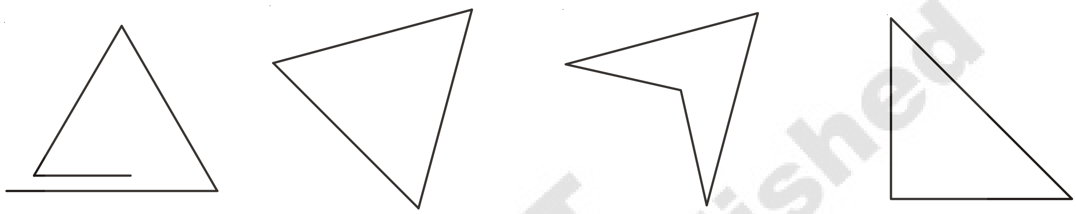
TRIANGLE AND ITS PROPERTIES 5

5.0 Introduction

You have been introduced to triangles in your previous class. Look at the figures given below. Which of these are triangles?

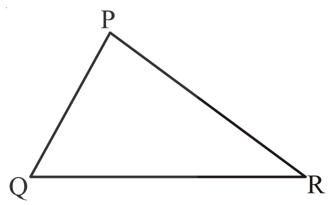


(i) (ii) (iii) (iv)

Discuss with your friends why you consider only some of these as triangles.

A triangle is a closed figure made up of three line segments.

In ΔPQR, the:



(i) Three sides are $\overline{PQ}, \overline{QR}, \overline{RP}$

(ii) Three angles are $\angle PQR$, $\angle QRP$, $\angle RPQ$

(iii) Three vertices are P, Q, R

The side opposite to vertex P is $\overline{QR}$ Can you name the sides which are opposite to vertices Q and R?

Likewise, the side opposite to $\angle QPR$ is $\overline{QR}$. Can you name the side which is opposite to $\angle PQR$

Try This

Uma felt that a triangle can be formed with three collinear points. Do you agree? Why?

Draw diagrams to justify your answer.

[If three more points lie on the same line, then they are called collinear points]

Note: LM = Length of Line segment of LM ; $\overline{LM}$ = Line segment LM

$\overrightarrow{LM}$ = Ray LM ; $\overleftrightarrow{LM}$ = Line LM

5.1 Classification of triangles

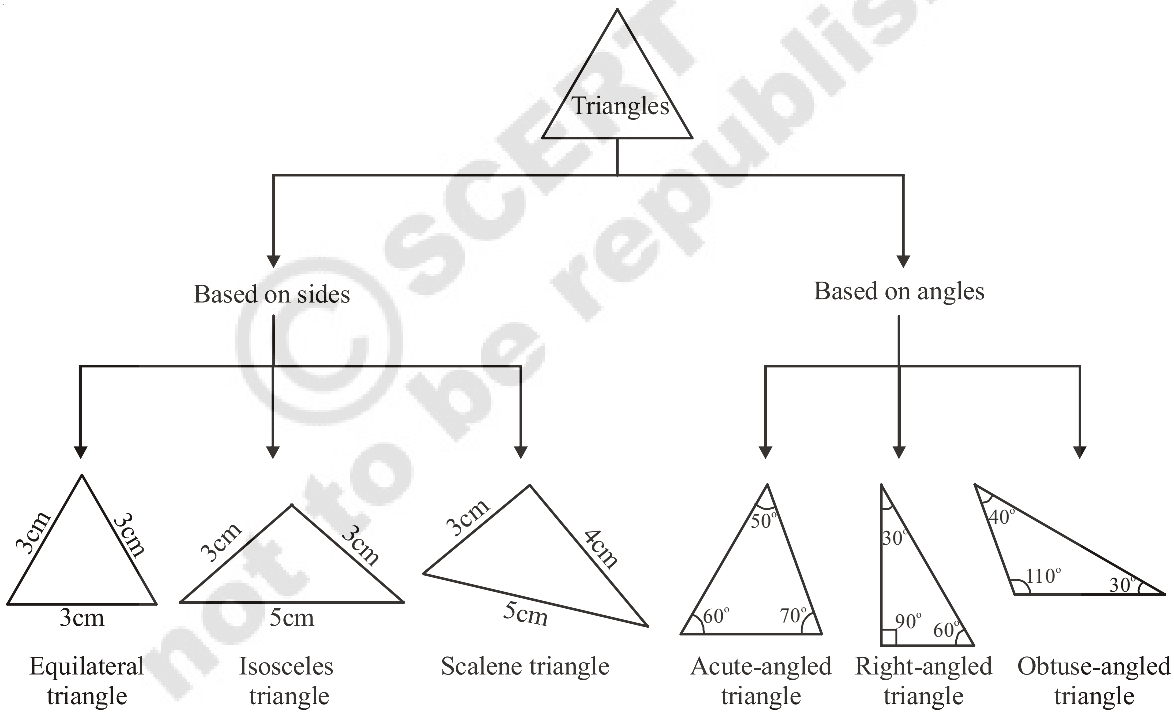
Triangles can be classified according to properties of their sides and angles.

Based on the sides, triangles are of three types:

- A triangle having all three sides of equal length is called an Equilateral Triangle.
- A triangle having two sides of equal length is called an Isosceles Triangle.
- If all the three sides of a triangle are of different length, the triangle is called a Scalene Triangle.

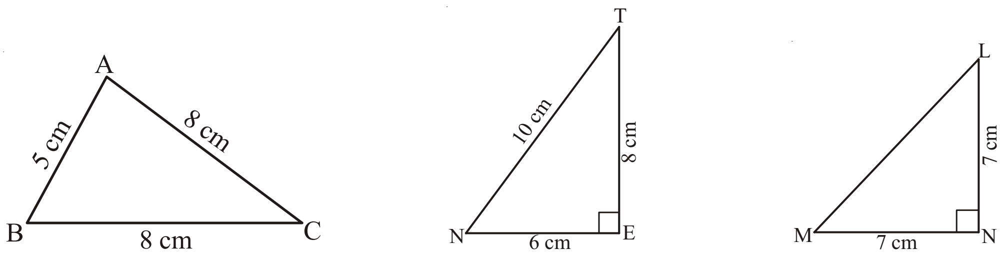
Based on the angles, triangles are again of three types:

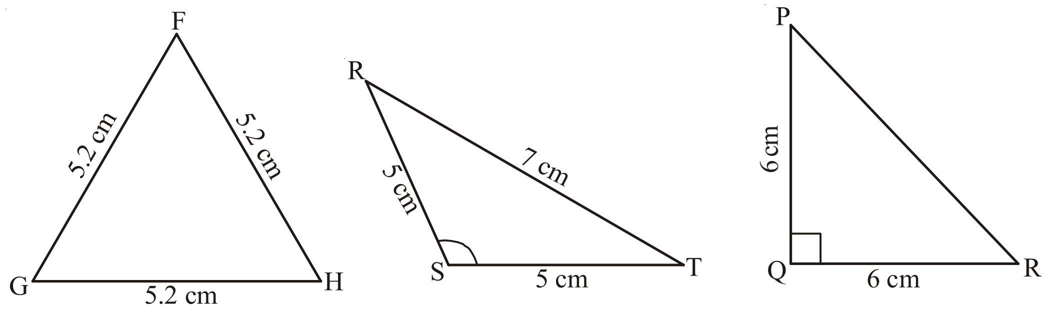
- A triangle whose all angles are acute is called an acute-angled triangle.
- A triangle whose one angle is obtuse is called an obtuse-angled triangle.
- A triangle whose one angle is a right angle is called a right-angled triangle.



Do This

1. Classify the following triangles according to their (i) sides and (ii) angles.





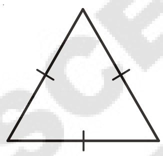
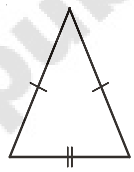
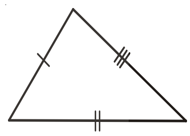
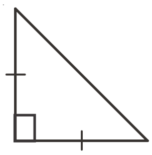
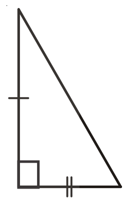
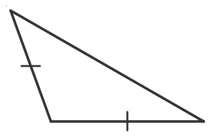
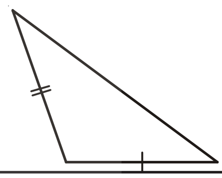
(2) Write the six elements (i.e. the 3 sides and 3 angles) of ΔABC.

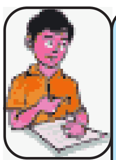
(3) Write the side opposite to vertex Q in ΔPQR.

(4) Write the angle opposite to side $\overline{LM}$ in ΔLMN.

(5) Write the vertex opposite to side $\overline{RT}$ in ΔRST.

If we consider triangles in terms of both sides and angles we can have the following types of triangles:

Type of Triangle	Equilaterial	Isosceles	Scalene
Acute-angled			
Right-angled			
Obtuse-angled			



Try This

1. Make paper-cut models of the various types of triangles discussed above. Compare your models with those of your friends.

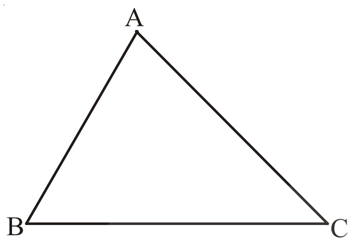
2. Rashmi claims that no triangle can have more than one right angle. Do you agree with her. Why?

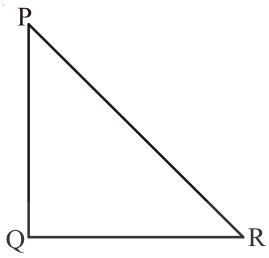
3. Kamal claims that no triangle can have more than two acute angles. Do you agree with him. Why?

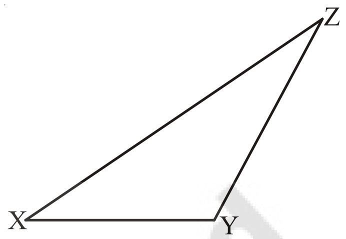
5.2 Relationship between the sides of a triangle

5.2.1 Sum of the lengths of two sides of a triangle

Draw any three triangles say ΔABC, ΔPQR and ΔXYZ as given below:







Use your ruler to find the lengths of their sides and tabulate your results as follows:

Name of Δ	Sides of Δ	Sum of two sides	Is this true?	Yes/No
ΔABC	$\overline{AB} =$	$\overline{AB} + \overline{BC} =$	$\overline{AB} + \overline{BC} > \overline{CA}$	
	$\overline{BC} =$	$\overline{BC} + \overline{CA} =$	$\overline{BC} + \overline{CA} > \overline{AB}$	
	$\overline{CA} =$	$\overline{CA} + \overline{AB} =$	$\overline{CA} + \overline{AB} > \overline{BC}$	
ΔPQR	$\overline{PQ} =$	$\overline{PQ} + \overline{QR} =$	$\overline{PQ} + \overline{QR} > \overline{RP}$	
	$\overline{QR} =$	$\overline{QR} + \overline{RP} =$	$\overline{QR} + \overline{RP} > \overline{PQ}$	
	$\overline{RP} =$	$\overline{RP} + \overline{PQ} =$	$\overline{RP} + \overline{PQ} > \overline{QR}$	
ΔXYZ	$\overline{XY} =$	$\overline{XY} + \overline{YZ} =$	$\overline{XY} + \overline{YZ} > \overline{ZX}$	
	$\overline{YZ} =$	$\overline{YZ} + \overline{ZX} =$	$\overline{YZ} + \overline{ZX} > \overline{XY}$	
	$\overline{ZX}$	$\overline{ZX} + \overline{XY} =$	$\overline{ZX} + \overline{XY} > \overline{YZ}$	

We can see that in all the above examples, the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

For eg. In ΔABC,

$$\overline{AB} + \overline{BC} > \overline{CA}$$

$$\overline{BC} + \overline{CA} > \overline{AB}$$

$$\overline{CA} + \overline{AB} > \overline{BC}$$

5.2.2 Difference between the lengths of two sides of a triangle

Take the same triangles as in the above example and tabulate your results as follows:

Name of Δ	Length of sides	Difference between two sides	Is this true?	Yes/No
ΔABC	AB =	BC–CA =	BC – AB <AC	
	BC =	CA–AB =	CA – AB <BC	
	CA=	AB–BC =	AB – BC <CA	
ΔPQR	PQ =	QR–RP =	QR – RP <PQ	
	QR =	RP–PQ =	RP – PQ <QR	
	RP =	PQ–QR =	PQ – QR <RP	
ΔXYZ	XY=	YZ–ZX =	YZ – ZX <XY	
	YZ =	ZX–XY =	ZX – XY <YZ	
	ZX =	XY–YZ =	XY – YZ <ZX	

From these observations we can conclude that the difference between the lengths of any two sides of a triangle is less than the length of the third side.

For eg. In ΔABC, AB – BC < CA ; BC – AB < CA

BC – CA < AB ; CA – BC < AB

CA – AB < BC ; AB – CA < BC

Try This

The lengths of two sides of a triangle are 6 cm and 9 cm. Write all the possible lengths of the third side.

Example 1: Can a triangle have sides with lengths 6 cm, 5 cm and 8 cm?

Solution : Let the sides of the triangle be AB = 6 cm

BC = 5 cm

CA = 8 cm

Sum of any two sides i.e, AB + BC = 6 + 5 = 11 > 8

BC + CA = 5 + 8 = 13 > 6

CA + AB = 8 + 6 = 14 > 5

Since, the sum of the lengths of any two sides is greater than the lenght of the third side. The triangle is possible.

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Exerciese - 1

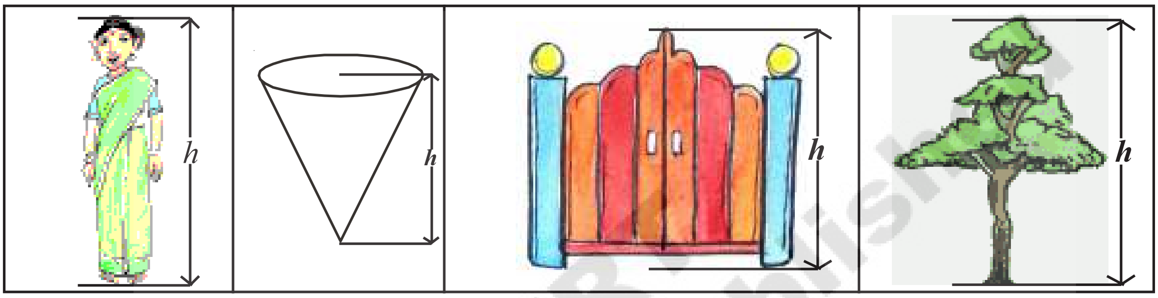
1. Is it possible to have a triangle with the following sides?

(i) 3 cm, 4 cm and 5 cm. (ii) 6 cm, 6 cm and 6 cm.

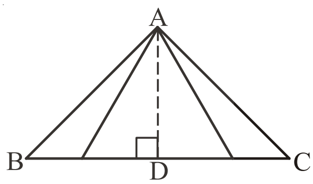
(iii) 4 cm, 4 cm and 8 cm. (iv) 3 cm, 5 cm and 7 cm.

5.3 Altitudes of a triangle

In your daily life you might have come across the word 'height' in different situations. How will you measure the height of different figures given below:



You will measure it from the top point of the object to its base as shown in the figures. Let us use this creteria to measure the height for a triangle.



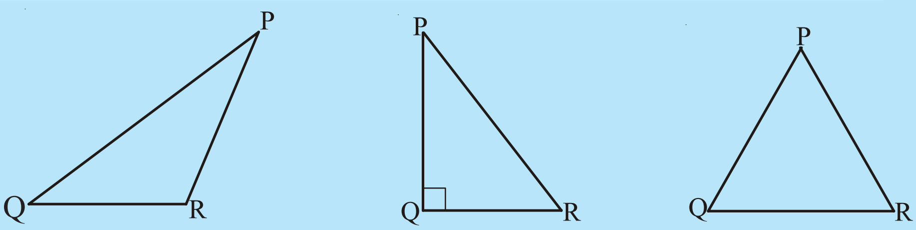
In a given ΔABC, the height is the distance from vertex A to the base $\overline{BC}$. However, you can think of many line segments from A to $\overline{BC}$. Which among them will represent the height?

The height is given by lenght of the line segment that starts from A and is perpendicular to $\overline{BC}$.

Thus, the line segment $\overline{AD}$ is the altitude of the triangle and its length is height. An altitude can be drawn from each vertex.

Try This

(i) Draw altitudes from P to $\overline{QR}$ for the following triangles. Also, draw altitudes from the other two vertices.(you can use a set squares if needed)

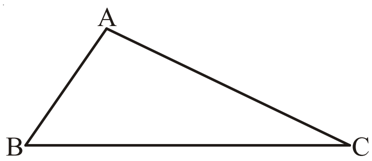


Obtuse-angled Right-angled Acute-angled

(ii) Will an altitude always lie in the interior of a triangle?

(iii) Can you think of a triangle in which the two altitudes of a triangle are two of its sides?

5.4 Medians of a triangle



Make a paper cut out of ΔABC.

Now fold the triangle in such a way that the vertex B falls on vertex C. The line along which the triangle has been folded will intersect side $\overline{BC}$ as shown in Figure 1. The point of intersection is the mid-point of side $\overline{BC}$ which we call D. Draw a line joining vertex A to this mid-point D (as can be seen in Figure 2).

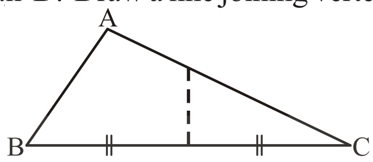


Figure 1

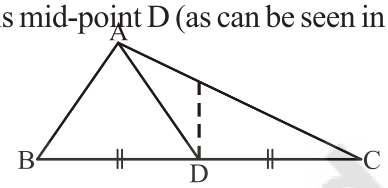
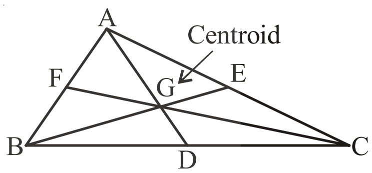


Figure 2

Similarly, fold the triangle in such a way that the vertex A falls on vertex C. The line along which the triangle has been folded will intersect side $\overline{AC}$. The point of intersection is the mid-point of side $\overline{AC}$. Draw a line joining vertex B to this mid-point, which we call E.

Lastly, fold the triangle in such a way that the vertex A falls on vertex B. The line along which the triangle has been folded will intersect side $\overline{AB}$. The point of intersection is the mid-point of side $\overline{AB}$. Draw a line joining vertex C to this mid-point, which we call F.



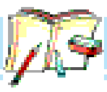
Line segments $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ join the vertices of the triangle to the mid-points of the opposite sides. They are called the medians of the triangle.

You will observe that the three medians intersect each other at a point in the interior of the triangle. This point of intersection is called the Centroid.

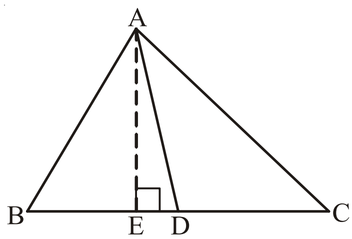
Thus, line segments which join the vertex of the triangle to the mid-point of the opposite side are called medians of the triangle. Their point of intersection is called the Centroid.

Try This

Take paper cut outs of right-angled triangles and obtuse-angled triangles and find their centroid.



Exercise - 2



1. In ΔABC, D is the midpoint of $\overline{BC}$.

(i) $\overline{AD}$ is the ______________________

(ii) $\overline{AE}$ is the ______________________

2. Name the triangle in which the two altitudes of the triangle are two of its sides.

3. Does a median always lie in the interior of the triangle?

4. Does an altitude always lie in the interior of a triangle?

5. (i) Write the side opposite to vertex Y in ΔXYZ.

(ii) Write the angle opposite to side $\overline{PQ}$ in ΔPQR.

(iii) Write the vertex opposite to side $\overline{AC}$ in ΔABC.

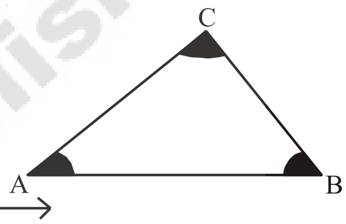
5.5 Properties of triangles

5.5.1 Angle-sum property of a triangle

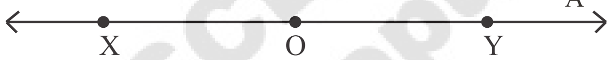
Let us learn about this property through the following four activities

Activity 1

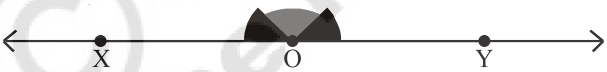
1. On a white sheet of paper, draw a triangle ABC. Using colour pencils mark its angles as shown.



2. Using a scissors, cut out the three angular regions.
3. Draw a line XY and mark a point 'O' on it.



4. Paste the three angular cut outs adjacent to each other to form one angle at 'O' as shown in the figure below.



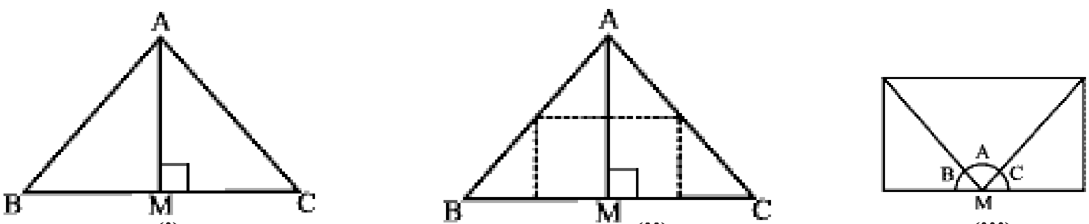
You will find that three angles now constitute a straight angle. Thus, the sum of the measures of angles of a triangle is equal to 180^o.

Activity 2

Take a piece of paper and cut out a triangle, say ABC.

Make the altitude $\overline{AM}$ by folding ΔABC.

Now, fold the three corners such that all the vertices A,B and C touch at M as shown in the following figures.

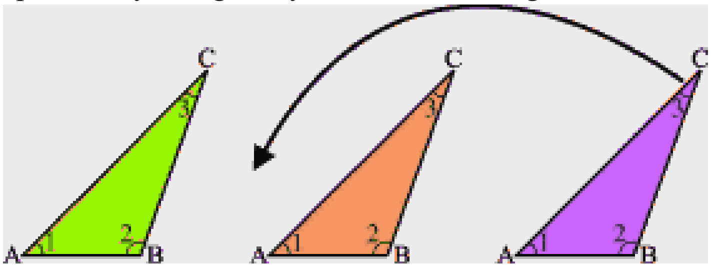


(i) (ii) (iii)

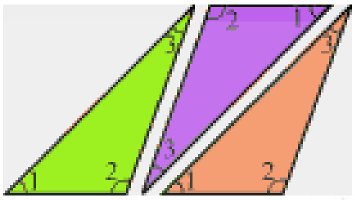
You will see that all the three angles A, B and C form a straight line and thus $\angle A + \angle B + \angle C = 180^o$.

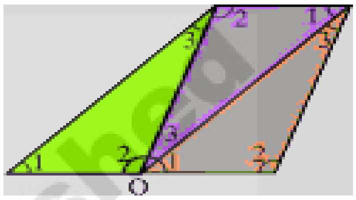
Activity 3

Take three copies of any triangle, say ABC. Mark its angles as 1,2 and 3 as shown below:



Arrange the triangular cut-outs as shown in the above and in the following figures.





What do you observe about $\angle 1 + \angle 2 + \angle 3$ at the point 'O'?

You will observe that three angles form a straight line and so measure 180°.

Activity 4

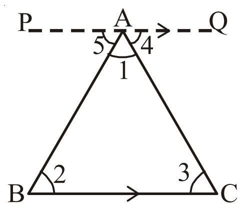
Draw any three triangles, say ΔABC, ΔPQR and ΔXYZ in your note book. Use your protractor and measure each of the angles of these triangles.

Name of the Triangle	Measure of angles	Sum of the measures of the three angles
ΔABC	$\angle A$ =....., $\angle B$ =....., $\angle C$ =.....,	$\angle A + \angle B + \angle C =$
ΔPQR	$\angle P$ =....., $\angle Q$ =....., $\angle R$ =.....,	$\angle P + \angle Q + \angle R =$
ΔXYZ	$\angle X$ =....., $\angle Y$ =....., $\angle Z$ =.....,	$\angle X + \angle Y + \angle Z =$

Allowing marginal errors in measurements, you will find that the sum of the three angles of a triangle is 180°.

You are now ready to give a formal justification of your assertion that the sum of the angles of a triangle is equal to 180° through logical argumentation.

Proof of angle-sum property of a triangle



Statement : The sum of the angles of a triangle is 180°

Given : A triangle ABC

To prove : $\angle A + \angle B + \angle C = 180^{\circ}$

Construction : Through A draw a line segment $\overleftrightarrow{PQ}$ parallel to BC.

Proof :

Mark the angles as indicated in the figure:

$\angle 2$	$=$	$\angle 5$	(alternate interior angles)
$\angle 3$	$=$	$\angle 4$	(alternate interior angles)
$\angle 2 + \angle 3$	$=$	$\angle 5 + \angle 4$	(adding (1) and (2))
$\angle 1 + \angle 2 + \angle 3$	$=$	$\angle 1 + \angle 5 + \angle 4$	(adding $\angle 1$ to both sides)
$\angle 1 + \angle 5 + \angle 4$	$=$	180°	(angles forming a straight line)

Therefore, $\angle 1 + \angle 2 + \angle 3 = 180^\circ$

$\therefore \angle A + \angle B + \angle C = 180^\circ$.

Thus, the sum of the angles of a triangle is 180°.

Example 1: In ΔABC, $\angle A = 30^\circ$, $\angle B = 45^\circ$, find $\angle C$.

Solution : In ΔABC, $\angle A + \angle B + \angle C = 180^\circ$ (angle-sum property of a triangle)

$30^\circ + 45^\circ + \angle C = 180^\circ$ (substituting given values in question)

$75^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 75^\circ$

Therefore, $\angle C = 105^\circ$

Example 2 : In ΔABC, if $\angle A = 3\angle B$ and $\angle C = 2\angle B$. Find all the three angles of ΔABC.

Solution : $\angle A + \angle B + \angle C = 180^\circ$ [angle-sum property of a triangle]

$3\angle B + \angle B + 2\angle B = 180^\circ$ [$\angle A = 3\angle B$, $\angle C = 2\angle B$]

$6\angle B = 180^\circ$

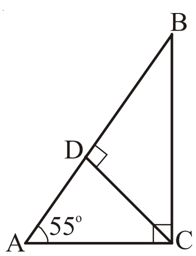
Therefore, $\angle B = 30^\circ$

Thus, $\angle A = 3\angle B = 3 \times 30^\circ = 90^\circ$

$\angle C = 2\angle B = 2 \times 30^\circ = 60^\circ$

Example 3 : ΔABC is right angled at C and $CD \perp AB$, $\angle A = 55^\circ$

Find (i) $\angle ACD$ (ii) $\angle BCD$ (iii) $\angle ABC$



Solution : In ΔACD,

$\angle CAD + \angle ADC + \angle ACD = 180^\circ$ (angle-sum property of a triangle)

$55^\circ + 90^\circ + \angle ACD = 180^\circ$ (substituting values given in question)

$$145^\circ + \angle ACD = 180^\circ$$

$$\angle ACD = 180^\circ - 145^\circ = 35^\circ$$

Therefore, $\angle ACD = 35^\circ$

(ii) In ΔABC,

$$\angle ACB = 90^\circ$$

Therefore, $\angle ACD + \angle BCD = 90^\circ$ (from the figure $\angle ACB = \angle ACD + \angle BCD$)

$35^\circ + \angle BCD = 90^\circ$ (from (i), $\angle ACD = 35^\circ$)

$$\angle BCD = 90^\circ - 35^\circ = 55^\circ$$

(iii) In ΔABC,

$\angle ABC + \angle BCA + \angle CAB = 180^\circ$ (angle-sum property of a triangle)

$\angle ABC + 90^\circ + 55^\circ = 180^\circ$ (given)

$$\angle ABC + 145^\circ = 180^\circ$$

$$\angle ABC = 180^\circ - 145^\circ$$

Therefore, $\angle ABC = 35^\circ$

Example 4 : The angles of a triangle are in the ratio 2 : 3 : 4. Find the angles.

Solution : The given ratio between the angles of the triangle = 2 : 3 : 4

Sum of the terms of the ratio = 2 + 3 + 4 = 9

Sum of the angles of a triangle = 180°

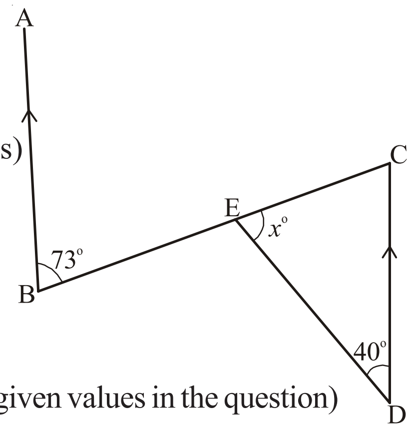
Therefore, 1^{st} angle $= \frac{2}{9} \times 180^\circ = 40^\circ$

2^{nd} angle $= \frac{3}{9} \times 180^\circ = 60^\circ$

3^{rd} angle $= \frac{4}{9} \times 180^\circ = 80^\circ$

Thus, the angles of the triangle are 40°, 60° and 80°.

Example 5 : Find the value of angle '*x*' in the figure.



Solution : $\angle ECD = \angle ABC = 73^o$

(Since AB || CD these two are alternate angles)

In ΔECD,

$\angle CED + \angle EDC + \angle DCE = 180^o$

(angle-sum property of a triangle)

$x^o + 40^o + 73^o = 180^o$ (substituting given values in the question)

$x^o + 113^o = 180^o$

$x^o = 180^o - 113^o$

$x^o = 67^o$

Example 6 : One angle of ΔABC is 40^o and the other two angles are equal. Find the measure (value) of each equal angle.

Solution : Let $\angle C = 40^o$ and $\angle A = \angle B = x^o$

$\angle A + \angle B + \angle C = 180^o$ (angle-sum property of a triangle)

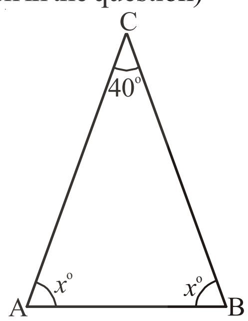
$x^o + x^o + 40^o = 180^o$ (substituting values given in the question)

$2x^o + 40^o = 180^o$

$2x = 180^o - 40^o$

$2x = 140^o$

$x^o = 70^o$

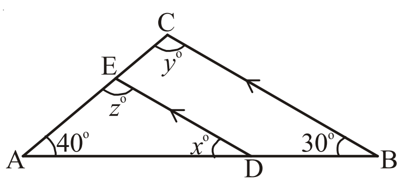


Thus, each angle is 70^o

Example 7 : In the figure, D and E are the points on sides AB and AC of ΔABC such that DE || BC. If $\angle B = 30^o$ and $\angle A = 40^o$, find (i) x (ii) y (iii) z

Solution : (i) $\angle ADE = \angle ABC$ (corresponding angle as DE || BC)

Therefore, $x^o = 30^o$



(ii) In ΔABC,

$\angle A + \angle B + \angle C = 180^o$ (angle sum property of a triangle)

$40^o + 30^o + y^o = 180^o$ (substituting values given inthe question)

$70^o + y^o = 180^o$

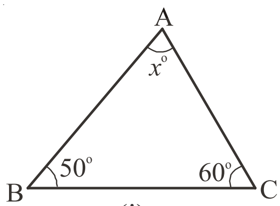
Therefore, $y^o = 180^o - 70^o = 110^o$

(iii) $y^{\circ} = z^{\circ} = 110^{\circ}$

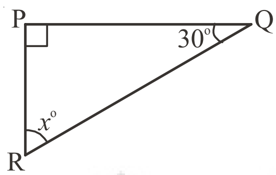
corresponding angles since DE || BC

Exercise - 3

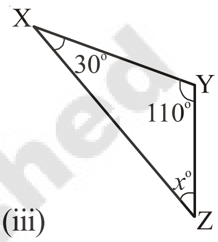
1. Find the value of the unknown 'x' in the following triangles.



(i)

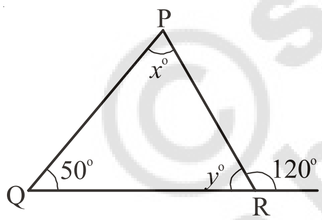


(ii)

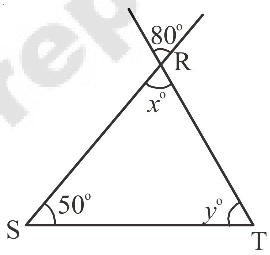


(iii)

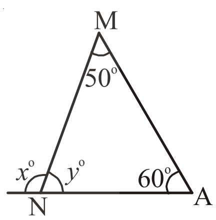
2. Find the values of the unknowns 'x' and 'y' in the following diagrams.



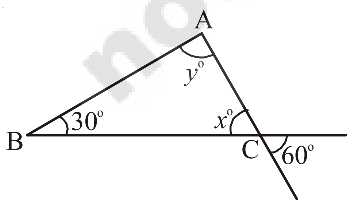
(i)



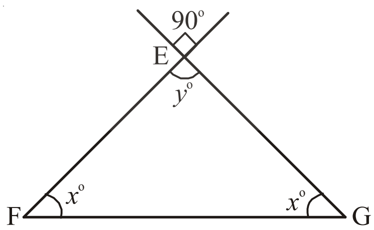
(ii)



(iii)



(iv)



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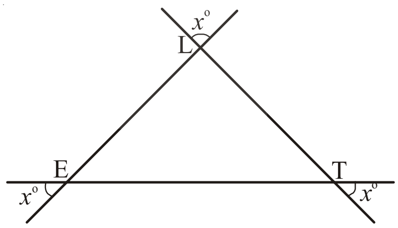


(vi)

3. Find the measure of the third angle of triangles whose two angles are given below:

(i) 38°, 102° (ii) 116°, 30° (iii) 40°, 80°

4. In a right-angled triangle, one acute angle is 30°. Find the other acute angle.

5. State true or false for each of the following statements.

(i) A triangle can have two right angles.

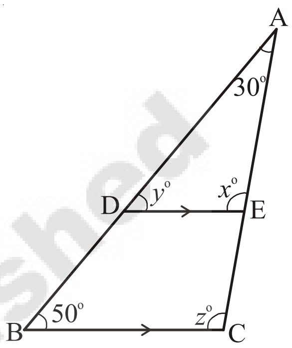
(ii) A triangle can have two acute angles.

(iii) A triangle can have two obtuse angles.

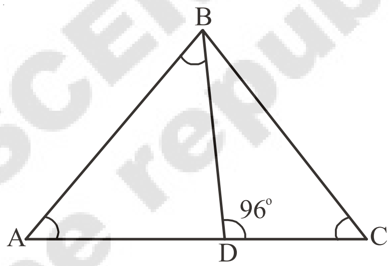
(iv) Each angle of a triangle can be less than 60°.

6. The angles of a triangle are in the ratio 1 : 2 : 3. Find the angles.

7. In the figure, $\overline{DE} \parallel \overline{BC}$, $\angle A = 30^{\circ}$ and $\angle B = 50^{\circ}$. Find the values of x, y and z.



8. In the figure, $\angle ABD = 3\ \angle DAB$ and $\angle BDC = 96^{\circ}$. Find $\angle ABD$.

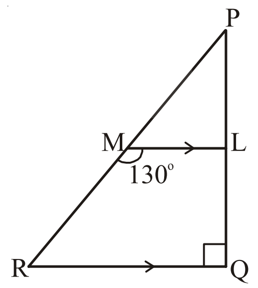


9. In ΔPQR $\angle P = 2\ \angle Q$ and $2\ \angle R = 3\ \angle Q$, calculate the angles of ΔPQR.

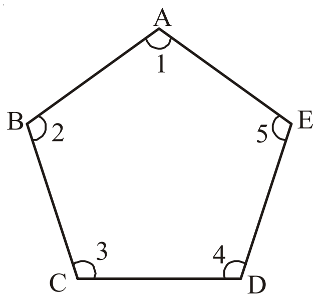
10. If the angles of a triangle are in the ratio 1 : 4 : 5, find the angles.

11. The acute angles of a right triangle are in the ratio 2 : 3. Find the angles of the triangle.

12. In the figure, ΔPQR is right angled at Q, $\overline{ML} \parallel \overline{RQ}$ and $\angle LMR = 130^{\circ}$. Find $\angle LPM$, $\angle PML$ and $\angle PRQ$.



13. In Figure ABCDE, find $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5$.



5.5.2 Exterior angle of a triangle

Draw ΔABC and produce one of its sides say $\overline{BC}$ as shown in the Figure 1. Observe the $\angle ACD$ formed at point C. This angle lies in the exterior of ΔABC. We call it the exterior angle of ΔABC formed at vertex C.

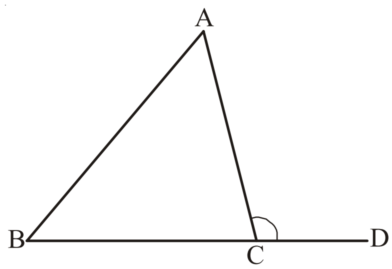


Figure 1

Clearly $\angle BCA$ is an adjacent angle to $\angle ACD$. The remaining two angles of the triangle namely $\angle BAC$ or $\angle A$ and $\angle CBA$ or $\angle B$ are called the two interior opposite angles of $\angle ACD$. Now cut out (or make trace copies of) $\angle A$ and $\angle B$ and place them adjacent to each other as shown in the Figure 2.

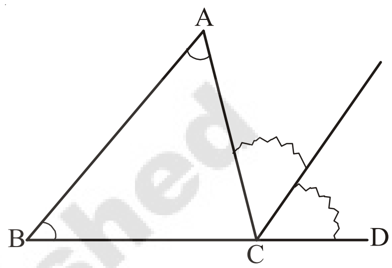


Figure 2

Do these two pieces together entirely cover $\angle ACD$?

Can you say that $\angle ACD = \angle A + \angle B$?

From the above activity, we can say that an exterior angle of a triangle is equal to the sum of two interior opposite angles.

Do This

Draw ΔABC and form an exterior $\angle ACD$. Now take a protractor and measure $\angle ACD$, $\angle A$ and $\angle B$.

Find the sum $\angle A + \angle B$ and compare it with the measure $\angle ACD$.

Do you observe that $\angle ACD$ is equal (or nearly equal) to $\angle A + \angle B$?

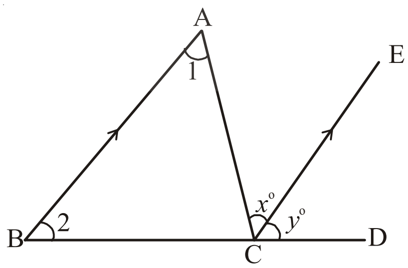
A logical step- by- step argument can further confirm that the exterior angle of a triangle is equal to the sum of the interior opposite angles.

Statement : An exterior angle of triangle is equal to the sum of its interior opposite angles.

Given : ΔABC with $\angle ACD$ as exterior angle

To prove : $\angle ACD = \angle A + \angle B$

Construction : Through C draw $\overline{CE}$ parallel to $\overline{BA}$



Justification

$\angle 1 = \angle x$ ($\overline{BA} \parallel \overline{CE}$ and $\overline{AC}$ is transversal therefore, alternate angles are equal)

$\angle 2 = \angle y$ ($\overline{BA} \parallel \overline{CE}$ and $\overline{BD}$ is transversal therefore, corresponding angles are equal)

$\angle 1 + \angle 2 = \angle x + \angle y$

Therefore , $\angle ACD = \angle 1 + \angle 2$ (from the figure $\angle x + \angle y = \angle ACD$)

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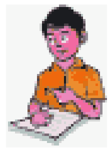
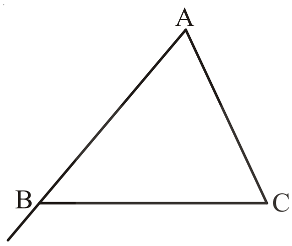
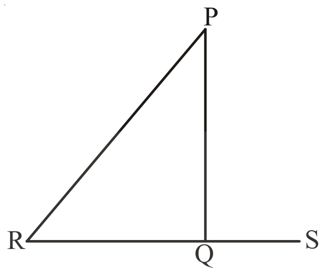
Thus, the exterior angle of a triangle is equal to the sum of the interior opposite angles. This property is called the exterior-angle property of a triangle.

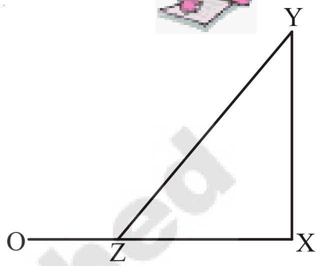
Do This

Copy each of the following triangles. In each case verify that an exterior angle of a triangle is equal to the sum of the two interior opposite angles.

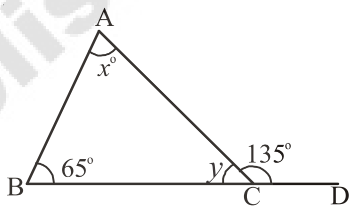








Example 8 : In the figure, find the value *x* and *y*.



Solution : $\angle ACD = \angle ABC + \angle BAC$

(exterior angle property)

$135^\circ = 65^\circ + x^\circ$

$135^\circ - 65^\circ = x^\circ$

Therefore, $x^\circ = 70^\circ$

$\angle ABC + \angle BAC + \angle BCA = 180^\circ$ (angle-sum property of a triangle)

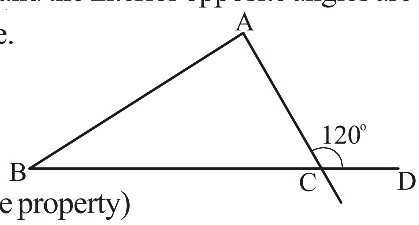
$65^\circ + 70^\circ + y^\circ = 180^\circ$

$135^\circ + y^\circ = 180^\circ$

$y^\circ = 180^\circ - 135^\circ$

Therefore, $y^\circ = 45^\circ$

Example 9 : One of the exterior angles of a triangle is 120° and the interior opposite angles are in the ratio 1 : 5. Find the angles of the triangle.



Solution : $\angle ACD = 120^\circ$ (from the question)

$\angle ACD = \angle A + \angle B$ (exterior angle property)

$\angle A + \angle B = 120^\circ$

$\angle B : \angle A = 1 : 5$

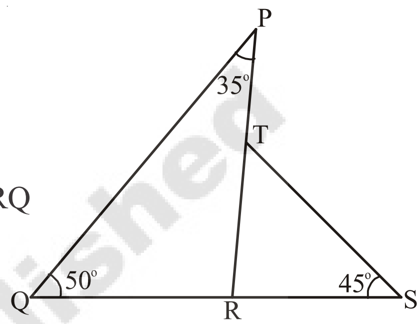
$$\angle B = \frac{1}{6} \times 120^{o} = 20^{o}$$

$$\angle A = \frac{5}{6} \times 120^{o} = 100^{o}$$

$\angle A + \angle B + \angle C = 180^{o}$ (angle-sum property of a triangle)

$100^{o} + 20^{o} + \angle C = 180^{o}$

Therefore, $\angle C = 180^{o} - 120^{o} = 60^{o}$



Example 10 : In the adjacent figure, find

(i) $\angle PRS$ (ii) $\angle PTS$ (iii) $\angle STR$ (iv) $\angle PRQ$

Solution : (i) In ΔPQR, $\angle PRS$ is the exterior angle and $\angle RQP$ and $\angle QPR$ are the interior opposite angles.

$\therefore \quad \angle PRS = \angle RQP + \angle QPR$ (exterior angle property)

$\angle PRS = 50^{o} + 35^{o} = 85^{o}$

(ii) In ΔRST, $\angle PTS$ is the exterior angle and $\angle SRT$ and $\angle RST$ are the interior opposite angles.

Therefore, $\angle PTS = \angle SRT + \angle RST$

$\angle PTS = 85^{o} + 45^{o}$ ($\angle SRT = \angle PRS = 85^{o}$)

$\angle PTS = 130^{o}$

(iii) In Δ RST we have

$\angle STR + \angle RST + \angle SRT = 180^{o}$ (angle-sum property of a triangle)

$\angle STR + 45^{o} + 85^{o} = 180^{o}$

$\angle STR + 130^{o} = 180^{o}$

Therefore, $\angle STR = 180^{o} - 130^{o} = 50^{o}$

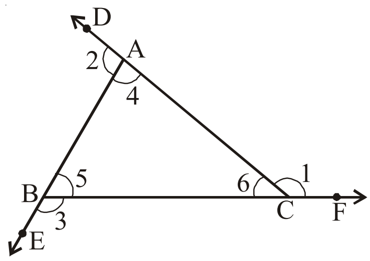
(iv) $\angle PRQ + \angle PRS = 180^{o}$ (liner pair property)

$\angle PRQ + 85^{o} = 180^{o}$

$\angle PRQ = 180^{o} - 85^{o}$

$\angle PRQ = 95^{o}$

Example 11 : Show that the sum of the exterior angles of ΔABC is 360°.



Solution : $\angle 2 + \angle 4 = 180^\circ$ (linear pair)

$\angle 3 + \angle 5 = 180^\circ$ (linear pair)

$\angle 6 + \angle 1 = 180^\circ$ (linear pair)

Adding the angles on both sides, we get-

$\angle 2 + \angle 4 + \angle 3 + \angle 5 + \angle 6 + \angle 1 = 180^\circ + 180^\circ + 180^\circ$

$(\angle 4 + \angle 5 + \angle 6) + (\angle 1 + \angle 2 + \angle 3) = 540^\circ$

We know that, $\angle 4 + \angle 5 + \angle 6 = 180^\circ$ (angle-sum property of a triangle)

Therefore, $180^\circ + \angle 1 + \angle 2 + \angle 3 = 540^\circ$

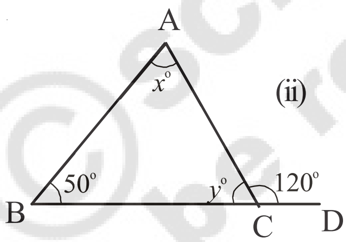
$\angle 1 + \angle 2 + \angle 3 = 540^\circ - 180^\circ$

$\angle 1 + \angle 2 + \angle 3 = 360^\circ$

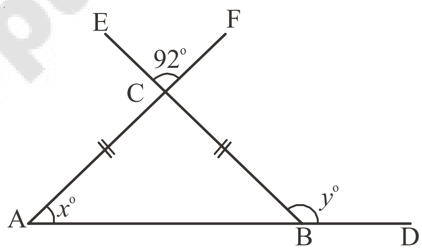
Thus, the sum of the exterior angles of a triangle is 360°.

Example 12 : Find the angles x and y in the following figures.

(i)



(ii)



Solution : (i) $\angle BAC + \angle ABC = \angle ACD$ (exterior angle property)

$x^\circ + 50^\circ = 120^\circ$

$x^\circ = 120^\circ - 50^\circ = 70^\circ$

$\angle ACB + \angle ACD = 180^\circ$ (linear pair)

$y^\circ + 120^\circ = 180^\circ$

$y^\circ = 180^\circ - 120^\circ = 60^\circ$

(ii) $\angle ACB = \angle ECF = 92^\circ$ (vertically opposite angles)

$\angle CAB = \angle CBA$ (opposite angles of equal sides)

In ΔABC, $\angle BAC + \angle CBA + \angle ACB = 180^\circ$ (angle-sum property)

$x^\circ + x^\circ + 92^\circ = 180^\circ$

$2x = 180^\circ - 92^\circ = 88^\circ$

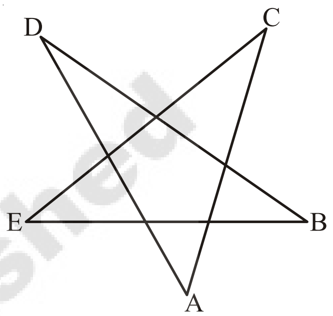
Therefore, $x^\circ = \dfrac{88}{2} = 44^\circ$

Also $\angle ABC + y^\circ = 180^\circ$ (linear pair)

$y^\circ = 180^\circ - x^\circ$

Therefore., $y^\circ = 180^\circ - 44^\circ = 136^\circ$

Example 13 : Find the value of $\angle A + \angle B + \angle C + \angle D + \angle E$ of the following figure.



Solution : Name the angles as shown in the figure.

In ΔGHC $\angle 3 + \angle 6 + \angle 7 = 180^\circ$ ....(1)

(angle-sum property of triangle)

In ΔEHB, $\angle 6 = \angle 5 + \angle 2$ ....(2)

In ΔAGD, $\angle 7 = \angle 1 + \angle 4$ ....(3)

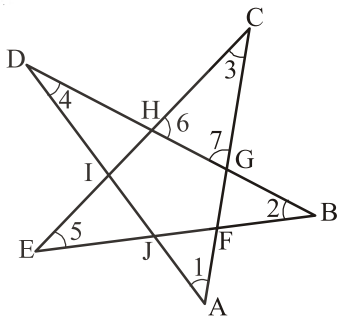
(exterior angle property of a triangle)

Substituting (2) and (3) in (1)

$\Rightarrow \quad \angle 3 + \angle 5 + \angle 1 + \angle 2 + \angle 4 = 180^\circ$

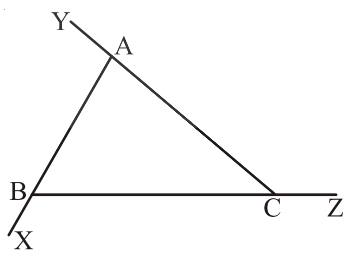
$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 180^\circ$

Therefore, $\angle A + \angle B + \angle C + \angle D + \angle E = 180^\circ$

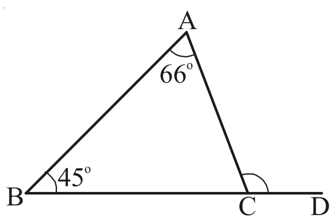


Exercise - 4

1. In ΔABC, name all the interior and exterior angles of the triangle.



2. For ΔABC, find the measure of $\angle ACD$.



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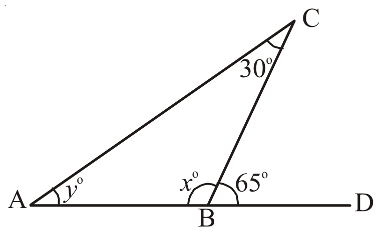
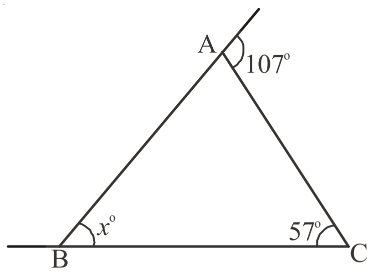
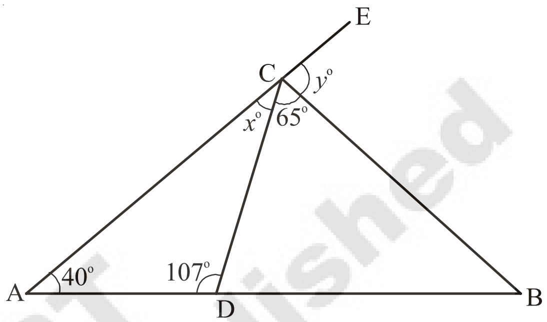
3. Find the measure of angles x and y.

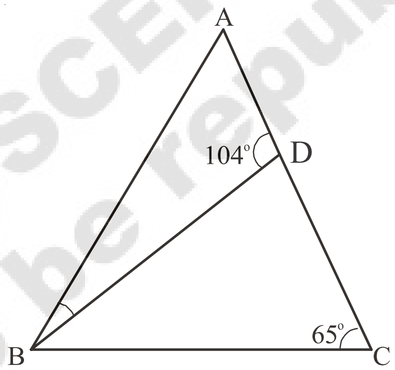


4. In the following figures, find the values of x and y.

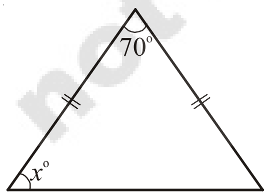




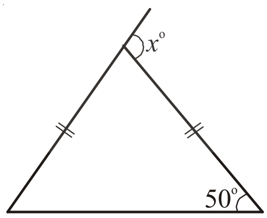
5. In the figure $\angle BAD = 3\ \angle DBA$, find $\angle CDB$, $\angle DBC$ and $\angle ABC$.



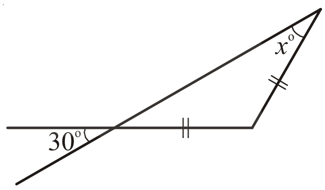
6. Find the values of x and y in the following figures.



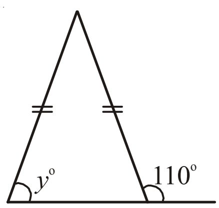
(i)



(ii)



(iii)



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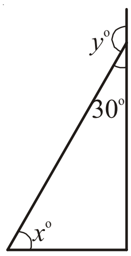

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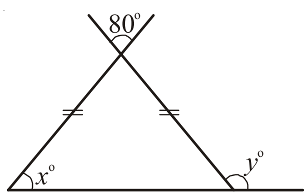


(vi)

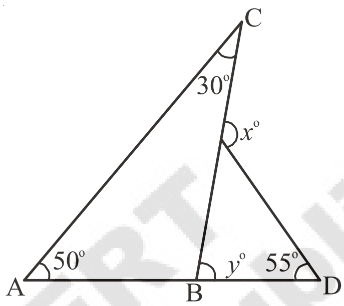
7. One of the exterior angles of a triangle is 125° and the interior opposite angles are in the ratio 2 : 3. Find the angles of the triangle.

8. The exterior $\angle PRS$ of ΔPQR is 105°. If $Q = 70^\circ$, find $\angle P$. Is $\angle PRS > \angle P$?

9. If an exterior angle of a triangle is 130° and one of the interior opposite angle is 60°. Find the other interior opposite angle.

10. One of the exterior angle of a triangle is 105° and the interior opposite angles are in the ratio 2 : 5. Find the angles of the triangle.

11. In the figure find the values of x and y.



Looking Back

1 (i) A triangle is a simple closed figure made up of three line segments.

(ii) Based on the sides, triangles are of three types

- A triangle having all three sides of same length is called an Equilateral Triangle.
- A triangle having at least two sides of equal length is called an Isosceles Triangle.
- If all the three sides of a triangle are of different length, the triangle is called a Scalene Triangle.

(iii) Based on the angles, triangles are of three types

- A triangle whose all angles are acute is called an acute-angled triangle.
- A triangle whose one angle is obtuse is called an obtuse-angled triangle.
- A triangle whose one angle is a right angle is called a right-angled triangle.

2. The six elements of a triangle are its three angles and the three sides.

3. Properties of the lengths of the sides of a triangle:

(i) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

(ii) The difference between the lengths of any two sides of a triangle is smaller than the length of the third side.

4. The line segment joining a vertex of a triangle to the mid-point of its opposite side is called a median of the triangle. A triangle has 3 medians.

5. The perpendicular line segment from a vertex of a triangle to its opposite side is called the altitude of the triangle.

6. The total measure of the three angles of a triangle is 180^{o}. This is called the angle sum property of a triangle.

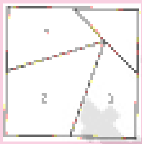
7. The measure of any exterior angle of a triangle is equal to the sum of its interior opposite angles. This is called the exterior angle property of the triangle.

8. Representation of line, line segment and ray

LM = Lenght of Line segment of LM ; $\overline{LM}$ = Line segment LM

$\overrightarrow{LM}$ = Ray LM ; $\overleftrightarrow{LM}$ = Line LM

Fun with Card board shapes



Take square card board sheet.
Mark the mid points of sides and draw lines as shown in the figure.
Cut the square into four parts and rearrange them to get a triangle