FRACTIONS, DECIMALS AND RATIONAL NUMBERS

2

2.0 Introduction

We come across many examples in our day-to-day life where we use fractions. Just try to recall them. We have learnt how to represent proper and improper fractions and their addition and subtraction in the previous class. Let us review what we have already learnt and then go further to multiplication and division of fractional numbers as well as of decimal fractions. We will conclude by an introduction to a bigger set of numbers called rational numbers.

The shaded portion of the figures given below have been represented using fractions. Are the representations correct?

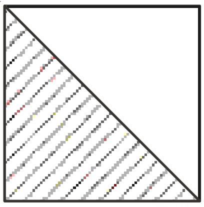


Figure 1

$\frac{1}{2}$

Y/N

Reason ....

Figure 2

$\frac{1}{2}$

Y/N

Reason ....

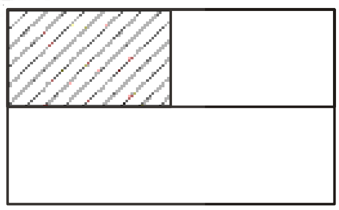


Figure 3

$\frac{1}{3}$

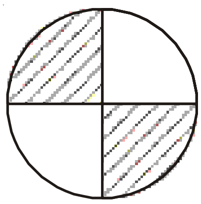
Y/N

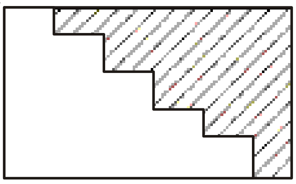
Reason ....

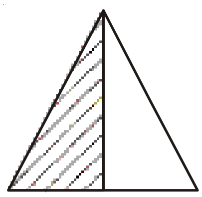
While considering the above you must have checked if each all parts are equal or not?.

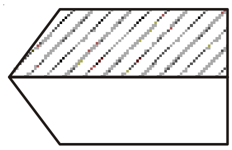
Make 5 more such examples and give them to your friends to verify.

Here is Neha's representation of $\frac{1}{2}$ in different figures.









Do you think that the shaded portions correctly represent $\frac{1}{2}$? Then what fractions are represented by unshaded portions?

Try This

Represent $\frac{3}{4}, \frac{1}{4}$ in different ways using different figures. Justify your representation. Share, and check it with your friends.

Proper and Improper fractions

You have learnt about proper and improper fractions. A proper fraction is a fraction that represents a part of a whole. Give five examples of proper fractions.

Is $\frac{3}{2}$ a proper fraction? How do you check if it is a proper fraction or not?

What are the properties of improper fractions? One of them is that in improper fractions the numerator is more or equal to than the denominator. What else do we know about these fractions.

We can see that all improper fractions can be written as mixed fractions. The improper fraction $\frac{3}{2}$ can be written as $1\frac{1}{2}$. This is a mixed fraction. This contains an integral part and a fractional part. The fractional part should be a proper fraction.

Do This

1. Write five examples, each of proper, improper and mixed fractions?

Try This

Represent $2\frac{1}{4}$ pictorically. How many units are needed for this.

Comparison of fractions

Do you remember how to compare like fractions, for e.g. $\frac{1}{5}$ and $\frac{3}{5}$? $\frac{3}{5}$ is bigger than $\frac{1}{5}$. Why?

Can you recall how to compare two unlike fractions, for e.g. $\frac{5}{7}$ and $\frac{3}{4}$?

We convert these into like fractions and then compare them.

$$\frac{5}{7} \times \frac{4}{4} = \frac{20}{28} \quad \text{and} \quad \frac{3}{4} \times \frac{7}{7} = \frac{21}{28}$$

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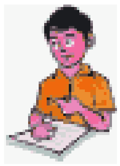
Since $\frac{5}{7} = \frac{20}{28}$ and $\frac{3}{4} = \frac{21}{28}$

Thus, $\frac{5}{7} < \frac{3}{4}$

Do These



1. Write five equivalent fractions for (i). $\frac{3}{5}$ (ii). $\frac{4}{7}$.

2. Which is bigger $\frac{5}{8}$ or $\frac{3}{5}$?

3. Determine if the following pairs are equal by writing each in their simplest form.

   (i) $\frac{3}{8}$ and $\frac{375}{1000}$ (ii) $\frac{18}{54}$ and $\frac{23}{69}$

   (iii) $\frac{6}{10}$ and $\frac{600}{1000}$ (iv) $\frac{17}{27}$ and $\frac{25}{45}$

You have already learnt about addition and subtraction of fractions in class VI. Let us solve some problems now.

Example 1 : Razia completes $\frac{3}{7}$ part of her homework while Rekha completed $\frac{4}{9}$ of it. Who has completed the lesser part?

Solution : To find this we have to compare $\frac{3}{7}$ and $\frac{4}{9}$.

Converting them to like fractions we have

$$\frac{27}{63} < \frac{28}{63}$$

Thus, $\frac{27}{63} < \frac{28}{63}$ and so $\frac{3}{7} < \frac{4}{9}$

Razia has completed a lesser part of her homework than Rekha.

Example 2 : Shankar's family consumed $3\frac{1}{2}$ kg sugar in the first 15 days of a month. For the next 15 days they consumed $3\frac{3}{4}$ kg sugar. How much sugar did they consume for the whole month?

Solution : The total weight of the sugar for the whole month

$$= \left(3\frac{1}{2}+3\frac{3}{4}\right) \text{ kg}$$

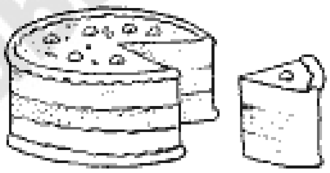
$$= \left(\frac{7}{2}+\frac{15}{4}\right) \text{ kg} = \left(\frac{14}{4}+\frac{15}{4}\right)$$

$$= \frac{29}{4} \text{ kg} = 7\frac{1}{4} \text{ kg.}$$

Example 3 : At Ahmed's birthday party, $\frac{5}{7}$ part of the total cake was distributed. Find how much cake is left?

Solution : Total cake is one $= 1$ or $\frac{1}{1}$

Cake distributed $= \frac{5}{7}$



The cake left $= 1-\frac{5}{7}$

$$= \frac{7}{7}-\frac{5}{7} = \frac{2}{7}$$

Thus, $\frac{2}{7}$ part of the total cake is left now.

Exercise - 1

1. Solve the following.

(i) $2+\frac{3}{4}$ (ii) $\frac{7}{9}+\frac{1}{3}$ (iii) $1-\frac{4}{7}$

(iv) $2\frac{2}{3}+\frac{1}{2}$ (v) $\frac{5}{8}-\frac{1}{6}$ (vi) $2\frac{2}{3}+3\frac{1}{2}$

2. Arrange the following in ascending order.

(i) $\frac{5}{8}, \frac{5}{6}, \frac{1}{2}$ (ii) $\frac{2}{5}, \frac{1}{3}, \frac{3}{10}$

3. Check whether in this square the sum of the numbers in each row and in each column and along the diagonals is the same.

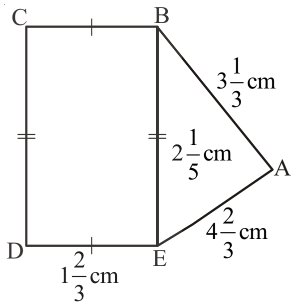
$\frac{6}{13}$	$\frac{13}{13}$	$\frac{2}{13}$
$\frac{3}{13}$	$\frac{7}{13}$	$\frac{11}{13}$
$\frac{12}{13}$	$\frac{1}{13}$	$\frac{8}{13}$

4. A rectangular sheet of paper is $5\frac{2}{3}$ cm long and $3\frac{1}{5}$ cm wide. Find its perimeter.

5. The recipe requires $3\frac{1}{4}$ cups of flour. Radha has $1\frac{3}{8}$ cups of flour. How many more cups of flour does she need?

6. Abdul is preparing for his final exam. He has completed $\frac{5}{12}$ part of his course content. Find out how much course content is left?

7. Find the perimeters of (i) ΔABE (ii) the rectangle BCDE in this figure. Which figure has greater perimetre and by how much?



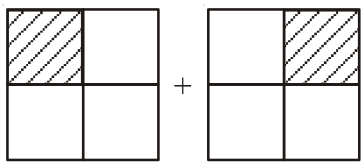
2.1 Multiplication of fractions

2.1.1 Multiplication of a fraction by a whole number

When we multiply whole numbers we know we are repeatedly adding a number. For example 5 x 4 means adding 5 groups of 4 each or 5 times 4.

Thus, when we say $2\times\frac{1}{4}$ it means adding $\frac{1}{4}$ twice or 2 times $\frac{1}{4}$.

Let us represnet this pictorially. Look at Figure 1. Each shaded part is $\frac{1}{4}$ part of a square. The two shaded parts together will represent $2\times\frac{1}{4}=\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$.



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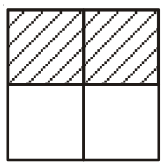


Figure 1 Figure 2

Let us now find $3\times\frac{1}{2}$. This means three times $\frac{1}{2}$ or three halves.

We have $$3\times\frac{1}{2}=\frac{1}{2}+\frac{1}{2}+\frac{1}{2}=\frac{3}{2}$$

Do This

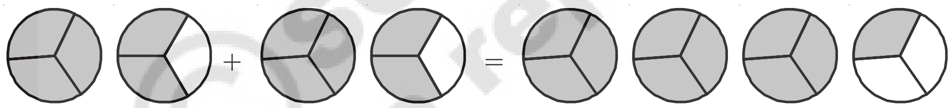
1. Find (i) $4\times\frac{2}{7}$ (ii) $4\times\frac{3}{5}$ (iii) $7\times\frac{1}{3}$

The fractions that we have considered till now, i.e., $\frac{1}{2},\frac{2}{3},\frac{2}{7}$ and $\frac{3}{5}$ are proper fractions.

Let us consider some improper fractions like $\frac{5}{3}$ .and how to multiply $2\times\frac{5}{3}$

$$2\times\frac{5}{3}=\frac{5}{3}+\frac{5}{3}=\frac{10}{3}=3\frac{1}{3}$$

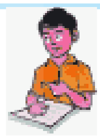
Pictorially



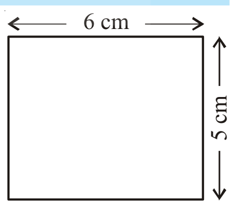
$$\frac{5}{3}+\frac{5}{3}=\frac{10}{3}$$

Do This

1. Find (i). $5\times\frac{3}{2}$ (ii) $4\times\frac{7}{5}$ (iii) $7\times\frac{8}{3}$



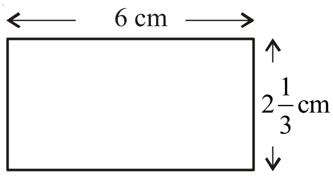
We know the area of a rectangle is equal to length × breadth. If the length and breadth of a rectangle are 6 cm and 5 cm respectively, then what will be its area? Obviously the area would be $6\times5=30$ cm^2.



If the length and breadth of a rectangle are 6 cm, $2\frac{1}{3}$ cm respectively, what would be the area of that rectangle.

Area of a rectangle is the product of its length and breadth. To multiply a mixed fraction with a whole number, first convert the mixed fractions to an improper fraction and then multiply.

Therefore, area of a rectangle $= 6 \times 2\frac{1}{3}$



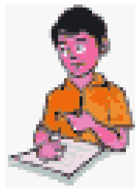
$= 6 \times \frac{7}{3} = \frac{42}{3}\ cm^2 = 14 cm^2$

You might have realised by now that to multiply a whole number with a proper or an improper fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator the same.

Do These

1. Find the following.

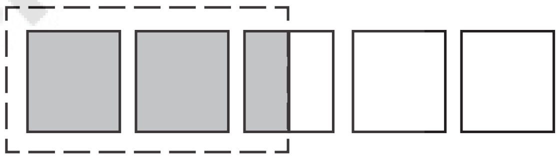
(i) $3 \times 2\frac{2}{7}$ (ii) $5 \times 2\frac{1}{3}$ (iii) $8 \times 4\frac{1}{7}$ (iv) $4 \times 1\frac{2}{9}$ (v) $5 \times 1\frac{1}{3}$



2. Represent pictorially $2 \times \frac{1}{5} = \frac{2}{5}$

Consider $\frac{1}{2} \times 5$. How do you understand it?

$\frac{1}{2} \times 5$ means half of 5, which is $\frac{5}{2}$ or $2\frac{1}{2}$



Thus, $\frac{1}{2}$ of $5 = \frac{1}{2} \times 5 = \frac{5}{2}$

Similarly $= \frac{1}{2}$ of $3 = \frac{1}{2} \times 3 = \frac{3}{2}$ or $1\frac{1}{2}$

Here onwards 'of' represents multiplication.

So what would $\frac{1}{4}$ of 16 mean? It tells us that the whole (16) is to be divided into 4 equal parts and one part out of that has to be taken. When we make 4 equal parts of 16, each part will be 4. So $\frac{1}{4}$ of 16 is 4.

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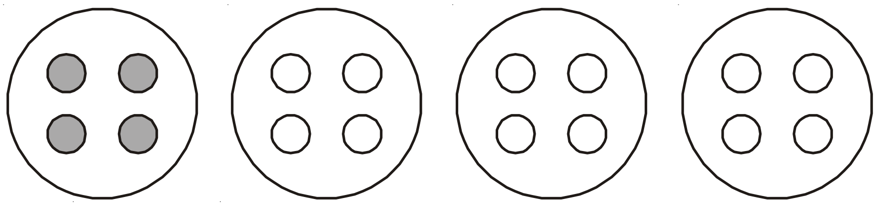
This can be illustrated with marbles as :



$\frac{1}{4}$ of 16 or $\frac{1}{4} \times 16 = \frac{16}{4} = 4$

Similarly, $\frac{1}{2}$ of $16 = \frac{1}{2} \times 16 = \frac{16}{2} = 8$.

Example 4 : Nazia has 20 marbles. Reshma has $\frac{1}{5}$ of the number of marbles that Nazia has. How many marbles does Reshma have?

Solution : Reshma has $\frac{1}{5} \times 20 = 4$ marbles.

Example 5 : In a family of four persons 15 chapaties were consumed in a day. $\frac{1}{5}$ of the chapaties were consumed by the mother and $\frac{3}{5}$ were consumed by the children and the remaining were eaten by the father.

(i) How many chapaties were eaten by the mother?

(ii) How many chapaties were eaten by the children?

(iii) What fraction of the total chapaties has been eaten by the father?

Solution : Total number of chapaties = 15

(i) Number of chapaties eaten by mother $\frac{1}{5} \times 15 = 3$ chapaties

(ii) $\frac{3}{5}$ of the total number is eaten by children, $\frac{3}{5} \times 15 = 9$ chapaties

(iii) The chapaties left for father = 15 – 3 – 9 = 3 chapaties

Fraction of chapathies eaten by father $= \frac{3}{15} = \frac{1}{5}$

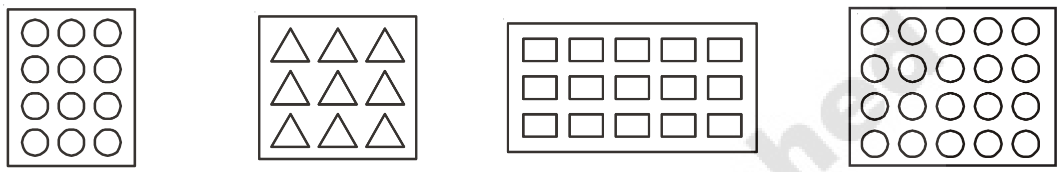
Exercise - 2

1. Multiply the following. Write the answer as a mixed fraction.

   (i) $\frac{3}{6}\times 10$ (ii) $\frac{1}{3}\times 4$ (iii) $\frac{6}{7}\times 2$ (iv) $\frac{2}{9}\times 5$ (v) $15\times\frac{2}{5}$

2. Shade : (i) $\frac{1}{2}$ of the circles in box (a) (ii) $\frac{2}{3}$ of the triangles in box (b)

   (iii) $\frac{3}{5}$ of the rectangles in box (c) (iv) $\frac{3}{4}$ of the circles in box (d)



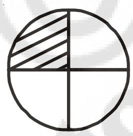
(a) (b) (c) (d)

3. Find (i) $\frac{1}{3}$ of 12 (ii) $\frac{2}{5}$ of 15

2.1.2 Multiplication of a fraction with a fraction

What does $\frac{1}{2}\times\frac{1}{4}$ mean? From the above we can understand that it means $\frac{1}{2}$ of $\frac{1}{4}$.

Consider $\frac{1}{4}$ -



How will we find $\frac{1}{2}$ of this shaded part? We can divide this one-fourth $\left(\frac{1}{4}\right)$ shaded part into two equal parts (Figure 1). Each of these two parts represents $\frac{1}{2}$ of $\frac{1}{4}$.

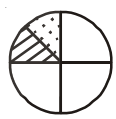


Figure 1

Let us call one of these parts this part 'A'. What fraction is 'A' of the whole circle? If we divide the remaining parts of the circle into two equal parts each, we get a total of eight equal parts. 'A' is one of these parts.

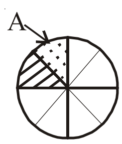
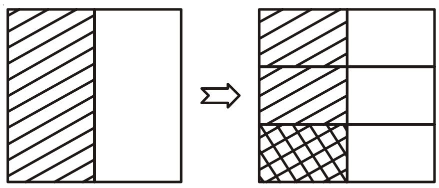
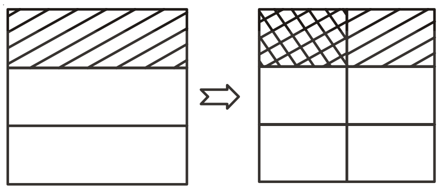


Figure 2

So, 'A' is $\frac{1}{8}$ of the whole. Thus, $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$

Find $\frac{1}{3}\times\frac{1}{2}$ and $\frac{1}{2}\times\frac{1}{3}$.

$\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{3}$ of  $= \frac{1}{6}$ thus $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{3}\times\frac{1}{2} = \frac{1}{6}$

$\frac{1}{2}$ of $\frac{1}{3}$ is $\frac{1}{2}$ of  $= \frac{1}{6}$ thus $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$

We can see that $\frac{1}{3}\times\frac{1}{2} = \frac{1}{2}\times\frac{1}{3}$

Do These

1. Fill in these boxes:

   (i) $\frac{1}{5}\times\frac{1}{7} = \frac{1\times 1}{5\times 7} = \Box$ (ii) $\frac{1}{2}\times\frac{1}{6} = \Box = \Box$

2. Find $\frac{1}{2}\times\frac{1}{5}$ and $\frac{1}{5}\times\frac{1}{2}$ using diagram check whether $\frac{1}{2}\times\frac{1}{5} = \frac{1}{5}\times\frac{1}{2}$

Consider one more example $\frac{2}{3}$ of $\frac{2}{5}$. We have shown $\frac{2}{5}$ in Figure 1 and $\frac{2}{3}\times\frac{2}{5}$ in Figure 2.

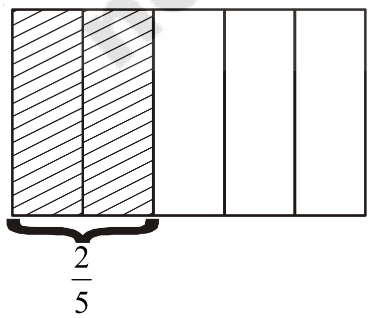


Figure 1

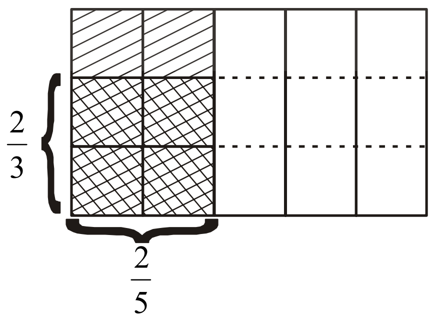


Figure 2

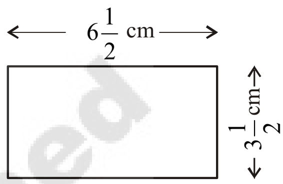
The cross hatched portion in figure (2) represents $\frac{2}{3}$ of $\frac{2}{5}$ or $\frac{2}{3}\times\frac{2}{5} = \frac{4}{15}$

To find the $\frac{2}{3}$ of $\frac{2}{5}$, we have made three equal parts of $\frac{2}{5}$ and then selected 2 out of the 3 parts.

This represent 4 parts out of a total 15 parts so $\frac{2}{3}$ of $\frac{2}{5} = \frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$.

Here, we can observe that Product of two fractions = $\dfrac{\textbf{Product of Numerators}}{\textbf{Product of Denominators}}$.

Now, what will be the area of the rectangle be if its length and breadth are $6\frac{1}{2}$ cm and $3\frac{1}{2}$ cm respectively.



The area = $6\frac{1}{2} \times 3\frac{1}{2} = \frac{13}{2} \times \frac{7}{2}$ cm^2. = $= \frac{91}{4} = 22\frac{3}{4}$ cm^2

Example 6 : Narendra reads $\frac{1}{4}$ of a short novel in 1 hour. What part of the book will he have read in $2\frac{1}{2}$ hours?

Solution : The part of the novel read by Narendra in 1 hour = $\frac{1}{4}$

So the part of the novel read by him in $2\frac{1}{2}$ hours $= 2\frac{1}{2} \times \frac{1}{4} \quad = \frac{5}{2} \times \frac{1}{4} = \frac{5}{8}$

So Narendra would read $\frac{5}{8}$ part of the novel in $2\frac{1}{2}$ hours.

Example 7 : A swimming pool is filled $\frac{3}{10}$ part in half an hour. How much will it be filled in $1\frac{1}{2}$ hour?

Solution : The part of the pool filled in half an hour = $\frac{3}{10}$.

So, the part of pool which is filled in $1\frac{1}{2}$ hour is 3 times the pool filled in half an hour.

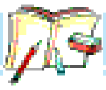
$= 3 \times \frac{3}{10} = \frac{9}{10}$ Thus, $\frac{9}{10}$ part of the pool will be filled in $1\frac{1}{2}$ hours.

Try This

You have seen that the product of two natural numbers is one or more than one is bigger than each of the two natural numbers. For example, $3 \times 4 = 12$, $12 > 4$ and $12 > 3$. What happens to the value of the product when we multiply two proper fractions?

Fill the following table and conclude your observations.

Eg: $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$	$\frac{8}{15} < \frac{2}{3}, \frac{8}{15} < \frac{4}{5}$	Product is less than each of the fractions
$\frac{1}{5} \times \frac{2}{7} = $ ------		
$\frac{3}{5} \times \frac{\square}{2} = \frac{21}{10}$		
$\frac{5}{\square} \times \frac{4}{3} = \frac{20}{6}$		



Exercise - 3

1. Find each of the following products.

   (i) $\frac{5}{6} \times \frac{7}{11}$ (ii) $6 \times \frac{1}{5}$ (iii) $2\frac{1}{3} \times 3\frac{1}{5}$

2. Multiply and reduce to lowest form.

   (i) $\frac{2}{3} \times 5\frac{1}{5}$ (ii) $\frac{2}{7} \times \frac{1}{3}$ (iii) $\frac{9}{3} \times \frac{5}{5}$

3. Which one is greater?

   (i) $\frac{2}{5}$ of $\frac{4}{7}$ or $\frac{3}{4}$ of $\frac{1}{2}$ (ii) $\frac{1}{2}$ of $\frac{4}{7}$ or $\frac{2}{3}$ of $\frac{3}{7}$

4. Rehana works $2\frac{1}{2}$ hours each day on her embroidery. She completes the work in 7 days. How many hours did she take to complete her work?

5. A truck runs 8 km using 1 litre of petrol. How much distance will it cover using $10\frac{2}{3}$ litres of petrol?

6. Raja walks $1\frac{1}{2}$ meters in 1 second. How much distance will he walk in 15 minutes?

7. Provide the number in the box $\square$ to make the statement true.

(i) $\frac{2}{3}\times\square = \frac{20}{21}$. (ii) $\frac{5}{7}\times\frac{\square}{5} = \frac{3}{\square}$

2.2 Division of fractions

Imagine you have 15 meters length of cloth and you want to make pieces of $1\frac{1}{2}$ metres length each from it. How many $1\frac{1}{2}$ meter pieces will you get? Here we will successively subtract $1\frac{1}{2}$ meters from 15 meters and see how many times we can do this, till we have no cloth left.

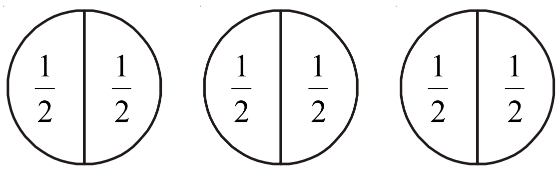
Look at one more example. A paper strip of length $\frac{21}{2}$ cm has to be cut into smaller strips of length $\frac{3}{2}$ cm each. How many pieces would we get? Clearly, we can cut $\frac{3}{2}$ cm each time or divide $\frac{21}{2}$ by $\frac{3}{2}$ i.e., $\frac{21}{2}\div\frac{3}{2}$.

Let us recall division with whole numbers. In $15\div 3$, we find out how many 3's are there in 15. The answer to this is 5. Similarly, to find the number of 2's in 18, we divide 18 by 2 or $18\div 2$. The answer to this is 9. Now correlate the same process in dividing whole numbers by fractions and fractions by fractions.

2.2.1 Division of whole number by a fraction

Let us find $3\div\frac{1}{2}$.

Kiran says we have to find how many halves $\left(\frac{1}{2}\right)$ are there in 3. To find the number of $\frac{1}{2}$ in 3, we draw the following.



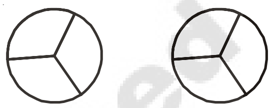
The figure above suggests that there are 6 halves in 3.

We can therefore say $3 \div \frac{1}{2} = 6$

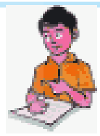
Think about $2 \div \frac{1}{3}$

This means finding how many one-thirds $\left(\frac{1}{3}\right)$ are there two wholes. How would you find these?

We can see that there are 6 one-thirds in two wholes or $2 \div \frac{1}{3} = 6$



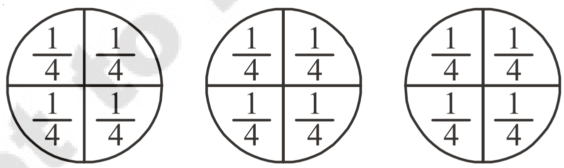
Do This



Find (i) $2 \div \frac{1}{4}$ (ii) $7 \div \frac{1}{2}$ (iii) $3 \div \frac{1}{5}$

2.2.1a) Reciprocal of a fraction

Now consider $3 \div \frac{1}{4}$. This means the number of $\frac{1}{4}$ parts obtained, when each of the three wholes, are divided into $\frac{1}{4}$ equal parts



The number of one-fourths is 12, or $3 \div \frac{1}{4} = 12$

We also see that, $3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 12$.

This suggests that $3 \div \frac{1}{4} = 3 \times \frac{4}{1}$

Also examine $2 \div \frac{1}{3}$.

We already found that $2 \div \frac{1}{3} = 6$

As inthe above example $2 \div \frac{1}{3} = 2 \times \frac{3}{1} = 6$

Similarly, $4 \div \frac{1}{4} = 16$ and $4 \times \frac{4}{1} = 16$.

The number $\frac{3}{1}$ can be obtained by interchanging the numerator and denominator of $\frac{1}{3}$ or by inverting $\frac{1}{3}$. Similarly, $\frac{4}{1}$ is obtained by inverting $\frac{1}{4}$.

Observe these products and fill in the blanks:

$7 \times \frac{1}{7} = 1$ $\qquad$ $\frac{2}{3} \times \frac{3}{2} = \frac{2 \times 3}{3 \times 2} = \frac{6}{6} = 1$

$\frac{1}{9} \times 9 =$ .... $\qquad$ $\frac{2}{7} \times$ .... $= 1$

$\frac{5}{4} \times \frac{4}{5} =$ .... $\qquad$ .... $\times \frac{5}{9} = 1$

Multiply five more such pairs.

Any two non-zero numbers whose product is 1, are called reciprocals of one another. So the reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$ and the reciprocal of $\frac{7}{4}$ is $\frac{4}{7}$.

What is the receiprocal of $\frac{5}{9}$ and $\frac{2}{5}$?

Try This

1. Will the reciprocal of a proper fraction be a proper fraction?
2. Will the reciprocal of an improper fraction be an improper fraction?

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Therefore, we can say that,

$1 \div \frac{1}{2} = 1 \times \frac{2}{1} = 1 \times \text{reciprocal of } \frac{1}{2}$.

$3 \div \frac{1}{4} = 3 \times \frac{4}{1} = 3 \times \text{ reciprocal of } \frac{1}{4}$.

$3 \div \frac{1}{2} = \text{........} = \text{......................}$

Raju applied this inverting procedure to mixed fractions and said that the reciprocal of $1\frac{1}{2}$ is $1\frac{2}{1}$. Is he right? Check.

So, $2 \div \frac{3}{4} = 2 \times \text{ reciprocal of } \frac{3}{4} = 2 \times \frac{4}{3}$.

$5 \div \frac{2}{4} = 5 \times \text{ .... } = 5 \times \text{......................}$

Thus dividing by a fraction is equivalent to multiplying the number by the reciprocal of that fraction.

Do This

Find (i) $9 \div \frac{2}{5}$ (ii) $3 \div \frac{4}{7}$ (iii) $2 \div \frac{8}{9}$

For dividing a whole number by a mixed fraction, we first convert the mixed fraction into an improper fraction and then solve it.

Example $4 \div 3\frac{2}{5} = 4 \div \frac{17}{5} = 4 \times \frac{5}{17} = \frac{20}{17}$ Find, $11 \div 3\frac{1}{3} = 11 \div \frac{10}{3} = ?$

Do This

Find (i) $7 \div 5\frac{1}{3}$ (ii) $5 \div 2\frac{4}{7}$

2.2.2 Division of a fraction by a whole number

What will $\frac{3}{4} \div 3$ be equal to?

Based on our earlier observations we have : $\frac{3}{4} \div 3 = \frac{3}{4} \div \frac{3}{1} = \frac{3}{4} \times \frac{1}{3} = \frac{3}{12} = \frac{1}{4}$

So, $\frac{2}{3} \div 5 = \frac{2}{3} \times \frac{1}{5} = ?$ What is $\frac{5}{7} \div 6$ and $\frac{2}{7} \div 8$?

For dividing mixed fractions by whole numbers, we convert the mixed fractions into improper fractions. For example

$2\frac{1}{3} \div 5 = \frac{7}{3} \div 5 = \frac{7}{3} \times \frac{1}{5} = \frac{7}{5}$. Find $4\frac{2}{5} \div 3 =$ .... = ...; $2\frac{3}{5} \div 2 =$ .... = ...

2.2.3 Division of a fraction by another fraction

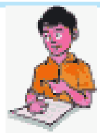
We can now find $\frac{1}{4} \div \frac{5}{6}$.

$\frac{1}{4} \div \frac{5}{6} = \frac{1}{4} \times$ reciprocal of $\frac{5}{6} = \frac{1}{4} \times \frac{6}{5} = \frac{6}{20} = \frac{3}{10}$.

Similarly, $\frac{8}{5} \div \frac{2}{3} = \frac{8}{5} \times$ reciprocal of $\frac{2}{3} =$ ....=........ and $\frac{1}{2} \div \frac{3}{4} =$............=.........

Do This

Find (i) $\frac{3}{5} \div \frac{1}{2}$ (ii) $\frac{1}{2} \div \frac{3}{5}$ (iii) $2\frac{1}{2} \div \frac{3}{5}$ (iv) $5\frac{1}{6} \div \frac{9}{2}$



Example 8 : An empty swimming pool is to be filled up to $\frac{9}{10}$ of its capacity. A pump takes half an hour to fill $\frac{3}{10}$ of the pool, how long will it take to fill $\frac{9}{10}$ of the pool?

Solution : We need to find how many $\frac{3}{10}$'s are there in $\frac{9}{10}$, solve the division problem $\frac{9}{10} \div \frac{3}{10}$.

$\frac{9}{10} \times \frac{10}{3} = 3$ Thus, it would take 3 half an hours, or $1\frac{1}{2}$ hours to fill the pool to its $\frac{9}{10}$.

Exercise 4

1. Find the reciprocal of each of the following fractions.

   (i) $\frac{5}{8}$ (ii) $\frac{8}{7}$ (iii) $\frac{13}{7}$ (iv) $\frac{3}{4}$

2. Find

   (i) $18 \div \frac{3}{4}$ (ii) $8 \div \frac{7}{3}$ (iii) $3 \div 2\frac{1}{3}$ (iv) $5 \div 3\frac{4}{7}$

3. Find

   (i) $\frac{2}{5} \div 3$ (ii) $\frac{7}{8} \div 5$ (iii) $\frac{4}{9} \div \frac{4}{5}$

4. Deepak can paint $\frac{2}{5}$ of a house in one day. If he continues working at this rate, how many days will he take to paint the whole house?

2.3 Decimal numbers or Fractional decimals

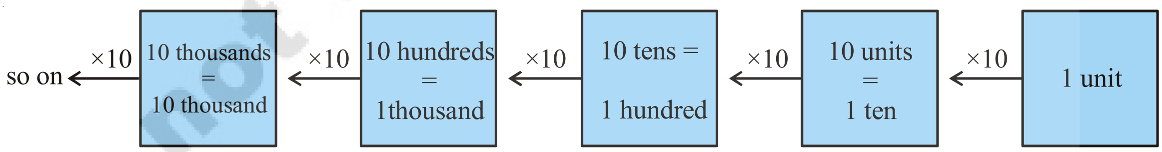
In class VI we have learnt about decimal numbers and their addition and subtraction. Let us review our understanding and then learn about multiplication and division.

Let us write 12714 in its expanded form:

$$12714 = 1 \times 10000 + 2 \times 1000 + 7 \times ......+ 1 \times ...... + 4 \times 1$$

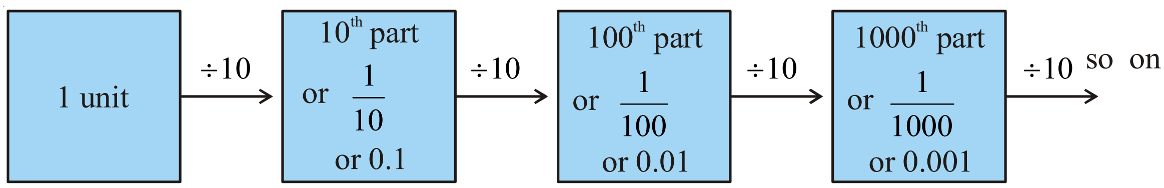
What will the expended form of 12714.2 be?

You will find that on moving from right to left, the value increase in multiples of 10.



Now, what happens when we move from left to right? You will find that the value gets, divided by 10. Now think, if the unit is divided by 10, what will happen? Remember you have learnt that

$$1 \div 10 = \frac{1}{10} = 0.1$$



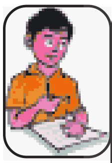
Thus, the expanded form of 12714.2 is-

$$12714.2 = 1\times 10000 + 2\times 1000 + 7\times \ldots\ldots + 1\times \ldots\ldots + 4\times 1 + 2\times \frac{1}{10}$$

Now find the place value of all the digits of 3.42. You might have noticed that a dot (.) or a decimal point separates whole part of the number from the fractional part. The part right side of the decimal point is called the decimal part of the number as it represents a part of 1. The part left to the decimal point is called the integral part of the number.

In the number 3.42-

	3 is at units place	4 is at the first place after the decimal point	2 is at the second place after the decimal point
Place value	$3\times 1 = 3$	$4\times\frac{1}{10}=\frac{4}{10}$ or .4	$2\times\frac{1}{100}=\frac{2}{100}$ or .02



Try This

1. Look at the following table and fill up the blank spaces.

Hundreds (100)	Tens (10)	Units (1)	Tenth $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$	Number
5	4	7	8	2	9	547.829
0	7	2	1	7	7	______
3	2	__	__	5	4	327.154
6	__	4	__	2	__	614.326
2	__	6	5	__	2	236.512

2. Write thefollowing numbers in their expanded form.

(i) 30.807 (ii) 968.038 (iii) 8370.705

To convert money, length, weight, etc from one unit to the other we often use decimals. For e.g. 5 paise = ₹ $\frac{5}{100}$ = ₹0.05; 220 g = $\frac{220}{1000}$ kg = 0.220 kg ; 5 cm = $\frac{5}{100}$ m = 0.05 m

Do This

Find (i) 50 paise = ₹ _____ (ii) 22 g = _____ kg (iii) 80 cm = m

2.3.1 Comparison of decimal numbers

Who has more money?

Abhishek and Neha have ₹ 375.50 and ₹375.75 respectively in their kiddy bank. To find who has more money, we first compare the digits on the left of the decimal point. Since both the children have ₹ 375 we compare the digits to the right of the decimal point starting from the tenth place. We find that Abhishek has 7 tenths and Neha has 5 tenths, 7 tenths >5 tenths, therefore, Abhishek has more money than Neha, i.e., 375.75 > 375.50.

Now compare quickly, which of the following pair of numbers is greater ?

(i) 37.65 and 37.60 (ii) 1.775 with 19.780 (iii) 364.10 and 363.10

Let us see how to add or subtract decimals.

(i) 221.85 + 37.10

$$\begin{array}{r} 221.85 \\ +37.10 \\ \hline 258.95 \\ \hline \end{array}$$

(ii) 39.70 - 6.85

$$\begin{array}{r} 39.70 \\ -\ 06.85 \\ \hline 32.85 \\ \hline \end{array}$$

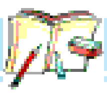
While adding or subtracting decimal numbers, the digits in the same places must be added or subtracted, i.e., while writing numbers one below the other, see that decimal points must come one below the other. Decimal places may be made equal by placing zeroes on the right side of the decimal number.

Do This

Find (i) 0.25 + 5.30 (ii) 29.75 - 25.97.

Example 9 : The equal sides of an isosceles triangle are 3.5 cm each and the other side is 2.5 cm. What is the perimeter of the triangle?

Solution : The sides of isosceles triangle are 3.5 cm, 3.5 cm and 2.5 cm. Therefore, the perimeter of the given triangle is = sum of lengths of three sides = 3.5 cm + 3.5 cm + 2.5 cm = 9.5 cm



Exercise - 5

1 cm = 10 mm
1 m = 100cm
1 km = 1000m
1 kg = 1000gm

1. Which one is greater?

   (i) 0.7 or 0.07 (ii) 7 or 8.5

   (iii) 1.47 or 1.51 (iv) 6 or 0.66

2. Express the following as rupees using decimals.

   (i) 9 paise (ii) 77 rupees 7 paise (iii) 235 paise

3. (i) Express 10 cm in metre and kilometre.

   (ii) Express 45 mm in centimeter, meter and kilometer.

4. Express the following in kilograms.

(i) 190 g (ii) 247 g (iii) 44 kg 80 gm

5. Write the following decimal numbers in expanded form.

(i) 55.5 (ii) 5.55 (iii) 303.03

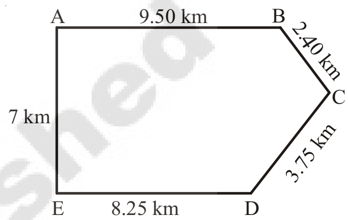
(iv) 30.303 (v) 1234.56

6. Write the place value of **3** in the following decimal numbers.

(i) 3.46 (ii) 32.46 (iii) 7.43

(iv) 90.30 (v) 794.037

7. Aruna and Radha start their journey from two different places. A and E. Aruna chose the path from A to B then to C, while Radha chose the path from E to D then to C.



Find who traveled more and by how much?

8. Upendra went to the market to buy vegetables. He brought 2 kg 250 gm tomatoes, 2 kg 500 gm potatoes, 750 gm lady fingers and 125 gm green chillies. How much weight did Upendra carry back to his house?

2.4 Multiplication of decimal numbers

Rajendra of class 7 went with her mother to the bazar to buy vegetables. There they purchased 2.5 kg potatoes at the rate of ₹ 8.50 per kg. How much money do they need to pay?

We come across various situations in day-to-day life where we need to know how to multiply two decimals. Let us now learn the multiplication of two decimal numbers.

Let us first multiply- 0.1×0.1

0.1 means one part of 10 parts. This is represented as $\frac{1}{10}$ using fractions and pictorially in Fig.1.

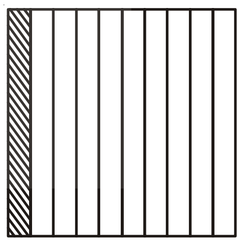


Figure 1

Thus, $0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10}$ which means $\frac{1}{10}$ of $\frac{1}{10}$. So here we are finding the 10th part of $\frac{1}{10}$. Thus, we divide $\frac{1}{10}$ into 10 equal parts and take one part. This is represented by one square in Figure 2. How many squares are there in Figure 2? There are 100 squares. So one square represents one out of 100 or 0.01. So we can conclude that

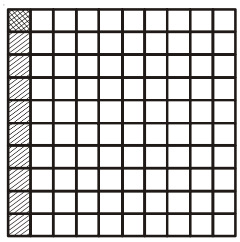


Figure 1

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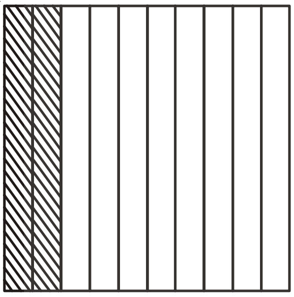
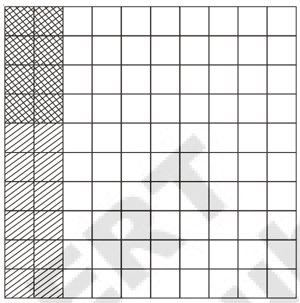
$$0.1 \times 0.1 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = 0.01$$

Let us now find 0.4 × 0.2

$$0.4 \times 0.2 = \frac{4}{10} \times \frac{2}{10} \text{ or } \frac{4}{10} \text{ of } \frac{2}{10}$$

Pictorially

$\frac{2}{10}$

$\frac{4}{10}$ of  =  $= \frac{8}{100}$

Since there are 8 double shaded squares out of 100, they represent 0.08.

While finding 0.1 × 0.1 and 0.4 × 0.2, you might have noticed that we first multiplied them as Whole numbers ignoring the decimal point. In 0.1 × 0.1, we found 01 × 01 or 1 × 1. Similarly in 0.4 × 0.2 we found 04 × 02 or 4 × 2. The products obtained are 1 and 8 respectively.

We then counted the total number of digits to the right of the decimal point in the numbers being multiplied. In both 0.1×0.1 and 0.4×0.2, the total number of digits to the right of the decimal point in the numbers being multiplied is 2 each. Thus, in each of their products we put the decimal point by counting two places from right to left.

For any decimal number which has no integral number part, we generally place a zero on the left side of decimal point to give prominance to decimal point.

Thus, 0.1 × 0.1 = .01 or 0.01

0.4 × 0.2 = .08 or 0.08

If we had multiplied 0.5 × 0.05 then we would have put the decimal point in the product by counting three places from right to left i.e. 0.5 × 0.05 = .025.

Let us now find 1.2 × 2.5

Multiply 12 and 25. We get 300. In both 1.2 and 2.5, there is 1 digit to the right of the decimal point. So, count 1 + 1 = 2 digits. From the rightmost digit (i.e., 0) in 300, move two places towards left. We get 3.00 or 3. Thus, 1.2 × 2.5 = 3

While multiplying 2.5 and 1.25 you will first multiply 25 and 125. For placing the decimal in the product obtained, we will count 1 + 2 = 3 (Why?). Thus, 2.5 × 1.25 = 3.225.

Do These

1. Find (i) 1.7×3 (ii) 2.0×1.5 (iii) 2.3×4.35
2. Arrange the products obtained in (1) in descending order.

Example 10 : The length of a rectangle is 7.1 cm and its breadth is 2.5 cm. What is the area of the rectangle?

Solution : Length of the rectangle = 7.1 cm

Breadth of the rectangle = 2.5 cm

Therefore, area of the rectangle = $7.1 \times 2.5 = 17.75$ cm^2

2.4.1 Multiplication of decimal number by 10, 100, 1000 etc.,

Reshma observed that $3.2 = \frac{32}{10}$ whereas $2.35 = \frac{235}{100}$. Thus, she found that depending on the position of the decimal point, the decimal number can be converted to a fraction with denominator 10 or 100etc., She wondered what would happen if a decimal number is multiplied by 10, 100 or 1000 etc.,

Let us see if we can find a pattern in multiplying numbers by 10 or 100 or 1000.

Have a look at the table given below and fill in the blanks :

$1.76 \times 10 = \frac{176}{100} \times 10 = 17.6$	$2.35 \times 10 =$	$12.356 \times 10 =$ ....
$1.76 \times 100 = \frac{176}{100} \times 100 = 176$ or 176.0	$2.35 \times 100 =$	$12.356 \times 100 =$ ....
$1.76 \times 1000 = \frac{176}{100} \times 1000 = 1760$ or 1760.0	$2.35 \times 1000 =$	$12.356 \times 1000 =$ ....
$0.5 \times 10 = \frac{5}{10} \times 10 = 5$; $0.5 \times 100 =$ .... ; $0.5 \times 1000 =$		

Look at your answers. Could you find any pattern? The decimal point in the products shifts to the right by as many zeroes as in 10, 100, 1000 etc,.

2.4.2 Division of decimal numbers

Gopal was preparing a design to decorate his classroom. He needed a few coloured strips of paper of length 1.6 cm each. He had a strip of coloured paper of length 9.6 cm . How many pieces of the required length will he get out of this strip? He thought it would be $\frac{9.6}{1.6}$ cm. Is he correct?

Both 9.6 and 1.6 are decimal numbers. So we need to know the division of decimal numbers too!

2.4.2 (a) Division by numbers like 10, 100, 1000 etc.,

Let us now divide a decimal number by 10, 100 and 1000.

Consider 31.5 ÷ 10.

$$31.5 \div 10 = \frac{315}{10} \div 10 = \frac{315}{10} \times \frac{1}{10} = \frac{315}{100} = 3.15$$

Similarly, $31.5 \div 100 = \frac{315}{10} \div 100 = \frac{315}{10} \times \frac{1}{100} = \frac{315}{1000} = 0.315$

Is there a pattern while dividing numbers by 10, 100 or 1000? This may help us in dividing numbers by 10, 100 or 1000 in a shorter way.

Observe the pattern in the table, given bellow and complete it.

29.5 ÷ 10 = 2.95	132.7 ÷ 10 = ...	1.5 ÷ 10 = ...	17.36 ÷ 10 = ...
29.5 ÷ 100 = 0.295	132.7 ÷ 10 = ...	1.5 ÷ 100 = ...	17.36 ÷ 100 = ...
29.5 ÷ 1000 = 0.0295	132.7 ÷ 1000 = ...	1.5 ÷ 1000 = ...	17.36 ÷ 1000 = ...

2.4.2 (b) Division of a decimal number by a whole number

Let us find $\frac{6.4}{2}$. Remember we also write it as $6.4 \div 2$.

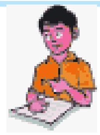
So, $6.4 \div 2 = \frac{64}{10} \div 2 = \frac{64}{10} \times \frac{1}{2}$ (as learnt in fractions)

$$= \frac{64 \times 1}{10 \times 2} = \frac{1 \times 64}{10 \times 2} = \frac{1}{10} \times \frac{64}{2} = \frac{1}{10} \times 32 = \frac{32}{10} = 3.2$$

Now, let us calculate $12.96 \div 4 = \frac{1296}{100} \div 4 = \frac{1296}{100} \times \frac{1}{4} = \frac{1}{100} \times \frac{1296}{4} = \frac{1}{100} \times 324 = 3.24$

Do This

1. Find (i) $35.7 \div 3$ (ii) $25.5 \div 3$



Example 11 : Find the average of 4.2, 3.8 and 7.6.

Solution : The average of 4.2, 3.8 and 7.6 is $\frac{4.2 + 3.8 + 7.6}{3} = \frac{15.6}{3} = 5.2$

2.4.2 (c) Division of a decimal number by another decimal number

Let us find how we divide a decimal number by another decimal number, For example $35.5 \div 0.5$.

$$= \frac{355}{10} \div \frac{5}{10} = \frac{355}{10} \times \frac{10}{5} = 71$$

Thus $35.5 \div 0.5 = 71$.

Example 12 : A truck covers a distance of 92.5 km in 2.5 hours. If the truck is travelling at the same speed through out the journey what is the distance covered by it in 1 hour?

Solution : Distance travelled by the truck $= 92.5$ km.

Time required to travel this distance $= 2.5$ hours.

So distance travelled by it in 1 hour $= \frac{92.5}{2.5} = \frac{925}{25} = 37$ km.

Exercise - 6

1. Solve the following.

   (i) 0.3×6 (ii) 7×2.7 (iii) 2.71×5

   (iv) 19.7×4 (v) 0.05×7 (vi) 210.01×5

   (vii) 2×0.86

2. Find the area of a rectangle whose length is 6.2 cm and breadth is 4 cm.

3. Solve the following.

(i) 21.3 × 10 (ii) 36.8 × 10 (iii) 53.7 × 10

(iv) 168.07 × 10 (v) 131.1 × 100 (vi) 156.1 × 100

(vii) 3.62 × 100 (viii) 43.07 × 100 (ix) 0.5 × 10

(x) 0.08 × 10 (xi) 0.9 × 100 (xii) 0.03 × 1000

4. A motor bike covers a distance of 62.5 km.consuming one litre of petrol. How much distance does it cover for 10 litres of petrol?

5. Solve the following.

(i) 1.5 × 0.3 (ii) 0.1 × 47.5 (iii) 0.2 × 210.8

(iv) 4.3 × 3.4 (v) 0.5 × 0.05 (vi) 11.2 × 0.10

(vii) 1.07 × 0.02 (viii) 10.05 × 1.05 (ix) 101.01 × 0.01

(x) 70.01 × 1.1

6. Solve the following.

(i) 2.3 ÷ 100 (ii) 0.45 ÷ 5 (iii) 44.3 ÷ 10

(iv) 127.1 ÷ 1000 (v) 7 ÷ 3.5 (vi) 88.5 ÷ 0.15

(vii) 0.4 ÷ 20

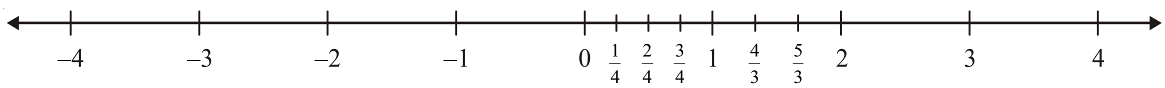
7. A side of a regular polygon is 3.5 cm in length. The perimeter of the polygon is 17.5 cm. How many sides does the polygon have?

8. A rain fall of 0.896 cm. was recorded in 7 hours, what was the average amount of rain per hour?

2.5 Introduction to Rational numbers

2.5.1 Positive fractional numbers:

We have learnt about integers and fractions. Let us see how the number line looks when both are marked on it.



We have $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$ ..... between 0 and 1 on the number line. All these are numbers that are less than one. We call them as proper fractions and say that all proper fractions lie between 0 and 1. Similarly, we know $\frac{4}{3}$ and $\frac{5}{3}$ would lie between 1 and 2. We can recall them as improper fractions. All these are called positive fractional numbers.

Do These

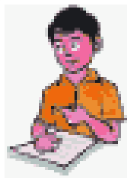
1. Write 5 more fractions between (i) 0 and 1 and (ii) 1 and 2.
2. Where does $4\frac{3}{5}$ lie on the number line?

On the left side of 0 we have integers –1, –2, –3 ....

Do the numbers increase or decrease as we move further left on the number line?

You know that number decreases as we move further left. The farther the number is from 0 on the left the smaller it is.

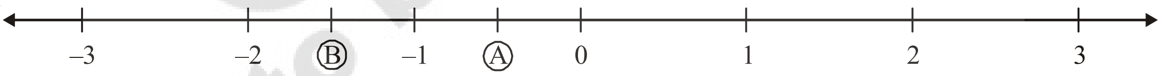
Do These



1. Find the greatest and the smallest numbers among the follwing groups?
   (i) 2, –2, –3, 4, 0, –5
   (ii) –3, –7, –8, 0, –5, –2
2. Write the following numbers in ascending order.
   (i) –5, –75, 3 – 2, 4, $\frac{3}{2}$ (ii) $\frac{2}{3}, \frac{3}{2}$, 0, –1, –2, 5

2.5.2 Negative fractional numbers

Consider the point A shown on the line.



It lies on the number line between 0 and –1. Is it more than 0 or less than 0?

Is it $\frac{1}{2}$? We cannot say it is $\frac{1}{2}$ as it is less than zero.

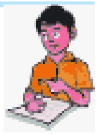
We write (A) as $-\frac{1}{2}$ and it is $\frac{1}{2}$ less than zero.

Similarly, (B) the mid point of –1 and –2 is. $-\frac{3}{2}$

Neha found an easy way to represent $\frac{-9}{4}$. She first wrote it in a mixed fraction $\frac{-9}{4} = -2\frac{1}{4}$ and then represented it between –2 and –3.

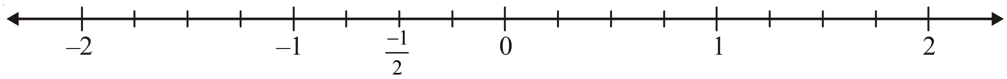
You can see that negative fractional numbers like $-\frac{1}{2}, -\frac{3}{2}, -\frac{9}{4}$ give us points in between any two negative integers or between zero and a negative integer.

Do These



1. On the number line given below represent the following numbers.

(i) $-\frac{7}{2}$ (ii) $\frac{3}{2}$ (iii) $\frac{7}{4}$ (iv) $\frac{-7}{4}$ (v) $\frac{-1}{4}$ (vi) $\frac{1}{4}$



2. Consider the following numbers on a number line.

$27, -\frac{7}{8}, \quad \frac{11}{943}, \quad \frac{54}{17}, \quad -68, \quad -3, \quad -\frac{9}{6}, \quad \frac{7}{2}$

(i) Which of these are to the left of

(a) 0 (b) –2 (c) 4 (d) 2

(ii) Which of these would be to the right of

(a) 0 (b) –5 (c) $3\frac{1}{2}$ (d) $\frac{-5}{2}$

2.5.3 Rational Numbers

We know 0, 1, 2, 3, 4, 5 are whole numbers. We also know that –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5 is a bigger collection of numbers called integers.

Rani says "All whole numbers are integers but the converse is not true." Do you agree with her? Rani is right as negative numbers like –6, –5, –4, –3, –2, –1 etc are integers but not whole numbers. Thus, all whole numbers are integers and all integers are not whole numbers.

We further know that positive fractional numbers like $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{5}{6}, \frac{11}{5}, \frac{8}{8}$ are ratios of whole numbers. All fractional numbers can in general be written as $\frac{w_1}{w_2}$ with the condition w_1 and w_2 are whole numbers and that w_2 is not equal to zero.

Try This

Write 5 fractional numbers and identify w_1 and w_2 in each of these.

Rational numbers are a bigger collection of numbers, which includes all integers, all positive fractional numbers and all negative fractional numbers.

The numbers $\frac{-7}{3}, \frac{-5}{2}, \frac{-7}{7}, \frac{-2}{7}, 0, \frac{1}{4}, \frac{4}{4}, \frac{17}{5}, \frac{6}{1}$ **etc. are all rational numbers.**

In all these we have a ratio of two integers, thus the numbers in the form of $\frac{p}{q}$**, where p and** q **are integers except that** q **is equal to zero are called as rational numbers.**
The set of rational numbers is denoted by Q

Try These

(i) Take any 5 integers and make all possible rational numbers with them.

(ii) Consider any 5 rational numbers. Find out which integers constitute them?

2.5.4 Comparing rational numbers

We know that $\frac{3}{4}$ and $\frac{9}{12}$ are equivalent fractional numbers. We also know that when we compare fractional numbers we convert each of them to equivalent fractional numbers and then compare the ones with a common denominator.

For example, to compare $\frac{3}{4}$ and $\frac{5}{7}$.

We write equivalent fractional numbers for both

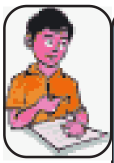
$\frac{3}{4} = \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}$ and

$\frac{5}{7} = \frac{10}{14}, \frac{15}{21}, \frac{20}{28}$ ....

We can compare $\frac{21}{28}$ with $\frac{20}{28}$ as they have same denominators.

$\frac{21}{28}$ is bigger than $\frac{20}{28}$

Therefore, $\frac{3}{4} > \frac{5}{7}$



Try These

1. Write three more equivalent fractions of $\frac{3}{4}$ and mark them on the number line.

   What do you observe?

2. Do all equivalent fractions of $\frac{6}{7}$ represent the same point on the number line?

Now compare $\frac{-1}{2}$ and $\frac{-2}{3}$

We write equivalent fractions for both

$$\frac{-1}{2} = \frac{-2}{4}, \frac{-3}{6}, \frac{-4}{8} \text{ ....}$$

$$\frac{-2}{3} = \frac{-4}{6}, \frac{-6}{9} \text{ ....}$$

We can compare $\frac{-3}{6}$ and $\frac{-4}{6}$ as they have same denominators. $\frac{-4}{6} < \frac{-3}{6}$

$\therefore \frac{-2}{3} < \frac{-1}{2}$

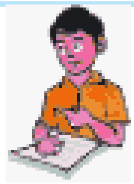
Try These

1. Are $\frac{-1}{2}$ and $\frac{-3}{6}$ represent same point on the number line?

2. Are $\frac{-2}{3}$ and $\frac{-4}{6}$ equivalent

Eg: When we place them on the number line we find that they occupy the same point. We can thus ay that $\frac{-1}{2}$ and $\frac{-2}{4}$ are equivalent rationals.

Do These



1. Write 5 equivalent rational numbers to (i) $\frac{5}{2}$ (ii) $\frac{-7}{9}$ (iii) $-\frac{3}{7}$

2. Identify the equivalent rational numbers in each question:

   (i) $\frac{-1}{2}, \frac{-3}{4}, \frac{-2}{4}, \frac{-4}{8}$

   (ii) $\frac{1}{4}, \frac{3}{4}, \frac{5}{3}, \frac{10}{6}, \frac{2}{4}, \frac{20}{12}$

We can say that to get equivalent rational numbers we multiply or divide the integer in the numerator and in the denominator by the same number.

For example,

For $\frac{1}{5}$ we would have $\frac{1\times 2}{5\times 2} = \frac{2}{10}$ as one equivalent number another is $\frac{1\times 3}{5\times 3} = \frac{3}{15}$.

For $\frac{-2}{7}$ we would have $\frac{-2\times 2}{7\times 2} = \frac{-4}{14}$ as one and $-\frac{2\times 3}{7\times 3} = \frac{-6}{21}$ as another.

We can go on to build more such equivalent rational numbers, just by multiplying with $\frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} \ldots$

Exercise - 7

1. Write any three equivalent rational numbers to each of the following

   (i) $\frac{2}{3}$ (ii) $-\frac{3}{8}$

2. What is the equivalent rational number for $\frac{-15}{36}$ with (i) denominator 12 (ii) numerator 75?

3. Mark the following rational numbers on the number line.

   (i) $\frac{1}{2}$ (ii) $\frac{3}{4}$ (iii) $\frac{3}{2}$ (iv) $\frac{10}{3}$

3. Find whether the following statements are true or false.

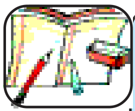
(i) Every integer is a rational number and vice versa ( )

(ii) In a rational number of the form $\frac{p}{q}$, q must be a non zero integer. ( )

(iii) Every decimal number can be represented as a rational number. ( )

(iv) $\frac{5}{7}, \frac{6}{7}, \frac{7}{7}$ are equivalent rational numbers. ( )

(v) Equivalent rational numbers of a positive rational numbers are all positive ( )



Looking back

1. We have learnt that for addition and subtraction of fractions; the fractions should be like fractions.

2. We have also learnt how to multiply fractions i.e, $\frac{\text{Product of numerators}}{\text{Product of denominators}}$

3. "of" can be used to represent multiplication. For example, $\frac{1}{3}$ of $6 = \frac{1}{\cancel{3}} \times \cancel{6}^{2} = 2$.

4. The product of two proper fractions is less than each of the fractions that are multiplied. The product of a proper and improper fraction is less than the improper fraction and greater than the proper fraction. The product of two improper fractions is greater than each of the fractions.

5. A reciprocal of a fraction is obtained by inverting the numerator and denominator.

6. We have seen how to divide two fractions.

   (i) While dividing a whole number with a fraction, we multiply the whole number with the reciprocal of that fraction.

   (ii) While dividing a fraction by a whole number we multiply the fraction with the reciprocal of the whole number.

   (iii) While dividing one fraction by another fraction, we multuiply the first fraction with the reciprocal of the second. So $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20}$.

7. We also learnt how to multiply two decimal numbers. While multiplying two decimal numbers, we first multiply them as whole numbers. We then count the total number of digits to the right of the decimal point in both the decimal numbers being multiplied. Lastly, we put the decimal point in the product by counting the digits from its rightmost place.

8. To multiply a decimal number by 10, 100, 1000 ... etc., we move the decimal point in the number to the right by as many places as there are zeros in the numbers 10, 100, 1000 . . .

9. We have learnt how to divide decimal numbers.

   (i) To divide a decimal number by a whole number, we first divide them as whole numbers. We then place the decimal point in the quotient as in the decimal number.

   Note that here we are considering only those divisions in which the remainder is zero.

   (ii) To divide a decimal number by 10, 100, 1000 or any other multiple of 10, we shift the decimal point in the decimal number to the left by as many places as there are zeros in 10, 100, 1000 etc.,

   (iii) While dividing two decimal numbers, first shift the decimal point to the right by equal number of places in both, to convert the divisor to a whole number.

10. Rational numbers are a bigger collection of numbers, which includes all integers, all positive fractional numbers and all negative fractional numbers. In all these we have a ratio of two integers, thus $\frac{p}{q}$ represents a rational number.

    In this i) p, q are integers and

    ii) $q \neq 0$

    The set of rational numbers is denoted by Q

John Napier (Scotland)
1550-1617 AD
Found logarithms.
Introduced napier rods for multiplictions.
Also introduced System of decimal fractions.