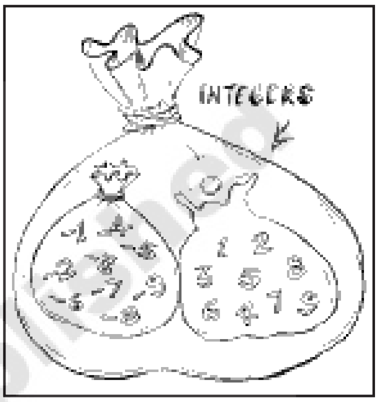
INTEGERS

1

1.0 Introduction



We start learning numbers like 1,2,3,4.... for counting objects around us. These numbers are thus called counting numbers or natural numbers. Let us think about these.

(i) Which is the smallest natural number?

(ii) Write five natural numbers between 100 and 10000.

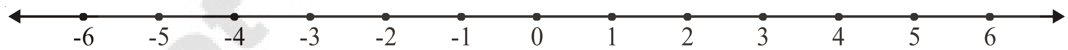
(iii) Can you find where the sequence of natural numbers ends?

(iv) What is the difference between any two consecutive natural numbers?

By including '0' to the collection of natural numbers, we get a new collection of numbers called whole numbers i.e., 0, 1, 2, 3, 4, ...

In class VI we also learnt about negative numbers. If we put whole number and negative numbers together we get a bigger collection of numbers called integers. In this chapter, we will learn more about integers, their operations and properties.

Let us now represent some integers on a number line.



(i) Which is the biggest integer represented on the above number line?

(ii) Which is the smallest integer?

(iii) Is 1 bigger than –3? Why?

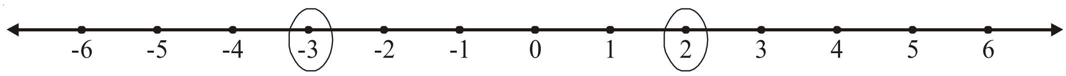
(iv) Is –6 bigger than –3? Why?

(v) Arrange 4, 6, –2, 0 and –5 in ascending order.

(vi) Compare the difference between (0 and 1) and (0 and –1) on the number line.

Exercise - 1

1. Some integers are marked on the number line. Which is the biggest and which is the smallest?



2. Write the integers between the pairs of integers given below. Also, choose the biggest and smallest intergers from them.

   (i) –5, –10 (ii) 3, –2 (iii) –8, 5

3. Write the following integers in ascending order (smallest to biggest).

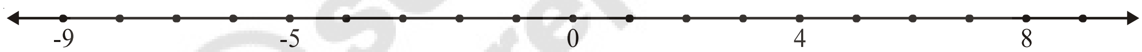
   (i) –5, 2, 1, –8 (ii) –4, –3, –5, 2 (iii) –10, –15, –7

4. Write the following integers in descending order (biggest to smallest).

   (i) –2, –3, –5 (ii) –8, –2, –1 (iii) 5, 8, –2

5. Represent 6, –4, 0 and 4 on a number line.

6. Fill the missing integers on the number line given below



7. The temperatures of 5 cities in India on a particular day are shown on the number line below.

   (i) Write the temperatures of the cities marked on it?

   (ii) Which city has the highest temperature?

   (iii) Which city has the lowest temperature?

   (iv) Which cities have temperature less than 0ºC?

   (v) Name the cities with temperature more than 0ºC?

1.1 Operations of integers

We have done addition and subtraction of integers in class VI. First we will review our understading of the same and then learn about multiplication and division of integers.

1.1.1 Addition of integers

Observe the additions given below.

$4 + 3 = 7$

$4 + 2 = 6$

$4 + 1 = 5$

$4 + 0 = 4$

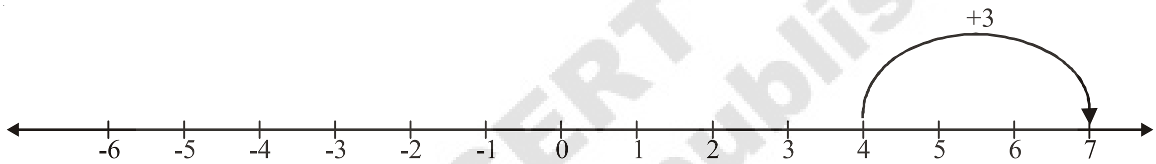
$4 + (-1) = 3$

$4 + (-2) = 2$

$4 + (-3) = 1$

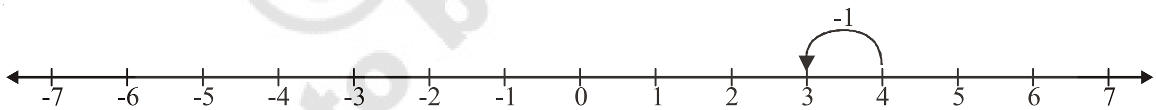
Do you see a pattern in the answers? You will find that when the number being added is decreased by one (3, 2, 1, 0, –1, –2, –3) then the value of the sum goes down by 1.

On the number line, when you add 3 to 4 you move right on the number line:



Similarly can you now show the additions of 2 and 1 to 4 on the number line drawn above? You will find that in each case you have moved right on the number line.

Now, let us see what is happening when we add –1 to 4. From the above pattern we find that the answer is 3. Thus, we have moved one step left on the number line:



You can now similarly show addition of –2 and –3 to 4 on the number line drawn above? You will find that in each case you are moving left on the number line.

Thus, each time you add a positive integer you move right on the number line. On the other hand, each time you add a negative number you move left on the number line.

Try This

1.	$9 + 7$	$= 16$		$9 + 1$	$=$
	$9 + 6$	$= 15$		$9 + 0$	$=$
	$9 + 5$	$=$		$9 + (-1)$	$=$
	$9 + 4$	$=$		$9 + (-2)$	$=$
	$9 + 3$	$=$		$9 + (-3)$	$=$
	$9 + 2$	$=$			

(i) Now represent 9 + 2, 9 + (–1) and 9 + (–3) on the number line.

(ii) When you added a positive integer, in which direction did you move on the number line?

(iii) When you added a negative integer, in which direction did you move on the number line?

2. Sangeetha said that each time you add two integers, the value of the sum is greater than the numbers. Is Sangeetha right? Give reasons for your answer.

Exercise - 2

1. Represent the following additions on a number line.

(i) 5 + 7 (ii) 5 + 2 (iii) 5 + (–2) (iv) 5 + (–7)

2. Compute the following.

(i) 7 + 4 (ii) 8 + (–3) (iii) 11 + 3

(iv) 14 + (–6) (v) 9 + (–7) (vi) 14 + (–10)

(vii) 13 + (–15) (viii) 4 + (–4) (ix) 10 + (–2)

(x) 100 + (–80) (xi) 225 + (–145)

1.1.2 Subtraction of integers

Now let us study the subtractions given below.

6 – 3 = 3

6 – 2 = 4

6 – 1 = 5

6 – 0 = 6

6 – (–1) = 7

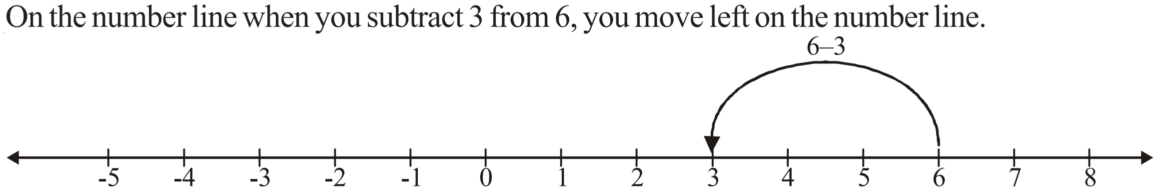
6 – (–2) = 8

6 – (–3) = 9

6 – (–4) = 10

Do you see a pattern in the answers? You will find that when the number being subtracted is decreased by one (3, 2, 1, 0, –1, –2, –3, –4) the value of the difference goes up by 1.

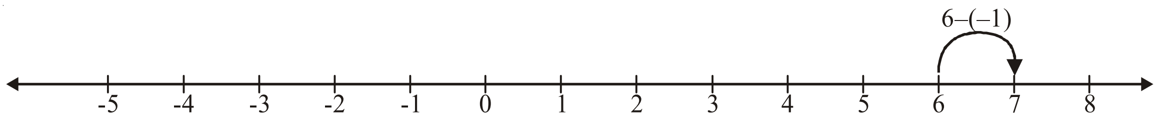
On the number line when you subtract 3 from 6, you move left on the number line.



You can now, similarly show subtraction of 2, 1 from 6 on the number line. You will find that in each case you have moved left on the number line.

Now, let us see what is happening when we subtract –1 from 6. As seen from the above pattern we find 6– (–1) = 7.

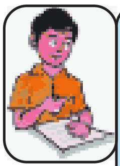
Thus, we have moved one step right on the number line .



You can now, similarly show subtraction of –2, –3, –4 from 6? You will find that in each case you are moving right on the number line.

Thus, each time you subtract a positive integer, you move left on the number line.

And each time you subtract a negative integer, you move right on the number line.



Try This

Complete the pattern.

1. 8 – 6 = 2
   8 – 5 = 3
   8 – 4 =
   8 – 3 =
   8 – 2 =
   8 – 1 =
   8 – 0 =
   8 – (–1) =
   8 – (–2) =
   8 – (–3) =
   8 – (–4) =

   (i) Now show 8 –6, 8–1, 8–0, 8–(–2), 8–(–4) on the number line.

   (ii) When you subtract a positive integer in which direction do you move on the number line?

   (iii) When you subtract a negative integer, in which direction do you move on the number line?

2. Richa felt that each time you subtract an integer from another integer, the value of the difference is lesser than the two numbers. Is Richa right? Give reasons for your answer.

Exercise - 3

1. Represent the following subtractions on the number line.

   (i) $7-2$ (ii) $8-(-7)$ (iii) $3-7$

   (iv) $15-14$ (v) $5-(-8)$ (vi) $(-2)-(-1)$

2. Solve the following.

   (i) $17-(-14)$ (ii) $13-(-8)$ (iii) $19-(-5)$

   (iv) $15-28$ (v) $25-33$ (vi) $80-(-50)$

   (vii) $150-75$ (viii) $32-(-18)$

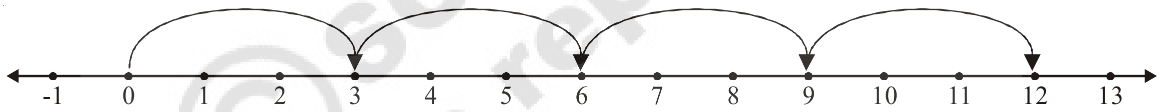
3. Express '–6' as the difference between a negative integer and a whole number.

1.1.3 Multiplication of integers

Now, let us multiply integers.

We know that $3+3+3+3 = 4\times 3$ (4 times 3)

On the number line, this can be seen as.

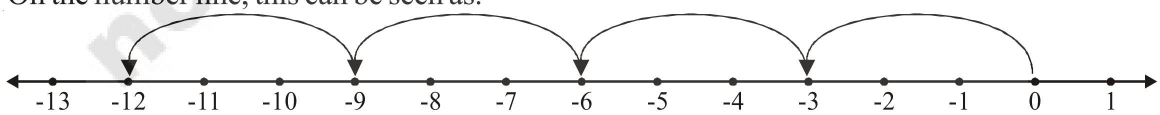


Thus, 4×3 means 4 jumps each of 3 steps from zero towards right on the number line and therefore $4\times 3 = 12$.

Now let us discuss $4\times(-3)$ i.e., 4 times (-3)

$4\times(-3) = (-3)+(-3)+(-3)+(-3) = -12$

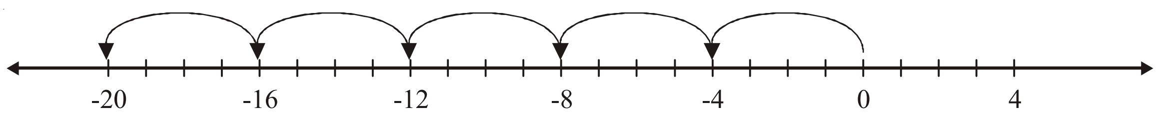
On the number line, this can be seen as.



Thus, $4\times(-3)$ means 4 jumps each of 3 steps from zero towards left on the number line and therefore $4\times(-3) = -12$

Similarly, $5\times(-4) = (-4)+(-4)+(-4)+(-4)+(-4) = -20$

On the number line, this can be seen as :



Thus, 5×-4 means 5 jumps each of 4 steps from zero towards left on the number line and therefore $5 \times -4 = -20$

Similarly, $2 \times -5 = (-5) + (-5) = -10$

$3 \times -6 = (-6) + (-6) + (-6) = -18$

$4 \times -8 = (-8) + (-8) + (-8) + (-8) = -32$

Do This

1. Solve the following.

   (i) 2×-6 (ii) 5×-4 (iii) 9×-4

Now, let us multiply -4×3

Study the following pattern-

$4 \times 3 = 12$

$3 \times 3 = 9$

$2 \times 3 = 6$

$1 \times 3 = 3$

$0 \times 3 = 0$

$-1 \times 3 = -3$

$-2 \times 3 = -6$

$-3 \times 3 = -9$

$-4 \times 3 = -12$

You see that as the multiplier decreases by 1, the product decreases by 3.

Thus, based on this pattern $-4 \times 3 = -12$.

We already know that $4 \times -3 = -12$

Thus, $-3 \times 4 = 3 \times -4 = -12$

Observe the symbol of the product as the negative sign differ in the multiplication.

Using this pattern we can say that

$4 \times -5 = -5 \times 4 = -20$

$2 \times -5 = -5 \times 2 = -10$

$3 \times -2 =$

$8 \times -4 =$

$6 \times -5 =$

From the above examples you would have noticed that product of possitive integer and a negative integer is always a negative integer.

1.1.3(a) Multiplication of two negative integers

Now, what if we were to multiply –3 and –4.

Study the following pattern.

$-3 \times 4 = -12$

$-3 \times 3 = -9$

$-3 \times 2 = -6$

$-3 \times 1 = -3$

$-3 \times 0 = 0$

$-3 \times -1 = 3$

$-3 \times -2 = 6$

$-3 \times -3 = 9$

$-3 \times -4 = 12$

Do you see a pattern? You will see that as the number of times we multiply goes down by 1, the product increases by 3.

Now let us multiply –4 and –3. Study the following products and fill the blanks.

$-4 \times 4 = -16$

$-4 \times 3 = -12$

$-4 \times 2 = -8$

$-4 \times 1 = -4$

$-4 \times 0 = 0$

$-4 \times -1 =$ ______

$-4 \times -2 =$ ______

$-4 \times -3 =$ ______

You will see that as the number of times we multiply goes down by 1, the product increases by 4.

According to the two patterns given above, $-3 \times -4 = -4 \times -3 = 12$

You have also observed that.

$-3 \times -1 = 3$ $\qquad$ $-4 \times -1 = 4$

$-3 \times -2 = 6$ $\qquad$ $-4 \times -2 = 8$

$-3 \times -3 = 9$ $\qquad$ $-4 \times -3 = 12$

Thus, every time we multiply two negative integers, the product is a positive integer.

Activity 1

Fill the grid by multiplying each number in the first column with each number in the first row.

×	3	2	1	0	–1	–2	–3
3	9	6	3	0	–3	–6	–9
2	6	4	2	0			
1							
0							
–1	–3	–2	–1	0	1	2	3
–2							
–3							

(i) Is the product of two positive integers always a positive integer?

(ii) Is the product of two negative integers always a positive integer?

(iii) Is the product of a negative and positive integer always a negative integer?

1.1.3(b) Multiplication of more than two negative integers

We observed that the product of two negative integers is a positive integer. What will be the product of three negative integers? Four negative integers? and so on .....

Let us study the following examples.

(i) $(-2) \times (-3) = 6$

(ii) $(-2) \times (-3) \times (-4) = [(-2) \times (-3)] \times (-4) = 6 \times (-4) = -24$

(iii) $(-2) \times (-3) \times (-4) \times (-5) = [(-2) \times (-3) \times (-4)] \times (-5) = (-24) \times (-5) = 120$

(iv) $[(-2) \times (-3) \times (-4) \times (-5) \times (-6)] = 120 \times (-6) = -720$

From the above products we can infer that.

(i) The product of two negative integers is a positive integer.

(ii) The product of three negative integers is negative integer.

(iii) The product of four negative integers is a positive integer.

(iv) The product of five negative integers is a negative integer.

Will the product of six negative integers be positive or negative? State reasons.

Try This

(a) $(-1) \times (-1) =$ ——

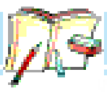
(b) $(-1) \times (-1) \times (-1) =$ ——

(c) $(-1) \times (-1) \times (-1) \times (-1) =$ ——

(d) $(-1) \times (-1) \times (-1) \times (-1) \times (-1) =$ ——

We further see that in (a) and (c) above, the number of negative integers that are multiplied are even (two and four respectively) and the products are positive integers. The number of negative integers that are multiplied in (b) and (d) are odd and the products are negative integers.

Thus, we find that if the number of negative integers being multiplied is even, then the product is a positive integer. And if the number of negative integers being multiplied is odd, the product is a negative integer.



Exercise - 4

1. Fill in the blanks.

(i) $(-100) \times (-6) =$ ....

(ii) $(-3) \times$ .... $= 3$

(iii) $100 \times (-6) =$ ....

(iv) $(-20) \times (-10) =$ ....

(v) $15 \times (-3) =$ ....

2. Find each of the following products.

(i) $3 \times (-1)$ (ii) $(-1) \times 225$

(iii) $(-21) \times (-30)$ (iv) $(-316) \times (-1)$

(v) $(-15) \times 0 \times (-18)$ (vi) $(-12) \times (-11) \times (10)$

(vii) $9 \times (-3) \times (-6)$ (viii) $(-18) \times (-5) \times (-4)$

(ix) $(-1) \times (-2) \times (-3) \times 4$ (x) $(-3) \times (-6) \times (-2) \times (-1)$

3. A certain freezing process requires that room temperature be lowered from 40ºC at the rate of 5ºC every hour. What will be the room temperature 10 hours after the process begins?

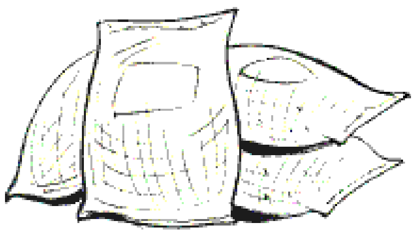
4. In a class test containing 10 questions, '3' marks are awarded for every correct answer and (–1) mark is for every incorrect answer and '0' for questions not attempted.

(i) Gopi gets 5 correct and 5 incorrect answers. What is his score?

(ii) Reshma gets 7 correct answers and 3 incorrect answers. What is her score?

(iii) Rashmi gets 3 correct and 4 incorrect answers out of seven questions she attempts. What is her score?

5. A merchant on selling rice earns a profit of ₹10 per bag of basmati rice sold and a loss of ₹5 per bag of non-basmati rice.



(i) He sells 3,000 bags of basmati rice and 5,000 bags of non-basmati rice in a month. What is his profit or loss in a month?

(ii) What is the number of basmati rice bags he must sell to have neither profit nor loss, if the number of bags of non-basmati rice sold is 6,400.

6. Replace the blank with an integer to make it a true statement.

(i) $(-3) \times$ ______ $= 27$ (ii) $5 \times$ ______ $= -35$

(iii) ______ $\times (-8) = -56$ (iv) ______ $\times (-12) = 132$

1.1.4 Division of integers

We know that division is the inverse operation of multiplication. Let us study some examples for natural numbers.

We know that $3 \times 5 = 15$

Therefore, $15 \div 5 = 3$ or $15 \div 3 = 5$

Similarly, $4 \times 3 = 12$

Therefore, $12 \div 4 = 3$ or $12 \div 3 = 4$

Thus, we can say that for each multiplication statement of natural numbers there are two corresponding division statements.

Can we write a multiplication statement and its corresponding division statements for integers?

Study the following and complete the table.

Multiplication statement	**Division statements**	
$2 \times (-6) = (-12)$	$(-12) \div (-6) = 2$,	$(-12) \div 2 = (-6)$
$(-4) \times 5 = (-20)$	$(-20) \div (5) = (-4)$,	$(-20) \div (-4) = 5$
$(-8) \times (-9) = 72$	$72 \div (-8) = (-9)$,	$72 \div (-9) = (-8)$
$(-3) \times (-7) =$ ________	______ $\div (-3) =$ _____ ,	____________________
$(-8) \times 4 =$ ___________	____________________ ,	____________________
$5 \times (-9) =$ ___________	____________________ ,	____________________
$(-10) \times (-5) =$	____________________ ,	____________________

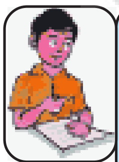
We can infer from the above that when we divide a negative integer by a positive integer or a positive integer by a negative integer, we divide them as whole numbers and then negative (–) sign for the quotient. We thus, get a negative integer as the quotient.

Do This

1. Solve the following.

(i) $(-100) \div 5$ (ii) $(-81) \div 9$ (iii) $(-75) \div 5$ (iv) $(-32) \div 2$

(v) $125 \div (-25)$ (vi) $80 \div (-5)$ (vii) $64 \div (-16)$



Try This

Can we say that $(-48) \div 8 = 48 \div (-8)$?

Check whether-

(i) $90 \div (-45)$ and $(-90) \div 45$ (ii) $(-136) \div 4$ and $136 \div (-4)$

We also observe that

$(-12) \div (-6) = 2$; $(-20) \div (-4) = 5$; $(-32) \div (-8) = 4$; $(-45) \div (-9) = 5$

So, we can say that when we divide a negative integer by a negative integer, we get a positive number as the quotient.

Do This

1. Compute the following.

(i) $-36 \div (-4)$ (ii) $(-201) \div (-3)$ (iii) $(-325) \div (-13)$

1.2 Properties of integers

In class VI we have learnt the properties of whole numbers. Here we will learn the properties of integers.

1.2.1 Properties of integers under addition

(i) Closure property

Study the following.

Statement	Conclusion
$5 + 8 = 13$	The sum is a whole number
$6 + 3 =$	
$13 + 0 =$	
$10 + 2 =$	
$0 + 6 = 6$	The sum is a whole number

Is the sum of two whole numbers always a whole number? You will find this to be true. Thus, we say that whole numbers follow the closure property of addition.

Do integers satisfy closure property of addition ? study the following additions and complete the balnks.

Statement	Conclusion
$6 + 3 = 9$	The sum is an integer
$-10 + 2 =$	
$-3 + 0 =$	
$-6 + 6 = 0$	
$(-2) + (-3) = -5$	
$7 + (-6) =$	The sum is an integer

Is the sum of two integers always an integer?

Can you give an example of a pair of integers whose sum is not an integer? You will not be able to find such a pair. **Therefore, integers are also closed under addition.**

In general, for any two integers a and b, a + b is also an integer.

(ii) Commutative property

Study the following and fill in the blanks.

Statement 1	Statement 2	Conclusion
4 + 3 = 7	3 + 4 = 7	4 + 3 = 3 + 4 = 7
3 + 5 =	5 + 3 =	
3 + 0 =	0 + 3 =	

Similarly, add as many pairs of whole numbers, as you wish. Did you find any pair of whole numbers for which the sum is different, when the order is changed. You will not find such a pair. Thus, we say that the addition of whole numbers is commutative.

Is addition of integers commutative? Study the following and fill in the blanks.

Statement 1	Statement 2	Conclusion
5 + (–6) = –1	(–6) + 5 = –1	5 + (–6) = (–6) + 5 = –1
–9 + 2 =	2 + (–9) =	
–4 + (–5) =	(–5) + (–4) =	

Did you find any pair of integers for which the sum is different when the order is changed? You would have not. **Therefore, addition is commutative for integers.**

In general, for any two integers a and b, a + b = b + a

(iii) Associative property

Let us study the following examples-

(i) (2 + 3) + 4 = 2 + (3 + 4)
5 + 4 = 2 + 7
9 = 9

(ii) (–2 + 3) + 5 = –2 + (3 + 5)
1 + 5 = –2 + 8
6 = 6

(iii) (–2 + 3) + (–5) = (- 2) + [3 + (–5)]
1 + (–5) = (- 2) + (-2)
–4 = –4

(iv) [(–2) + (–3)] + (-5) = –2 + [(–3) + (–5)]
–5 + (–5) = –2 + (–8)
–10 = –10

7th Maths EM Final 6.9.12.pdf 20 06-09-2012 12:48:32

Is the sum in each case equal? You will find this to be true.

Therefore, integers follow the associative property under addition.

Try This

1. (i) $(2 + 5) + 4 \quad = \quad 2 + (5 + 4)$

   (ii) $(2 + 0) + 4 \quad = \quad 2 + (0 + 4)$

Does the associative property hold for whole numbers? Take two more examples and write your answer.

In general, for any three integers a, b and c, $(a + b) + c = a + (b + c)$

(iv) Additive identity

Carefully study the following.

$-2 \;+\; 0 \;=\; -2$

$5 \;+\; 0 \;=\; 5$

$8 \;+\; 0 \;=$

$-10 \;+\; 0 \;=$

On adding zero to integers, do you get the same integer? Yes, you do.

Therefore, '0' is the additive identity for integers.

In general, for any integer a, $a+0 = 0 + a= a$

Try This

1. Add the following
   (i) $2 + 0 =$
   (ii) $0 + 3 =$
   (iii) $5 + 0 =$
2. Similarly, add zero to as many whole numbers as possible.
   Is zero the additive identity for whole numbers?

(v) Additive Inverse

What should be added to 3 to get its additive identity '0'?

Study the following-

$3 + (-3) = 0$

$7 + (-7) = 0$

$(-10) + 10 = 0$

Check whether we get similar pairs for other integers.

In each pair given above, one integer is called the additive inverse of the other integer.

In general, for any integer 'a' there exists an integer (–a) such that a + (–a) = 0. Both the integers are called additive inverse of each other.

1.2.2 Properties of integers under multiplication

(i) Closure property

Study the following and complete the table

Statement	Conclusion
$9 \times 8 = 72$	The product is an integer
$10 \times 0 =$	
$-15 \times 2 =$	
$-15 \times 3 = -45$	
$-11 \times -8 =$	
$10 \times 10 =$	
$5 \times -3 =$	

Is it possible to find a pair of integers whose product is not an integer? You will not find this to be possible.

Note: Do you remember fractions and decimals are not Integers.

Therefore, integers follow the closure property of multiplication.

In general, if a and b are two integers, a × b is also an integer.

7th Maths EM Final 6.9.12.pdf 22 06-09-2012 12:48:32

Try This

1. Multiply the following
   (i) $2 \times 3 =$ ________
   (ii) $5 \times 4 =$ ________
   (iii) $3 \times 6 =$ ________
2. Similarly, multiply any two whole numbers of your choice.
   Is the product of two whole numbers always a whole number?

(ii) Commutative property

We know that multiplication is commutative for whole numbers. Is it also commutative for integers?

Statement 1	Statement 2	Conclusion
$5 \times (-2) = -10$;	$(-2) \times 5 = -10$	$5 \times (-2) = (-2) \times 5 = -10$
$(-3) \times 6 =$	$6 \times (-3) =$	
$-20 \times 10 =$	$10 \times (-20) =$	

Therefore, multiplication of integers follows the commutative property.

In general, for any two integers a and b, $a \times b = b \times a$

(iii) Associative property

Consider the multiplication of 2, –3, –4 grouped as follows.

$[2 \times (-3)] \times (-4)$ and $2 \times [(-3) \times (-4)]$

We see that-

$[2 \times (-3)] \times (-4)$ and $2 \times [(-3) \times (-4)]$
$= (-6) \times (-4)$ $= 2 \times 12$
$= 24$ $= 24$

In first case 2, –3 are grouped together and in the second –3, –4 are grouped together. In both cases the product is the same.

Thus, $[2 \times (-3)] \times [(-4)] = 2 \times [(-3) \times (-4)]$

Does the grouping of integers affect the product of integers? No, it does not.

The product of three integers does not depend upon the grouping of integers. Therefore, the multiplication of integers is associative.

In general, for any integers, a, b and c, $(a \times b) \times c = a \times (b \times c)$

Do This

1. Is $[(-5) \times 2)] \times 3 = (-5) \times [(2 \times 3)]$?

2. Is $[(-2) \times 6] \times 4 = (-2) \times [(6 \times 4)]$?

Try This

$(5 \times 2) \times 3 = 5 \times (2 \times 3)$

Is the associative property true for whole numbers? Take many more examples and verify.

(iv) Distributive property

We know that, $9 \times (10 + 2) = (9 \times 10) + (9 \times 2)$

Thus, multiplication distributes over addition is true for whole numbers.

Let us see if this is true for integers-

(i) $-2 \times (1 + 3) = [(-2) \text{ x } 1] + [(-2) \times 3]$

$-2 \times 4 = -2 + (-6)$

$-8 = -8$

(ii) $-1 \times [3 + (-5)] = [(-1) \times 3] + [(-1) \times (-5)]$

$-1 \times (-2) = -3 + (+5)$

$2 = 2$

Verify $-3 \times (-4+2) = [(-3) \times (-4)] + [-3 \times (2)]$

You will find that in each case, the left hand side is equal to the right hand side.

Thus, multiplication distributes over addition of integers too.

In general, for any integers a, b and c, $a \times (b + c) = a \times b + a \times c$

7th Maths EM Final 6.9.12.pdf 24 06-09-2012 12:48:32

(v) Multiplicative identity

$2 \times 1 = 2$

$-5 \times 1 = -5$

$-3 \times 1 =$ ______

$-8 \times 1 =$ ______

$1 \times -5 =$ ______

0 is the additive identity of integers

You will find that multiplying an integer by 1 does not change the integer. Thus, 1 is called the multiplicative identity for integers.

In general, for any integer 'a', $a \times 1 = 1 \times a = a$

(vi) Multiplication by zero

We know that any whole number when multiplied by zero gives zero. What happens in case of integers? Study the following-

$(-3) \times 0 = 0$

$0 \times (-8) =$ ______

$9 \times 0 =$ ______

This shows that the product of an integer and zero is zero.

In general for any integer a, $a \times 0 = 0 \times a = 0$

Exercise 5

1. Verify the following.
   (i) $18 \times [7 + (-3)] = [18 \times 7] + [18 \times (-3)]$
   (ii) $(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$
2. (i) For any integer a, what is (–1) x a equal to?
   (ii) Determine the integer whose product with (–1) is 5
3. Find the product, using suitable properties.
   (i) $26 \times (-48) + (-48) \times (-36)$ (ii) $8 \times 53 \times (-125)$
   (iii) $15 \times (-25) \times (-4) \times (-10)$ (iv) $(-41) \times 102$
   (v) $625 \times (-35) + (-625) \times 65$ (vi) $7 \times (50 - 2)$
   (vii) $(-17) \times (-29)$ (viii) $(-57) \times (-19) + 57$

1.2.3 Properties of integers under subtraction

(i) Closure under subtraction

Do we always get an integer, when subtracting an integer from an integer?

Do the following.

$9 - 7 =$ ______

$7 - 10 =$ ______

$2 - 3 =$ ______

$-2 - 3 =$ ______

$-2 - (-5) =$ ______

$0 - 4 =$ ______

What did you find? Can we say that integers follow the closure property for subtraction?

Therefore, for any integers a and b, a – b is also an integer.

(ii) Commutativity under subtraction

Let us take an example. Consider the integers 6 and –4

$$6 - (-4) = 6 + 4 = 10 \quad \text{and}$$

$$-4 - (6) = -4 - 6 = -10$$

Therefore, $6 - (-4) \neq -4 - (6)$

Thus, subtraction is not commutative for integers.

Try This

Take atleast 5 different pairs of integers and see if subtraction is commutative.

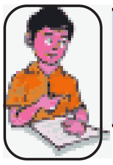
1.2.4 Properties of integers under division

(i) Closure Property

Study the following table and complete it.

Statement	Inference	Statement	Inference
$(-8) \div (-4) = 2$	Result is an integer	$(-8) \div 4 = \dfrac{-8}{4} = -2$	
$(-4) \div (-8) = \dfrac{-4}{-8} = \dfrac{1}{2}$	Result is not an integer	$4 \div (-8) = \dfrac{4}{-8} = \dfrac{-1}{2}$	

What can you infer? You will infer that integers are not closed under division.



Try This

Take atleast five pairs of integers and check whether they are closed under division.

(ii) Commutative Property

We know that division is not commutative for whole numbers. Let us check it for integers also.

You can see from the table given above that $(-8) \div (-4) \neq (-4) \div (-8)$.

Is $(-9) \div 3$ the same as $3 \div (-9)$?

Is $(-30) \div (6)$ the same as $(-6) \div (-30)$?

Thus, we can say that **division of integers is not commutative.**

Try This

Take atleast 5 pairs of integers and see if division of integers is commutative.

(iii) Division by Zero

Like whole numbers, any integer divided by zero is meaningless and zero divided by an integer other than zero is equal to zero.

For any integer a, $a \div 0$ is not defined but $0 \div a = 0$ for $a \neq 0$.

(iv) Identity in division

When we divide a whole number by 1 it gives the same whole number. Let us check whether this is true for negative integers also.

Observe the following-

$(-8) \div 1 = (-8)$ $\quad$ $(11) \div 1 = +11$ $\quad$ $(-13) \div 1 =$ _____ $\quad$ $(-25) \div 1 =$ _______

Thus, a negative integer or a positive integer divided by 1 gives the same integer as quotient.

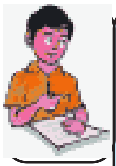
1 is the identity in division of integers.

In general, for any integer a, $a \div 1 = a$.

What happens when we divide any integer by (-1)? Complete the following table-

$(-8) \div (-1) = 8$ $\quad$ $11 \div (-1) = -11$ $\quad$ $13 \div (-1) =$ _____ $\quad$ $(-25) \div (-1) =$ _____

We can say that if any integer is divided by (-1) it does not give the same integer, but gives its additive identity.



Try This

1. For any integer a, is

   (i) $a \div 1 = 1$?

   (ii) $a \div (-1) = -a$?

   Take different values of 'a' and check.

(iii) Associative property

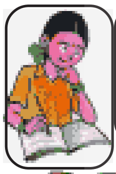
Is $[(-16) \div 41] \div (-2) = (-16) \div [4 \div (-2)]$?

$[(-16) \div 4] \div (-2) = (-4) \div (-2) = 2$

$(-16) \div [4 \div (-2)] = (-16) \div (-2) = 8$

Therefore, $[(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

Thus, division of integers is not associative.



Try This

Take atleast five more examples and check whether division is associative for integers.

Exercise - 6

1. Fill the following blanks.

   (i) $-25 \div$ .... = 25

   (ii) .... $\div 1$ = -49

   (iii) $50 \div 0$ = ...

   (iv) $0 \div 1$ = ...

1.3 Some practical problems using negative numbers

Example 1 : In a test (+5) marks are given for every correct answer and (–2) marks are given for every incorrect answer. (i) Radhika answered all the questions and scored 30 marks through 10 correct answers. (ii) Jaya also answered all the questions and scored (–12) marks through 4 correct answers. How many incorrect answers had both Radhika and Jaya attempted?

Solution : (i) Marks given for one correct answer $= 5$

So marks given for 10 correct answers $= 5 \times 10 = 50$

Radhika's score $= 30$

Marks obtained for incorrect answers $= 30 - 50 = -20$

Marks given for one incorrect answer $= (-2)$

Therefore, number of incorrect answers $= (-20) \div (-2) = 10$

(ii) Marks given for 4 correct answers $= 5 \times 4 = 20$

Jaya's score $= -12$

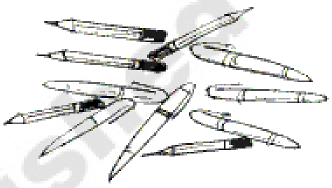
Marks obtained for incorrect answers $= -12 - 20 = -32$

Marks given for one incorrect answer $= (-2)$

Therefore number of incorrect answers $= (-32) \div (-2) = 16$

Example 2 : A shopkeeper earns a profit of ₹ 1 by selling one pen and incurs a loss of 40 paise per pencil while selling pencils of her old stock.

(i) In a particular month she incurs a loss of ₹ 5. In this period, she sold 45 pens. How many pencils did she sell in this period?



(ii) In the next month she earns neither profit nor loss. If she sold 70 pens, how many pencils did she sell?

Solution : (i) Profit earned by selling one pen ₹ 1

Profit earned by selling 45 pens = ₹ 45, which we denote by 45

Total loss given = ₹ 5, which we denote by -5.

Profit earned + Loss incurred = Total loss

Therefore, Loss incurred = Total loss-Profit earned

$= -5 - (45) = (-50) = -$ ₹ $50 = -5000$ paise

Loss incurred by selling one pencil = 40 paise which we write as -40 paise

So, number of pencils sold $= (-5000) \div (-40) = 125$ pencils.

(ii) In the next month there is neither profit nor loss.

So, profit earned + loss incurred $= 0$

i.e., Profit earned $= -$ Loss incurred.

Now, profit earned by selling 70 pens = ₹ 70

Hence, loss incurred by selling pencils = $-$₹ 70 or -7000 paise.

Total number of pencils sold $= (-7000) \div (-40) = 175$ pencils.

Exercise - 7

1. In a class test containing 15 questions, 4 marks are given for every correct answer and (-2) marks are given for every incorrect answer. (i) Bharathi attempts all questions but only 9 answers are correct. What is her total score? (ii) One of her friends Hema answers only 5 questions correct. What will be her total score?

2. A cement company earns a profit of ₹ 9 per bag of white cement sold and a loss of ₹ 5 per bag of grey cement sold.

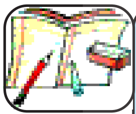
(i) The company sells 7000 bags of white cement and 6000 bags of grey cement in a month. What is its profit or loss?

(ii) What is the number of white cement bags it must sell to have neither profit nor loss, if the number of grey bags sold is 5400.

3. The temperature at 12 noon was 10° C above zero. If it decreases at the rate of 2°C per hour until midnight, at what time would the temperature be 8°C below zero? What would be the temperature at midnight?

4. In a class test (+3) marks are given for every correct answer and (–2) marks are given for every incorrect answer and no marks for not attempting any question. (i) Radhika scored 20 marks. If she has got 12 correct answers, how many questions has she attempted incorrectly? (ii) Mohini scores (–5) marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?

5. An elevator descends into a mine shaft at the rate of 6 meters per minute. If the decent starts from 10 m above the ground level, how long will it take to reach – 350 m.



Looking Back

1. **N** (natural numbers) = 1, 2, 3, 4, 5 . . .

   W (whole numbers) = 0, 1, 2, 3, 4, 5 . . .

   Z (Integers) = ..., –4, –3, –2, –1, 0, 1, 2, 3, 4 . . .

   also 0, ±1, ±2, ±3 (set of integers also represented as I.)

2. (i) Each time you add a positive integer, you move right on the number line.

   (ii) Each time you add a negative integer, you move left on the number line.

3. (ii) Each time you subtract a positivie integer, you move left on the number line.

   (iii) Each time you subtract a negative integer, you move right on the number line.

4. (i) Each time you multiply a negative integer by a positive integer or a positive integer by a negative integer, the product is a negative integer.

(ii) Each time you multiply two negative integers, the product is a positive integer.

(iii) Product of even number of negative integers is positive (+ve), product of odd number of negative integers is negative (–ve).

5. (i) Each time you divide a negative integer by a positive integer or a positive integer by a negative integer the quotient is negative integer.

(ii) Each time you divide negative integer by a negative integer the quotient is positive integer.

(iii) When you multiply or divide two integers of same sign the result is always positive; if they are of opposite signs the result is negative.

6. The following are the propeties satisfied by addition and subtraction of integers-

(i) Integers are closed for addition and subtraction both. i.e., $a + b$ and $a–b$ are integers, where a and b are any integers.

(ii) Addition is commutative for integers, i.e., $a + b = b + a$, for all integers a and b.

(iii) Addition is associative for integers, i.e., $(a+b) + c = a + (b + c)$, for all integers a, b, and c.

(iv) Integer 0 is the identity under addition, i.e., $a + 0 = 0 + a = a$, for every integer a.

7. Integers show some properties under multiplication.

(i) Integers are closed under multiplication. i.e., $a \times b$ is an integer for any two integers a and b.

(ii) Multiplication is commutative for integers. i.e., $a \times b = b \times a$ for any integers a and b.

(iii) The integer 1 is the identity under multiplication, i.e., $1 \times a = a \times 1 = a$, for any integer a.

(iv) Multiplication is associative for integers, i.e., $(a \times b) \times c = a \times (b \times c)$ for any three integers a, b, and c.

8. In intergers multiplication distributes over addition. i.e., $a \times (b+c) = a \times b + a \times c$ for any three integers a, b and c. This is called distributive property.

9. The properties of commutativity and associativity under addition and multiplication and the distributive property help us make our calculations easier.

10. For any integer a, we have

(i) $a \div 0$ is not defined or meaningless

(ii) $0 \div a = 0$ (for $a \neq 0$)

(iii) $a \div 1 = a$