, AC and AD as the concurrent edges.

ii. Discuss the statement pattern, using truth table: $\sim(\sim p \wedge \sim q) \vee q$

iii. If point C($\bar{c}$) divides the segment joining the points A($\bar{a}$) and B ($\bar{b}$) internally in the ratio m : n, then prove that $\bar{c} = \frac{m\bar{b} + n\bar{a}}{m + n}$.

(B) Attempt any TWO of the following:

i. Find the direction cosines of the line perpendicular to the lines whose direction ratios are –2, 1, –1 and –3, –4, 1.

ii. In any ΔABC, if a^2, b^2, c^2 are in arithmetic progression, then prove that cot A, cot B, cot C are in arithmetic progression.

iii. The sum of three numbers is 6. When second number is subtracted from thrice the sum of first and third number, we get number 10. Four times the third number is subtracted from five times the sum of first and second number, the result is 3. Using above information, find these three numbers by matrix method.

Q.3. (A) Attempt any TWO of the following:

i. If θ is the acute angle between the lines represented by equation $ax^2 + 2hxy + by^2 = 0$, then prove that $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$, $a + b \neq 0$.

ii. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other, then find the value of 'k'.

iii. Construct the switching circuit for the following statement:
$[p \vee (\sim p \wedge q)] \vee [(\sim q \wedge r) \vee \sim p]$

(B) Attempt any TWO of the following:

i. Find the general solution of $\cos x - \sin x = 1$.

ii. Find the equations of the planes parallel to the plane $x - 2y + 2z - 4 = 0$, which are at a unit distance from the point (1, 2, 3).

iii. A diet of a sick person must contain at least 48 units of vitamin A and 64 units of vitamin B. Two foods F_1 and F_2 are available. Food F_1 costs ` 6 per unit and food $F_2$ costs ` 10 per unit. One unit of food F_1 contains 6 units of vitamin A and 7 units of vitamin B. One unit of food F_2 contains 8 units of vitamin A and 12 units of vitamin B. Find the minimum cost for the diet that consists of mixture of these two foods and also meeting the minimal nutritional requirements.

SECTION – II

Q.4. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions:

i. A random variable X has the following probability distribution:

$X = x$	–2	–1	0	1	2	3
$P(x)$	0.1	0.1	0.2	0.2	0.3	0.1

Then $E(x) =$

(A) 0.8 (B) 0.9
(C) 0.7 (D) 1.1

ii. If $\int_0^\alpha 3x^2\,dx = 8$, then the value of α is

(A) 0 (B) –2
(C) 2 (D) ±2

iii. The differential equation of $y = \frac{c}{x} + c^2$ is

(A) $x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = y$ (B) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(C) $x^3\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = y$ (D) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

(B) Attempt any THREE of the following:

i. Evaluate: $\int e^x\left[\frac{\sqrt{1-x^2}\cdot\sin^{-1}x+1}{\sqrt{1-x^2}}\right]dx$

ii. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots\infty}}}$, then show that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

iii. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x}\,dx$

iv. If $y = e^{ax}$, show that $x\frac{dy}{dx} = y\log y$.

v. A fair coin is tossed five times. Find the probability that it shows exactly three times head.

Q.5. (A) Attempt any TWO of the following:

i. Integrate: $\sec^3 x$ w.r.t. x.

ii. If $y = (\tan^{-1}x)^2$, show that

$(1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} - 2 = 0$

iii. If $f(x) = \left[\tan\left(\frac{\pi}{4}+x\right)\right]^{\frac{1}{x}}$, for $x \neq 0$

$= k$, for $x = 0$

is continuous at $x = 0$, find k.

(B) Attempt any TWO of the following:

i. Find the co-ordinates of the points on the curve $y = x - \frac{4}{x}$, where the tangents are parallel to the line $y = 2x$.

ii. Prove that: $\int\sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$

iii. Evaluate: $\int_0^{\pi}\frac{x\sin x}{1+\sin x}dx$

Q.6. (A) Attempt any TWO of the following:

i. Find a and b, so that the function f(x) defined by

$f(x) = -2\sin x$, for $-\pi \le x \le -\frac{\pi}{2}$

$= a\sin x + b$, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$= \cos x$, for $\frac{\pi}{2} \le x \le \pi$ is continuous on $[-\pi, \pi]$.

ii. If $\log_{10}\left(\frac{x^3 - y^3}{x^3 + y^3}\right) = 2$, then show that $\frac{dy}{dx} = -\frac{99x^2}{101y^2}$.

iii. Let the p.m.f. (probability mass function) of random variable x be

$P(x) = \binom{4}{x}\left(\frac{5}{9}\right)^x \left(\frac{4}{9}\right)^{4-x}$, $x = 0, 1, 2, 3, 4$.

$= 0$, otherwise

Find $E(x)$ and $Var(x)$.

(B) Attempt any TWO of the following:

i. Examine the maxima and minima of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$. Also, find the maximum and minimum values of $f(x)$.

ii. Solve the differential equation $(x^2 + y^2)dx - 2xydy = 0$.

iii. Given the p.d.f. (probability density function) of a continuous random variable x as:

$f(x) = \frac{x^2}{3}$, $-1 < x < 2$

$= 0$, otherwise

Determine the c.d.f. (cumulative distribution function) of x and hence find $P(x < 1)$, $P(x \le -2)$, $P(x > 0)$, $P(1 < x < 2)$.

SECTION –I

Q.1. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions:

i. If $p \wedge q = F$, $p \rightarrow q = F$, then the truth value of p and q is :

(A) T, T (B) T, F

(C) F, T (D) F, F

ii. If $A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$ and $|A| = 3$, then (adj. A) = ________

(A) $\frac{1}{9}\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -5 & -2 \\ -2 & 4 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 4 & -2 \\ -2 & -5 & 4 \\ 1 & -2 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & -4 & 2 \\ 2 & 5 & -4 \\ 1 & -2 & 1 \end{bmatrix}$

iii. The slopes of the lines given by $12x^2 + bxy - y^2 = 0$ differ by 7. Then the value of b is :

(A) 2 (B) ± 2

(C) ± 1 (D) 1

(B) Attempt any THREE of the following:

i. In a Δ ABC, with usual notations prove that:

$$\frac{a - b\cos C}{b - a\cos C} = \frac{\cos B}{\cos A}$$

ii. Find 'k', if the equation $kxy + 10x + 6y + 4 = 0$ represents a pair of straight lines.

iii. If A, B, C, D are four non-collinear points in the plane such that $\overline{AD} + \overline{BD} + \overline{CD} = \overline{O}$, then prove that point D is the centroid of the ΔABC.

iv. Find the direction cosines of the line

$$\frac{x+2}{2} = \frac{2y-5}{3}; \; z = -1.$$

v. Show that the points (1, 1, 1) and (–3, 0, 1) are equidistant from the plane $\hat{r}.\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0$.

Q.2. (A) Attempt any TWO of the following:

i. Using truth table prove that $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$.

ii. Prove that a homogeneous equation of second degree, $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines passing through the origin, if $h^2 - ab \geq 0$.

iii. Prove that the volume of a parallelopiped with coterminal edges as $\bar{a}, \bar{b}, \bar{c}$ is $[\bar{a}, \bar{b}, \bar{c}]$. Hence find the volume of the parallelopiped with coterminal edges $\bar{i}+\bar{j}, \bar{j}+\bar{k}$ and $\bar{k}+\bar{i}$.

(B) Attempt any TWO of the following:

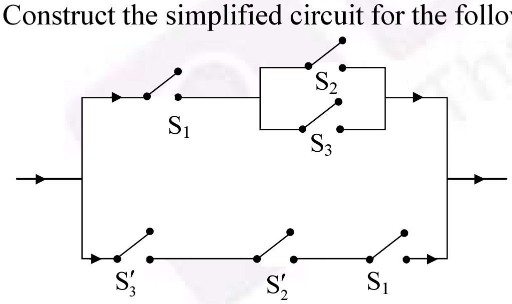
i. Find the inverse of the matrix, $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by using column transformations.

ii. In ΔABC, prove that : $\tan\frac{(A-B)}{2} = \left(\frac{a-b}{a+b}\right).\cot\frac{C}{2}$.

iii. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$; and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar. Find the equation of the plane containing them.

Q.3. (A) Attempt any TWO of the following:

i. Construct the simplified circuit for the following circuit:



ii. Express $-\hat{i}-3\hat{j}+4\hat{k}$ as a linear combination of vectors $2\hat{i}+\hat{j}-4\hat{k}, 2\hat{i}-\hat{j}+3\hat{k}$ and $3\hat{i}+\hat{j}-2\hat{k}$.

iii. Find the length of the perpendicular from the point (3, 2, 1) to the line $\frac{x-7}{-2} = \frac{y-7}{2} = \frac{z-6}{3}$.

(B) Attempt any TWO of the following:

i. Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

ii. Minimize : $Z = 6x + 4y$
Subject to the conditions:
$3x + 2y \geq 12$,
$x + y \geq 5$,
$0 \leq x \leq 4$,
$0 \leq y \leq 4$

iii. If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \cot^{-1}\left(\frac{x+2}{x+1}\right) = \frac{\pi}{4}$; find x.

SECTION – II

Q.4. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions:

i. If $y = \sec^{-1}\left(\frac{\sqrt{x}-1}{x+\sqrt{x}}\right) + \sin^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right)$, then $\frac{dy}{dx} = \ldots$

(A) x (B) $\frac{1}{x}$

(C) 1 (D) 0

ii. If $I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$, then the value of I is:

(A) 0 (B) π

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

iii. The solution of the differential equation $\frac{dy}{dx} = \sec x - y \tan x$ is:

(A) $y \sec x = \tan x + c$ (B) $y \sec x + \tan x = c$

(C) $\sec x = y \tan x + c$ (D) $\sec x + y \tan x = c$

(B) Attempt any THREE of the following:

i. Evaluate: $\int \frac{1}{x \log x \log(\log x)} dx$

ii. Find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.

iii. Find k, such that the function

$$P(x) = \begin{cases} k\binom{4}{x}; x = 0,1,2,3,4. k > 0 \\ 0 \quad ;\text{otherwise;} \end{cases}$$

Is a probability mass function (p.m.f.).

iv. Given is X ~ B (n, p). If E(X) = 6, and Var(X) = 4.2, find the value of n.

v. Solve the differential equation $y - x\frac{dy}{dx} = 0$

Q.5. (A) Attempt any TWO of the following:

i. Discuss the continuity of the function

$$f(x) = \frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}, \text{ for } x \neq \frac{\pi}{2}$$

$$= 3, \quad \text{for } x = \frac{\pi}{2},$$

at $x = \frac{\pi}{2}$

ii. If $f'(x) = k(\cos x - \sin x), f'(0) = 3$ and $f\left(\frac{\pi}{2}\right) = 15$,

find $f'(x)$.

iii. Differentiate $\cos^{-1}\left(\frac{3\cos x - 2\sin x}{\sqrt{13}}\right)$ w. r. t. x.

(B) Attempt any TWO of the following:

i. Show that: $\int \frac{1}{x^2\sqrt{a^2+x^2}}\,dx = \frac{-1}{a^2}\frac{\sqrt{a^2+x^2}}{x} + c$

ii. A rectangle has area 50 cm^2. Find its dimensions when its perimeter is the least.

iii. Prove that : $\int_{-a}^{a} f(x)\,dx = 2.\int_{0}^{a} f(x)\,dx$, if $f(x)$ is an even function.

$= 0$, if $f(x)$ is an odd function.

Q.6. (A) Attempt any TWO of the following:

i. If $y = f(u)$ is a differential function of u and $u = g(x)$ is a differential function of x, then prove that $y = f[g(x)]$ is a differential function of x and $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$

ii. Each of the total five questions in a multiple choice examination has four choices, only one of which is correct. A student is attempting to guess the answer. The random variable x is the number of questions answered correctly. What is the probability that the student will give atleast one correct answer?

iii. If $f(x) = x^2 + a$, for $x \geq 0$

$= 2\sqrt{x^2+1} + b$, for $x < 0$ and $f\left(\frac{1}{2}\right) = 2$,

is continuous at $x = 0$, find a and b.

(B) Attempt any TWO of the following:

i. Find the approximate value of cos (89°, 30′). [Given is: 1° = 0·0175°C]

ii. Solve the differential equation:

$x + y\frac{dy}{dx} = \sec(x^2 + y^2)$. Also find the particular solution if $x = y = 0$.

iii. Find the expected value, variance and standard deviation of random variable X whose probability mass function (p.m.f.) is given below:

$X = x$	1	2	3
$P(X = x)$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

SECTION – I

Q.1. (A) Select and write the correct answer from the given alternatives in each of the following sub-questions:

i. Inverse of the statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is

(A) $(p \wedge q) \rightarrow (p \vee q)$ (B) $\sim(p \vee q) \rightarrow (p \wedge q)$
(C) $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$ (D) $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$

ii. If the vectors $2\hat{i} - q\hat{j} + 3\hat{k}$ and $4\hat{i} - 5\hat{j} + 6\hat{k}$ are collinear, then value of q is

(A) 5 (B) 10 (C) $\frac{5}{2}$ (D) $\frac{5}{4}$

iii. If in ΔABC with usual notations a = 18, b = 24, c = 30 then $\sin\frac{A}{2}$ is equal to

(A) $\frac{1}{\sqrt{5}}$ (B) $\frac{1}{\sqrt{10}}$ (C) $\frac{1}{\sqrt{15}}$ (D) $\frac{1}{2\sqrt{5}}$

(B) Attempt any THREE of the following:

i. Find the angle between the lines
$\bar{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and
$\bar{r} = 5\hat{i} - 2\hat{k} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

ii. If p, q, r are the statements with truth values T, F, T, respectively then find the truth value of $(r \wedge q) \leftrightarrow \sim p$

iii. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 5 \end{bmatrix}$ then find A^{-1} by adjoint method.

iv. By vector method show that the quadrilateral with vertices A (1, 2, –1), B (8, –3, –4), C (5, –4, 1), D (–2, 1, 4) is a parallelogram.

v. Find the general solution of the equation $\sin x = \tan x$.

Q.2. (A) Attempt any TWO of the following:

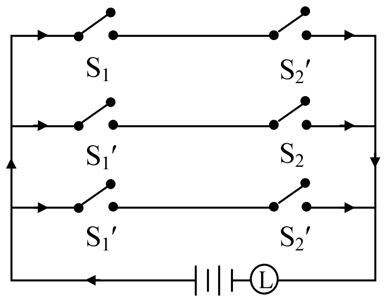
i. Find the joint equation of pair of lines passing through the origin and perpendicular to the lines represented by $ax^2 + 2hxy + by^2 = 0$

ii. Find the principal value of $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

iii. Find the cartesian form of the equation of the plane $\bar{r} = \left(\hat{i}+\hat{j}\right)+ s\left(\hat{i}-\hat{j}+2\hat{k}\right)+t\left(\hat{i}+2\hat{j}+\hat{k}\right)$

(B) Attempt any TWO of the following:

i. Simplify the following circuit so that new circuit has minimum number of switches. Also draw simplified circuit.



ii. A line makes angles of measures 45° and 60° with positive direction of y and z axes respectively. Find the d.c.s. of the line and also find the vector of magnitude 5 along the direction of line.

iii. Maximize:
$z = 3x + 5y$
Subject to: $x + 4y \le 24$
$3x + y \le 21$
$x + y \le 9$
$x \ge 0, y \ge 0$

Q.3. (A) Attempt any TWO of the following:

i. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

ii. Show that the points (1, –1, 3) and (3, 4, 3) are equidistant from the plane $5x + 2y - 7z + 8 = 0$

iii. In any triangle ABC with usual notations prove $c = a \cos B + b \cos A$

(B) Attempt any TWO of the following:

i. Find p and k if the equation $px^2 - 8xy + 3y^2 + 14x + 2y + k = 0$ represents a pair of perpendicular lines.

ii. The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is ` 60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is ` 90 whereas the cost of 6 dozen pencils, 2 dozen pens and 3 dozen erasers is ` 70. Find the cost of each item per dozen by using matrices.

iii. Prove that the volume of the parallelopiped with coterminus edges as $\bar{a}, \bar{b}, \bar{c}$ is $\left[\bar{a}, \bar{b}, \bar{c}\right]$ and hence find the volume of the parallelopiped with its coterminus edges $2\hat{i}+5\hat{j}-4\hat{k}$, $5\hat{i}+7\hat{j}+5\hat{k}$, and $4\hat{i}+5\hat{j}-2\hat{k}$.

SECTION – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following sub-questions:

i. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7\left(\frac{d^2y}{dx^2}\right)$ are respectively.

(A) 2, 3 (B) 3, 2
(C) 2, 2 (D) 3, 3

ii. $\int_4^9 \frac{1}{\sqrt{x}} dx =$ _______

(A) 1 (B) –2
(C) 2 (D) –1

iii. If the p.d.f. of a continuous random variable X is given as

$f(x) = \frac{x^2}{3}$ for $-1 < x < 2$

$= 0$ otherwise.

then c.d.f. fo X is

(A) $\frac{x^3}{9} + \frac{1}{9}$ (B) $\frac{x^3}{9} - \frac{1}{9}$
(C) $\frac{x^2}{4} + \frac{1}{4}$ (D) $\frac{1}{9x^3} + \frac{1}{9}$

(B) Attempt any THREE of the following:

i. If $y = \sec\sqrt{x}$ then find $\frac{dy}{dx}$.

ii. Evaluate : $\int \frac{(x+1)}{(x+2)(x+3)} dx$

iii. Find the area of the region lying in the first quadrant bounded by the curve $y^2 = 4x$, X axis and the lines $x = 1$, $x = 4$.

iv. Solve the differential equation $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

v. Given X ~ B(n, p) if E(X) = 6, Var (X) = 4·2, find the value of n and p.

Q.5. (A) Attempt any TWO of the following:

i. If the function $f(x) = \frac{(4^{\sin x} - 1)^2}{x \cdot \log(1+2x)}$, for $x \neq 0$ is continuous at $x = 0$, find $f(0)$.

ii. Evaluate : $\int \frac{1}{3 + 2\sin x + \cos x} dx$

iii. If $y = f(x)$ is differentiable function of x such that inverse function $x = f^{-1}(y)$ exists then prove that x is a differentiable function of y and $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$, where $\frac{dy}{dx} \neq 0$

(B) Attempt any TWO of the following:

i. A point source of light is hung 30 feet directly above a straight horizontal path on which a man of 6 feet in height is walking. How fast is the man's shadow lengthening and how fast the tip of shadow is moving when he is walking away from the light at the rate of 100 ft/min?

ii. The p.m.f. for X = number of major defects in a randomly selected appliance of a certain type is

X = x	0	1	2	3	4
P (x)	0·08	0·15	0·45	0·27	0·05

Find the expected value and variance of X.

iii. Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

hence evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} \cdot dx$

Q.6. (A) Attempt any TWO of the following:

i. If $y = e^{\tan x} + (\log x)^{\tan x}$ then find $\frac{dy}{dx}$

ii. If the probability that a fluorescent light has a useful life of at least 800 hours is 0·9, find the probabilities that among 20 such lights at least 2 will not have a useful life of at least 800 hours. [Given : $(0{\cdot}9)^{19} = 0{\cdot}1348$]

iii. Find α and β, so that the function f (x) defined by

$f(x) = -2\sin x$, for $-\pi \le x \le -\frac{\pi}{2}$

$= \alpha \sin x + \beta$, for $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$= \cos x$, for $\frac{\pi}{2} \le x \le \pi$ is continuous on $[-\pi, \pi]$

(B) Attempt any TWO of the following:

i. Find the equation of a curve passing through the point (0, 2), given that the sum of the coordinates of any point on the curve exceeds the slope of the tangent to the curve at that point by 5.

ii. If u and v are two functions of x then prove that :

$\int uv\,dx = u\int v\,dx - \int\left[\frac{du}{dx}\int v\,dx\right]dx$

Hence evaluate $\int x e^x\,dx$

iii. Find the approximate value of $\log_{10}(1016)$ given that $\log_{10}e = 0{\cdot}4343$.

SECTION – I

Q.1. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions:

i. The negation of $p \wedge (q \rightarrow r)$ is

(A) $p \vee (\sim q \vee r)$ (B) $\sim p \wedge (q \rightarrow r)$

(C) $\sim p \wedge (\sim q \rightarrow \sim r)$ (D) $\sim p \vee (q \wedge \sim r)$

ii. If $\sin^{-1}(1 - x) - 2\sin^{-1}x = \frac{\pi}{2}$ then x is

(A) $-\frac{1}{2}$ (B) 1

(C) 0 (D) $\frac{1}{2}$

iii. The joint equation of the pair of lines passing through (2, 3) and parallel to the coordinate axes is

(A) $xy - 3x - 2y + 6 = 0$ (B) $xy + 3x + 2y + 6 = 0$

(C) $xy = 0$ (D) $xy - 3x - 2y - 6 = 0$

(B) Attempt any THREE of the following:

i. Find $(AB)^{-1}$ if $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ 1 & -2 \end{bmatrix}$

ii. Find the vector equation of the plane passing through a point having position vector $3\hat{i} - 2\hat{j} + \hat{k}$ and perpendicular to the vector $4\hat{i} + 3\hat{j} + 2\hat{k}$.

iii. If $\bar{p} = \hat{i} - 2\hat{j} + \hat{k}$ and $\bar{q} = \hat{i} + 4\hat{j} - 2\hat{k}$ are position vector (P.V.) of points P and Q, find the position vector of the point R which divides segment PQ internally in the ratio 2:1.

iv. Find k, if one of the lines given by $6x^2 + kxy + y^2 = 0$ is $2x + y = 0$.

v. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are at right angle then find the value of k.

Q.2. (A) Attempt any TWO of the following:

i. Examine whether the following logical statement pattern is tautology, contradiction or contingency.
$[(p \rightarrow q) \wedge q] \rightarrow p$

ii. By vector method prove that the medians of a triangle are concurrent.

iii. Find the shortest distance between the lines $\bar{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$ where λ and μ are parameters.

(B) Attempt any TWO of the following:

i. In ΔABC with the usual notations prove that

$$(a-b)^2 \cos^2\left(\frac{C}{2}\right) + (a+b)^2 \sin^2\left(\frac{C}{2}\right) = c^2.$$

ii. Minimize $z = 4x + 5y$ subject to $2x + y \geq 7$, $2x + 3y \leq 15$, $x \leq 3$, $x \geq 0$, $y \geq 0$. Solve using graphical method.

iii. The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is ` 60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is ` 90 whereas the cost of 6 dozen pencils, 2 dozen pens and 3 dozen erasers is ` 70. Find the cost of each item per dozen by using matrices.

Q.3. (A) Attempt any TWO of the following:

i. Find the volume of tetrahedron whose coterminus edges are $7\hat{i} + \hat{k}$, $2\hat{i} + 5\hat{j} - 3\hat{k}$ and $4\hat{i} + 3\hat{j} + \hat{k}$.

ii. Without using truth table show that

$\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

iii. Show that every homogeneous equation of degree two in x and y, i.e., $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through origin if $h^2 - ab \geq 0$.

(B) Attempt any TWO of the following:

i. If a line drawn from the point A(1, 2, 1) is perpendicular to the line joining P(1, 4, 6) and Q(5, 4, 4) then find the co-ordinates of the foot of the perpendicular.

ii. Find the vector equation of the plane passing through the points $\hat{i} + \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$. Hence find the cartesian equation of the plane.

iii. Find the general solution of $\sin x + \sin 3x + \sin 5x = 0$.

SECTION – II

Q.4. (A) Select and write the most appropriate answer from the given alternatives in each of the following sub-questions:

i. If the function

$f(x) = k + x$, for $x < 1$

$= 4x + 3$, for $x \geq 1$

is continuous at $x = 1$ then $k =$

(A) 7 (B) 8 (C) 6 (D) –6

ii. The equation of tangent to the curve $y = x^2 + 4x + 1$ at $(-1, -2)$ is

(A) $2x - y = 0$ (B) $2x + y - 5 = 0$

(C) $2x - y - 1 = 0$ (D) $x + y - 1 = 0$

iii. Given that $X \sim B(n = 10, p)$. If $E(X) = 8$ then the value of p is

(A) 0.6 (B) 0.7 (C) 0.8 (D) 0.4

(B) Attempt any THREE of the following:

i. If $y = x^x$, find $\frac{dy}{dx}$.

ii. The displacement ‘s’ of a moving particle at time ‘t’ is given by $s = 5 + 20t - 2t^2$. Find its acceleration when the velocity is zero.

iii. Find the area bounded by the curve $y^2 = 4ax$, X-axis and the lines $x = 0$ and $x = a$.

iv. The probability distribution of a discrete random variable X is:

$X = x$	1	2	3	4	5
$P(X = x)$	k	2k	3k	4k	5k

Find $P(X \leq 4)$.

v. Evaluate: $\int \frac{\sin x}{\sqrt{36-\cos^2 x}}dx$

Q.5. (A) Attempt any TWO of the following:

i. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x then prove that $y = f(g(x))$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

ii. The probability that a person who undergoes kidney operation will recover is 0.5. Find the probability that of the six patients who undergo similar operations.

a. None will recover.

b. Half of them will recover.

iii. Evaluate: $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x}dx$

(B) Attempt any TWO of the following:

i. Discuss the continuity of the following functions. If the function have a removable discontinuity, redefine the function so as to remove the discontinuity.

$$\left.\begin{aligned} f(x) &= \frac{4^x - e^x}{6^x - 1}, \text{ for } x \neq 0 \\ &= \log\left(\frac{2}{3}\right), \text{ for } x = 0 \end{aligned}\right\} \text{at } x = 0$$

ii. Prove that:

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$$

iii. A body is heated at 110°C and placed in air at 10°C. After 1 hour its temperature is 60°C. How much additional time is required for it to cool to 35°C?

Q.6. (A) Attempt any TWO of the following:

i. Prove that: $\int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(2a - x)dx$

ii. Evaluate: $\int \frac{1 + \log x}{x(2 + \log x)(3 + \log x)}dx$

iii. If $y = \cos^{-1}\left(2x\sqrt{1-x^2}\right)$, find $\frac{dy}{dx}$

(B) Attempt any TWO of the following:

i. Solve the differential equation $\cos(x + y)dy = dx$

Hence find the particular solution for $x = 0$ and $y = 0$.

ii. A wire of length l is cut into two parts. One part is bent into a circle and other into a square. Show that the sum of areas of the circle and square is the least, if the radius of circle is half the side of the square.

iii. The following is the p.d.f. (Probability Density Function) of a continuous random variable X:

$$\begin{aligned} f(x) &= \frac{x}{32}, \quad 0 < x < 8 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

a. Find the expression for c.d.f. (Cumulative Distribution Function) of X.

b. Also find its value at $x = 0.5$ and 9.