CHAPTER - 9

Introduction to Algebra

9.1 INTRODUCTION

Our study so far has been with numbers and shapes. What we have learnt so far comes under arithmetic and geometry. Now we begin the study of another branch of mathematics called Algebra.

The main feature of algebra is the use of letters or alphabet to represent numbers. Letter can represent any number, not just a particular number. It may stand for an unknown quantity. By learning the method of determining unknowns we develop powerful tools for solving puzzles and many problems in daily life.

Consider the following

Damini and Kowshik are playing a game.

Kowshik : If you follow my instructions and tell me the final result, then I will tell you your age.

Dhamini : But you know my age so what is new?

Kowshik : Ok, take the age of person who is unknown to me. Do not reveal me the age but still I will tell you the age.

Dhamini : Alright, what are your instructions? Let me see how you do it.

Kowshik : First, double the age.

Dhamini : Done.

Kowshik : Add 5 to the result and tell me the final result.

Dhamini : Ok, the result is '27'.

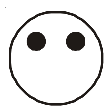
Kowshik : Good! Your friend's age is 11 years.

Dhamini was surprised. She thought for a while and said 'I know how you found the age'.

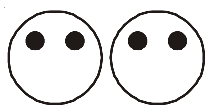
Do you know how it was done? You too can try!!!

9.2 PATTERNS - MAKING RULES

9.2.1 Pattern-1

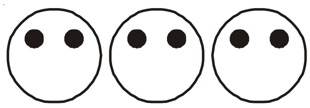


Praveen and Moulika were making human faces as shown in the following figure. They use black stickers for eyes. Moulika took two black stickers and formed a human face as shown in the figure.

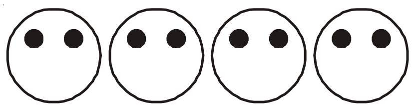


Praveen also took two black stickers to form a human face and put it next to the one made by Moulika.

Then Moulika added one more



and Praveen also



Soon after their friend Rahim joined them. He asked them, " How many black stickers will be required to form 8 such shapes". Immediately Moulika counts the number of black stickers in four shapes, doubles the number and says 16.

"Well" Rahim said and asks them, " How many black stickers will be required to form 69 such human faces". Moulika and Praveen feel this method of counting stickers is a bit laborious and time consuming, specially when the number of faces are very large. They decide to find a new way. They think a while and make the following table.

Number of human faces formed	1	2	3 . . .	
Number of black stickers required	2	4	6	. .
Also represented as (pattern formation)	2×1	2×2	2×3	. . .

Do you notice a relation between the number of faces formed and the number of black stickers required?

Moulika says that there is a relationship between the number of faces to be formed and the number of black stickers required.

For example to make 1 face, the required stickers are 2 i.e. 2×1 or $2 \times$ the number of faces formed. Let us see if it works for larger number of faces.

For 2 faces, the required stickers are $4 = 2 \times 2 = 2 \times$ number of faces formed.

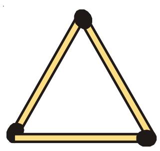
For 3 faces, the required stickers are $6 = 2 \times 3 = 2 \times$ number of faces formed.

Moulika said that the number of black stickers required is twice the number of faces formed i.e. number of black stickers required = 2 times the number of faces formed.

Now for the number of faces to be 69 we require.

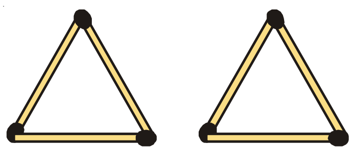
$2 \times 69 = 138$ black stickers.

9.2.2 Pattern-2



To make a triangle, 3 match sticks are used.

If we want to make two triangles we need 6 match sticks.



The following table gives the number of match sticks required and the number of triangles to be formed:

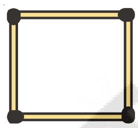
Number of triangles to be formed	1	2	3	4	5	6	...
Number of match sticks required	3	6	9	12	15	18	...
Observation (Pattern)	3×1	3×2	3×3	3×4	3×5	3×6	...

What is the rule for the number of triangles formed and the match sticks needed?

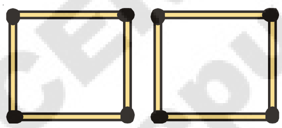
The rule is number of match sticks required = 3 times the number of triangles to be formed.

9.2.3 Pattern-3

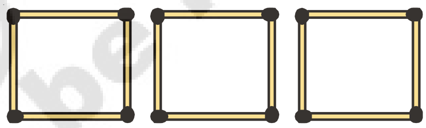
To make a square, 4 match sticks are needed.



If we want to make two squares we need 8 match sticks



If we want to make three squares we need 12 match sticks



Let us arrange the above information in the following table

Number of Squares to be formed	1	2	3	....
Number of match sticks required	4	8	12	. ...
Observation (Pattern)	4×1	4×2	4×3	....

i.e. number of match sticks required = 4 times number of squares to be formed.

9.3 Variable

Let us consider the table in pattern-1

Number of human faces to be formed	1	2	3	. . .
Number of black stickers required	2	4	6	. . .
Pattern	2×1	2×2	2×3	. . .

In the table as the number of human faces formed goes on increasing the number of black stickers required also goes on increasing. Also notice that in each case the number of stickers required is twice the number of human faces formed.

For the sake of convenience, let us write a letter say 'm' for the number of faces formed.

Therefore number of black stickers required = $2 \times m$

Instead of writing "$2 \times m$" we write "2m". Note that "2m" is same as "$2 \times m$" not as $2 + m$.

$\therefore$ The number of black stickers required = 2m.

If we want to make one human face,the value of m = 1. Therefore according to the rule the number of stickers required is $2 \times 1 = 2$.

If we want to make two faces, the value of 'm' becomes 2. Therefore the number of stickers required is $2 \times 2 = 4$.

Now, can you guess the number of stickers required for three faces? Obviously 6.

From the above example we found relation between the number of stickers required and the number of faces.

Number of stickers required $= 2\,m$

Here m is the number of faces and it can take any value i.e. 1, 2, 3, 4, ...

The 'm' here is an example of a variable, the value of 'm' is not fixed and it can take different values. Accordingly the number of stickers also changes.

Now consider the table of pattern-2

Number of triangles to be formed	1	2	3	4	5	6	....
Number of match sticks required	3	6	9	12	15	18	....
Observation (Pattern)	3×1	3×2	3×3	3×4	3×5	3×6	.....

Now can you frame the rule for the number of match sticks required for a given number of triangles to be formed?

Obviously number of match sticks required = 3 y, where 'y' is number of triangles .

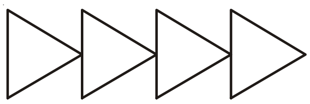
Here also 'y' takes different values. $y = 1, 2, ...$

i.e. the value of 'y' changes. Hence 'y' is an example of a variable.

Go back to the table of pattern -3 and make the rule for the number of match sticks required for a given number of squares. Take n to denote the number of squares and m to denote the matchsticks needed.

TRY THESE

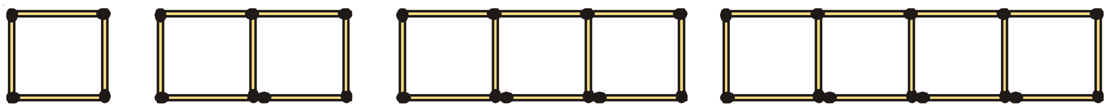
1. Can you now write the rule to form the following pattern with match sticks?



2. Find the rule for required number of match sticks to from a pattern repeating 'H' . How would the rule be for repeating the shape 'L'?

9.4 More Patterns

Consider the match stick pattern constructing squares



Shape-1 Shape-2 Shape-3 Shape-4

The number of squares and the match sticks required are given below:

Number of squares	1	2	3	4	5
Number of match sticks (m)	4	7	10	13	---
Pattern	$(3\times1)+1$	$(3\times2)+1$	$(3\times3)+1$	$(3\times4)+1$	---

Then the rule is

Number of match sticks = 3 × (number of squares) + 1

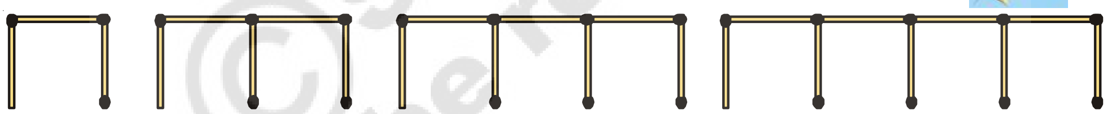
let S = number of squares

Therefore number of match sticks used = $(3\times S) + 1 = 3S + 1$

Here the letter 's' is an example for a variable.

Try These

A line of shapes is constructed using matchsticks .



Shape-1 Shape-2 Shape-3 Shape-4

(i) Find the rule that shows how many sticks are needed to make a group of such shapes?

(ii) How many match sticks are needed to form a group of 12 shapes?

We can use any letter eg. m, n, p, s, x, y, z etc. to denote a variable. Variable does not have a fixed value or a fixed letter attached to it. A letter can denote any quantity. In the above examples we have used m, y, s to denote the number of matchsticks.

Example-1. Number of pencils with Rama is 3 more than Rahim. Find the number of pencils Rama has in terms of what Rahim has?

***Solution*:** If Rahim has 2 pencils then Rama has 2 + 3 = 5 pencils.

If Rahim has 5 pencils than Rama has 5 + 3 = 8 pencils.

We do not know how many pencils Rahim has.

But we know that Rama's pencils = Rahim's pencils + 3

If we denote the number of pencils Rahim has as n, then the number of pencils of Rama are n+3

Here n = 1, 2, 3 ....therefore 'n' is a variable.

Example-2. Hema and Madhavi are sisters. Madhavi is 3 years younger than Hema. Write Madhavi's age in terms of Hema's age?

***Solution*:** Given that Madhavi is younger than Hema by 3 years, if Hema is 10 years old then Madhavi is 10-3 = 7 years old.

If Hema is 16 years old, Madhavi is 16-3= 13 years old.

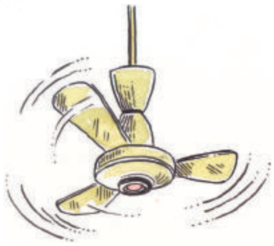
Here we don't know the exact age of Hema. It may take any value. So let the age of Hema be 'p' years, then Madhavi's age is "p - 3" years.

Here 'p' is also an example of a variable. It takes different values like 1,2,3.........

As you would expect when 'p' is 10, 'p-3' is 7 and when 'p' is 16, p-3 is 13.

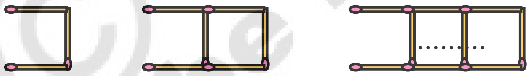
Exercise - 9.1

1. Find the rule which gives the number of match sticks required to make the following match sticks patterns.
   (i) A pattern of letter 'T' (ii) A pattern of letter 'E'
   (iii) A pattern of letter 'Z'
2. Make a rule between the number of blades required and the number of fans (say n) in a hall?



3. Find a rule for the following patterns between number of shapes formed and number of match sticks required.
   (a)

   (b)

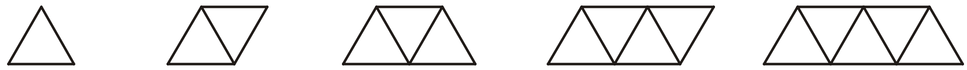


4. The cost of one pen is ₹ 7 then what is the rule for the cost of 'n' pens.
5. The cost of one bag is ₹ 90 what is the rule for the cost of 'm' bags?
6. The rule for purchase of books is that the cost of q books is ₹ 23q ; then find the price of one book?
7. John says that he has two books less than Gayathri has. Write the relationship using letter x.
8. Rekha has 3 books more than twice the books with Suresh. Write the relationship using letter y.
9. A teacher distributes 6 pencils per student. Can you find how many pencils are needed for the given number of students (use 'z' for the number of students).
10. Complete each table to generate the given functional relationship.

(i)	x	1	2	3	4	5	9	....
	$3x+2$	5	....	....	....	....	....	38

(ii)	a	1	3	6	7	9	8	....
	$5a-1$	4	....	....	....	....	....	49

11. Observe the following pattern.



Shape-1 Shape-2 Shape-3 Shape-4 Shape-5

Count the number of line segments in each shape.

(i) How many line segments will 9 such shapes contain?

(ii) Write the rule for the above pattern.

9. 5 Expressions with variables

Recall that in arithmetic we have come across expression like 5 + 4, 11 - 9 etc. These are all formed using numbers.

Observe the following

Ram says that he has scored five marks more than Tony. Can you find the marks scored by Ram ? Here we do not know the marks of Tony. We proceed by supposing Tony's marks.

Suppose Tony scored 45 marks. Then marks scored by Ram would be 45+5= 50

If Tony scored 56 marks. Then marks scored by Ram would be 56+5= 61

Now let us suppose Tony scored 'x' marks. Can you say the marks scored by Ram?

The marks scored by Ram would be x+5.

This x+5 is known as an expression with variable 'x'.

In fact, we have seen expressions like 2m, 3y, 4z, 2s+1, 3s+1, 8p, n+3, p-3 in the earlier discussion. Those expressions are obtained by using operations of addition, subtraction, multiplication and division of variables. For example, the expression 'p-3' is formed by subtracting 3 from the variable 'p' and the expression '8 p' is formed by multiplying the variable 'p' by '8'.

We know that variables can take different values; they have no fixed value, but they are numbers. That is why operations of addition, subtraction, multiplication and division can be done on them.

We have already come across daily life situation in which expressions are useful. Let us recall some of them:

S.No.	Situation	Variable	Statement using Expression
(i)	'n' divided by 7		
(ii)	₹ 5 more than what Geeta has	Geeta has ₹ y	$y + 5$
(iii)	Perimeter is 4 times the side in a square		
(iv)	Price of apple is twice the price of guava		
(v)	Renu's height is 3 feet less than Leela's height		
(vi)	I have scored $\frac{1}{3}$ of the runs scored by you.		

Example-3. Write statement for the following expressions:

(i) $2p$ (ii) $7 + x$

Solution: (i) Raju has twice the money then Seema.

(ii) I have 7 marbles more than Dilip.

Example-4. Madhu plants 5 more Groundnut seeds than Bean seeds. How many Groundnut seeds does he plant (take number of Bean seeds as 'm')

Solutioin: Let the number of Bean seeds = m

Therefore number of Groundnut seeds = 'm+ 5'

Exercise -9.2

1. Write the expressions for the following statements
   (i) q is multiplied by 5
   (ii) y is divided by 4
   (iii) One fourth of the product of numbers p and q
   (iv) 5 is added to the three times z
   (v) 9 times 'n' is added to '10'
   (vi) 16 is subtracted from two times 'y'
   (vii) 'y' is multiplied by 10 and then x is added to the product
2. Write two statements each for the following expressions

   (i) $y - 11$ (ii) $10a$ (iii) $\frac{x}{5}$ (iv) $3m + 11$ (v) $2y - 5$
3. Peter has 'p' number of balls. Number of balls with David is 3 times the balls with Peter. Write this as an expression.
4. Sita has 3 more note books than Githa. Find the number of books that Sita has? Use any letter for the number of books that Gita has.
5. Cadets are marching in a parade. There are 5 cadets in each row. What is the rule for the number of cadets, for a given number of rows? Use 'n' for the number of rows.

9.6 Rules from Geometry/Mensuration

Perimeter of a square

We know that perimeter of a polygon is the sum of the lengths of all its sides.

A square has 4 sides and they are equal in length.

Therefore the perimeter of a square = Sum of the length of the sides of the square.

= 4 × length of the side.(side + side +side + side) = 4 × s = 4s

Thus we get the rule for the perimeter of the square. The length of the square can have any value, its value is not fixed. It is also a variable. The use of the variable allows us to write the general rule in a way that is concise and easy to remember. We wrote the rule for perimeter of a square. What would be the rule for perimeter of an equilateral triangle?

Try These

1. Find the general rule for the perimeter of a rectangle. Use variables 'l' and 'b' for length and breadth of the rectangle respectively.
2. Find the general rule for the area of a square by using the variable 's' for the side of a square.
3. What would be the rule for perimeter of an is ocelese triangle?

9.7 Rule from arithmetic

Observe the following number pattern

2, 4 , 6 , 8 , 10 , ...

To find the nth term in the given pattern, we put the sequence in a table

Even Number	2	4	6	8	10	12	14	16	18	20
Pattern	2×1	2×2	2×3	2×4	2×5	.....	2×7	.....	2×9	.....

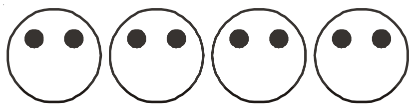
From the table it is clear that the first even number is 2×1, the second even number is 2×2 and so on. Using the above logic, we can fill up the blanks in the table and find the pattern for 'n^{th}' even number. It is $2 \times n$ i.e., '$2n$'.

So the n^{th} term of the pattern 2,4,6,8,10,…… is $2n$.

Do This

1. Find the n^{th} term in the following sequences
   (i) 3 , 6 , 9 , 12 , …………..
   (ii) 2 , 5 , 8 , 11 , …………...
   (iii) 1 , 8 , 27 , 64 , 125 …………...

9.8 Simple Equations

Let us recall the face pattern. 

We know that the number of black stickers required is given by the rule $2m$, if m is taken to be the number of faces to be formed.

We can find the number of stickers required for a given number of faces. What about the other way? How to find the number of faces formed when the number of stickers are given.

This mean, we have to find the number of faces (i.e. m) for the given number of stickers 10. For 10 stickers we know $2m = 10$

Here we have a condition to be satisfied by the variable m

The condition to be satisifed that 2 times m must be 10 is example of an equation.

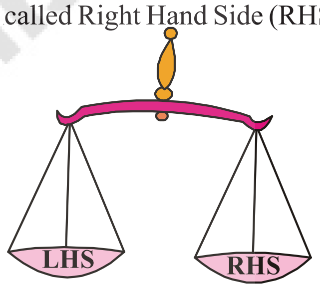
Our question can be answered by observing the table.

m	$2m$	Condition satisfied? Yes/No.
2	4	No
3	6	No
4	8	No
5	10	Yes
6	12	No
7	14	No

The equation $2m = 10$ is satisfied only when $m = 5$.

9.8.1 L.H.S & R.H.S of an Equation

If we observe the equation $2m = 10$ we can find that equation has sign of equality between its two sides. The value of expression to the left of the equal its sign in an equation is called Left Hand Side (LHS) and the value of which is right side of the equal its sign is called Right Hand Side (RHS).



An equation says that the value of the LHS is equal to the value of RHS. This condition of an equation is often compared with a simple balance with equal weights on both pans.

If LHS is not equal to RHS we do not get an equation. For example $4 + 5$ on one side and 7 on the other side is not an equation. We would write $4 + 5 \neq 7$ or $4 + 5 > 7$. Similarly $x + 5 > 6$, $y - 1 < 10$ are not equations.

Do This

1. Write LHS and RHS of following simple equations:

   (i) $2x + 1 = 10$ (ii) $9 = y - 2$ (iii) $3p + 5 = 2p + 10$

2. Write any two simple equations and give their LHS & RHS.

9.8.2 Solution of an equation (Root of the equation)- Trial & Error Method

Let us take the other example considered at the beginning of the chapter. We observed a conversation between Damini and Kowshik. In that conversation Damini said that the final result was 27 and Kowshik told her friend's age as 11 years.

Let us see how he found the age.

Let the Damini friend's age be 'x' years. Doubling it we get '$2x$'. After adding 5 to it, it becomes '$2x + 5$'.

Therefore the final result is '$2x + 5$'. Damini said that final result was 27.

This tells us $\mathbf{2x + 5 = 27}$

Let us take the above equation $2x + 5 = 27$ is the condition to be satisfied by 'x'

Here 'x' is a variable and can take any value like 1, 2, 3, ….

If $x = 1$ then the value of $2x + 5 = 2 \times 1 + 5 = 7$

If $x = 2$ then the value of $2x + 5 = 2 \times 2 + 5 = 9$

If $x = 3$ then the value of $2x + 5 = 2 \times 3 + 5 = 11$ and so on

Writing 1,2,3 ..... in the place of 'x' is called "**Substitution**".

Let us examine the values of LHS and RHS by substituting values for the variable 'x'

Substituting value (x)	Value of LHS (2x+5)	Value of RHS 27	Whether LHS and RHS are equal
1	$2 \times 1+5 = 7$	27	Not equal
2	$2 \times 2+5 = 9$	27	Not equal
3	$2 \times 3+5 = 11$	27	Not equal
4	$2 \times 4+5 = 13$	27	Not equal
5	$2 \times 5+5 = 15$	27	Not equal
6	$2 \times 6+5 = 17$	27	Not equal
7	$2 \times 7+5 = 19$	27	Not equal
8	$2 \times 8+5 = 21$	27	Not equal
9	$2 \times 9+5 = 23$	27	Not equal
10	$2 \times 10+5 = 25$	27	Not equal
11	**$2 \times 11+5 = 27$**	**27**	**Equal**
12	$2 \times 12+5 = 29$	27	Not equal

From the table it is obvious that when '$x = 11$' the both LHS and RHS are equal. Therefore $x = 11$ is called the solution of equation $2x + 5 = 27$.

Solution of an equation is the value of the variable for which LHS and RHS are equal. The solution is also called as root of the equation.

Algebra is a powerful tool for solving puzzles, riddles and problems in our daily life.

Consider the second equation $3m = 15$

The following table shows for different values of 'm', the value of LHS and the comparison with the RHS.

Substituting value (m)	Value of LHS (3m)	Value of RHS (15)	Whether LHS and RHS are equal
1	$3 \times 1 = 2$	15	Not equal
2	$3 \times 2 = 6$	15	Not equal
3	$3 \times 3 = 9$	15	Not equal
4	$3 \times 4 = 12$	15	Not equal
5	**$3 \times 5 = 15$**	**15**	**Equal**
6	$3 \times 6 = 18$	15	Not equal

From the table we find that for m=5 both LHS and RHS are equal. Therefore m = 5 is the solution of the equation. The method we followed in the above is called Trial and Error Method.

Do This

Find the solution of the equation '$x - 4 = 2$' by Trial and Error method.

Exercise - 9.3

1. State which of the following are equations.

(i)	$x - 3 = 7$	(ii)	$l + 5 > 9$	(iii)	$p-4 < 10$
(iv)	$5 + m = -6$	(v)	$2s - 2 = 12$	(vi)	$3x +5 > 13$
(vii)	$3x < 15$	(viii)	$2x - 5 = 3$	(ix)	$7y + 1 < 22$
(x)	$-3z + 6 = 12$	(xi)	$2x - 3y = 3$	(xii)	$z^2 = 4$

2. Write LHS and RHS of the following equations.

(i)	$x-5 =6$	(ii)	$4y = 12$	(iii)	$2z + 3 = 7$
(iv)	$3p = 24$	(v)	$4 = x - 2$	(vi)	$2a - 3 = -5$

3. Solve the following equation by Trial & Error Method.

(i)	$x + 3 = 5$	(ii)	$y - 2 = 7$	(iii)	$a- 2 = 6$
(iv)	$5y = 15$	(v)	$6n = 30$	(vi)	$3z = 27$

What We Have Discussed?

1. We looked at the patterns arising from making of many identical letters or shapes using match sticks. We learnt to write general realtion between the number of matchsticks required for making a number of identical shapes. Since the number of times the shape is repeated is a variable, we denote it by an alphabet in writing the rule.
2. A variable takes different values. Its value is not fixed.
3. We may use any letter a, b, m, n, p, q, x, y, z etc., to represent a variable.
4. A variable allows us to express relations in any practical situation.
5. Variables are numbers, although their value is not fixed. We can do operations on them just as in the case of fixed numbers.
6. We can form expressions with variables using different operations. Some examples are $2m$, $3s+1$, $8p$, $x/3$ etc.
7. Variables allow us to express many common rules of geometry and arithmetic in a general way.
8. An equation is a condition on a variables. Such a condition limits the values the variable can have.
9. An equation has two sides, L.H.S. and R.H.S., on both sides of equal to sign.
10. The L.H.S. of an equation is equal to its R.H.S. only for definite values of the variable in the equation.
11. To get the solution of an equation, one of the methods used is the Trial and Error method.