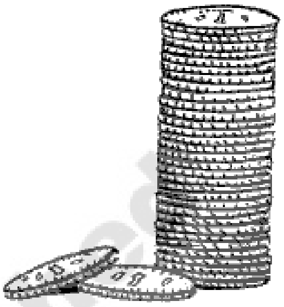
CONGRUENCY OF TRIANGLES 8

8.0 Introduction



If we take a pile of one rupee coins and place them one on top of the other, they would match perfectly. Do you know why this happens? This is because all the coins have the same size and shape. In the same way papers of a blank note book have the same size and shape.

Look around you and find some examples of objects that share this kind of similarity i.e. they are identical in shape and size. Think of at least 5 such examples.

When we talk about objects of the same size and shape we say that the objects are congruent. A practical test of congruence is to place one object over the other and see if they superimpose exactly.

Activity:

Are all ten rupee notes congruent? How will you check?

Similarly, check if the five rupee note you find are congruent.

We see many examples of congruent objects all around us. Now, think of some shapes that are congruent.

Do This

1. Here are some shapes. See whether all the shapes given in a row are congruent to each other or not. You can trace the figures and check.

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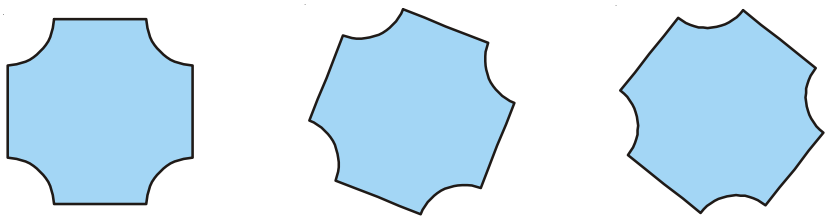

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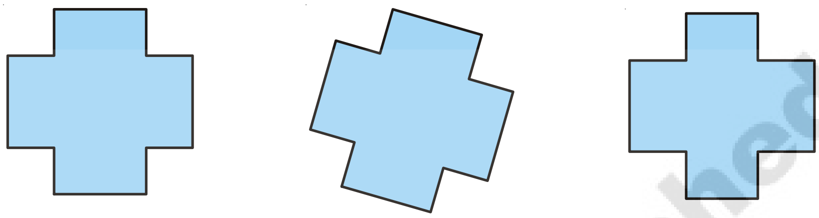

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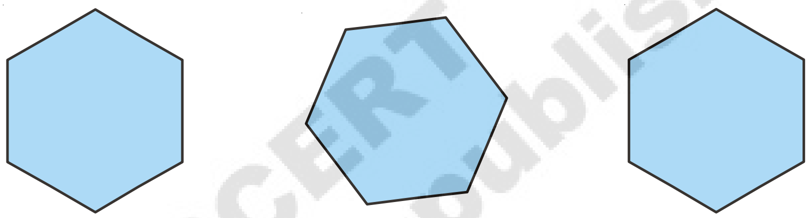
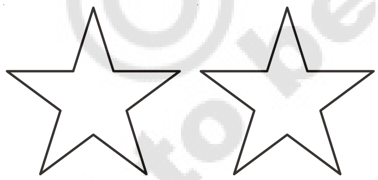


2. Which of the following pairs of figures are congruent?



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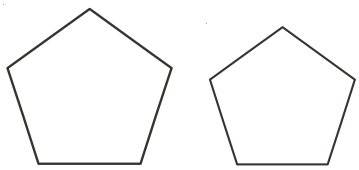

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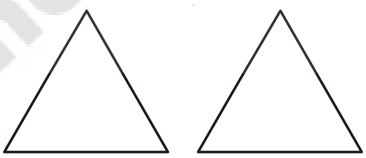

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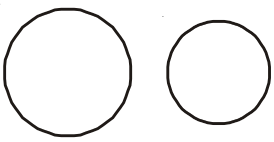

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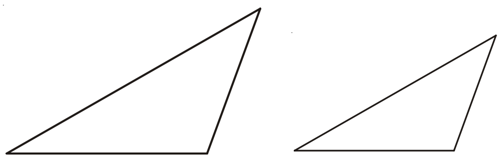

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8.1 Congruency of line segments

Observe the two pairs of line segments given below.

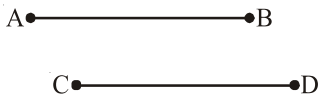


Figure 1

Figure 2

Copy the line segment AB on a tracing paper. Place it on CD. You will find that AB covers CD. Hence the line segments are congruent. We write $\overline{AB} \cong \overline{CD}$.

Repeat this activity for the pair of line segments in Figure 2. What do you find? Are they congruent?

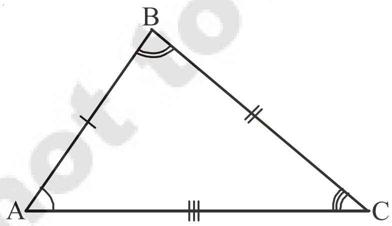
You will notice that the pair of line segments in Figure 1 match with each other because they have same length and this is not the case in Figure 2.

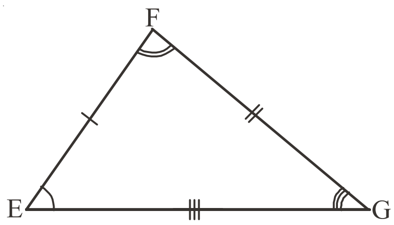
The line segment has only one dimension i.e., length. So if two line segments have the same length, they are congruent. Conversely, if two line segments are congruent, they have the same length.

When we write AB = CD, what we actually mean is $\overline{AB} \cong \overline{CD}$.

8.2 Congruency of triangles

We learnt that two line segments are congruent where one of them, is the copy of the other. We extend this idea to triangles. Two triangles are congruent if they are copies of one another and when superimposed, they cover each other exactly.





ΔABC and ΔEFG cover each other exactly i.e. they are of the same size and shape. They are congruent triangles. We express congruency of the two triangles as $\Delta ABC \cong \Delta EFG$.

If two triangles are congruent then all the corresponding six elements of the two triangles i.e. the three angles and three sides are congruent. We also say that if the corresponding parts of two triangles are congruent, then the triangles are congruent. This means that, when you place ΔABC on ΔEFG, their corresponding corners fall on each other. A lies on E, B lies on F and C lies on G. Also $\angle$A falls on $\angle$E, $\angle$B falls on $\angle$F and $\angle$C falls on $\angle$G and lastly AB falls on EF, BC falls on FG and AC falls on EG.

Thus, for two triangles that are congruent, their corresponding parts i.e. vertices, angles and sides match one another or are equal.

In ΔABC and ΔEFG

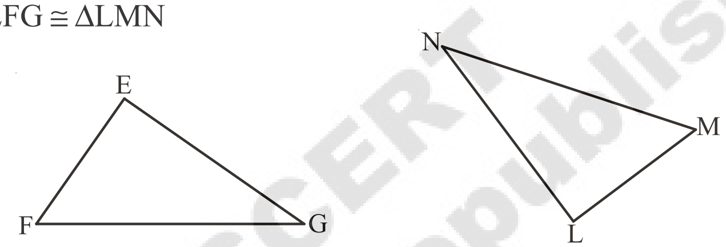
$A \rightarrow E$	$B \rightarrow F$	$C \rightarrow G$	(corresponding vertices)
$\angle A \cong \angle E$	$\angle B \cong \angle F$	$\angle C \cong \angle G$	(corresponding angles)
$\overline{AB} \cong \overline{EF}$	$\overline{BC} \cong \overline{FG}$	$\overline{AC} \cong \overline{EG}$	(corresponding sides)

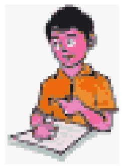
The order of the alphabet in the names of congruent triangles displays the corresponding relationships. Thus, when we say that $\Delta ABC \cong \Delta EFG$

Verify if $\Delta ABC \cong \Delta EFG$, their corresponding verticies, sides and angles.

Do This

1. $\Delta EFG \cong \Delta LMN$



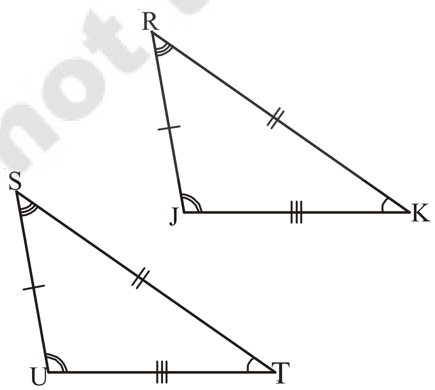


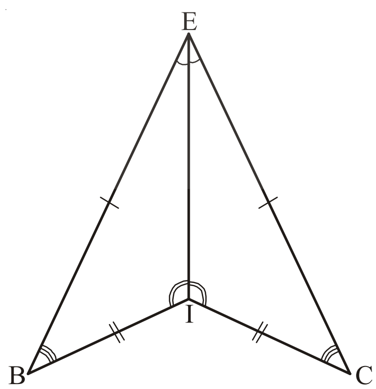
Write the corresponding vertices, angles and sides of the two triangles.

2. If $\Delta ABC \cong \Delta DEF$ write the parts of ΔABC that correspond to-

(i) DE (ii) $\angle E$ (iii) DF (iv) EF (v) $\angle F$

3. Name the congruent triangles in each of the following pairs. Write the statement using '$\cong$'.





4 . Name the congruent angles and sides for each pair of congruent triangles.

1. $\Delta TUV \cong \Delta XYZ$ 2. $\Delta CDG \cong \Delta RSW$

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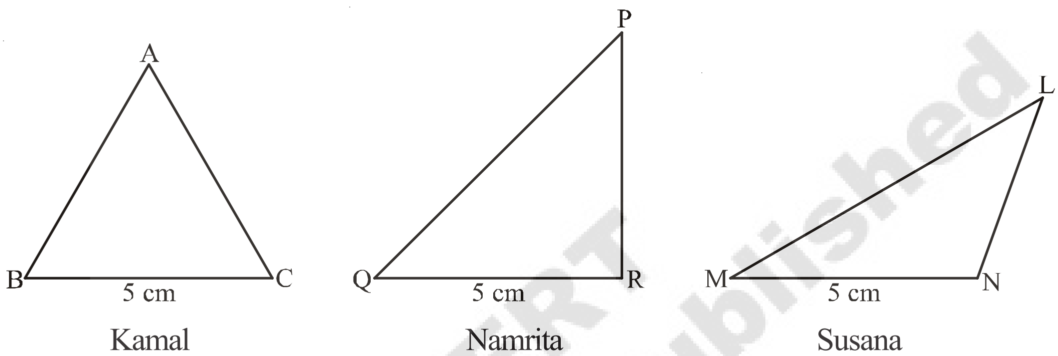
8.3 Criterion for congruency of triangles

Is it necessary for congruency to check whether all the corresponding parts of two triangles are congruent? How can we check if the given triangles are congruent using a minimum number of measurements. Let us explore and find out.

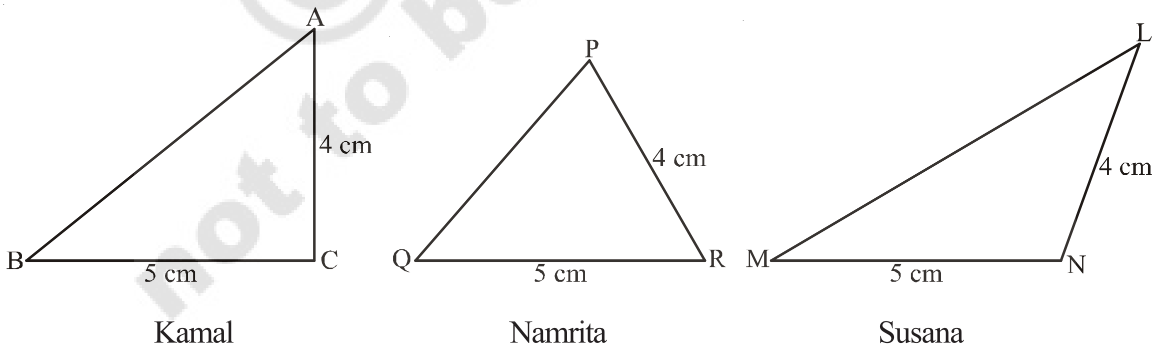
8.3.1 Side-Side-Side congruency (SSS)

Will all of you draw the same triangle if you only knew that the measure of one side of the triangle is 5 cm? Kamal, Namrita and Susana have drawn them like this.

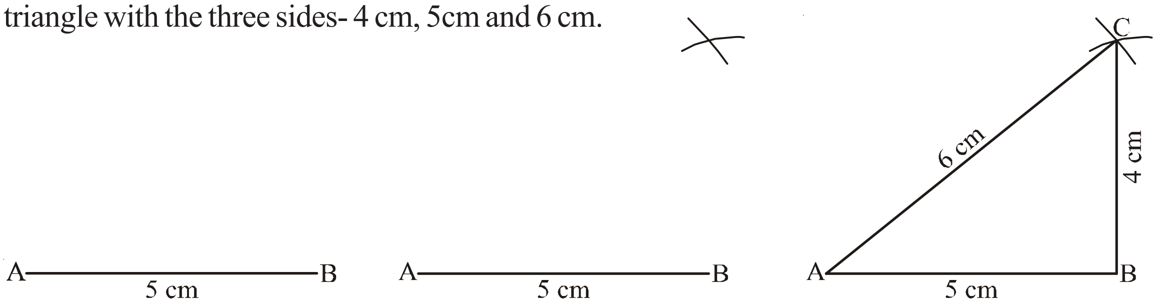


As you can see all the triangles are different. Kamal drew an equilateral triangle, Namrita drew a right-angled triangle and Susana drew an obtuse-angled triangle.

Now will all of you draw the same triangle, if you knew that the two sides of a triangle are say, 4 cm and 5 cm. Again Kamal, Namrita and Susana drew different triangles.



What if you know all the sides of the triangle? Kamal, Namrita and Susana all drew the same triangle with the three sides- 4 cm, 5cm and 6 cm.



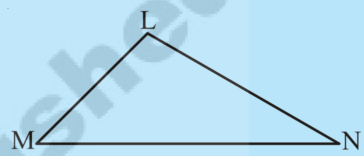
Thus, if we want to make a copy of ABC or a triangle congruent to ABC, we need the lengths of the three sides. This is referred to as the Side-Side-Side(SSS) criterion for congruency of triangles.

If two triangles are congruent because the lengths of their corresponding sides are equal, then will their angles also be equal?

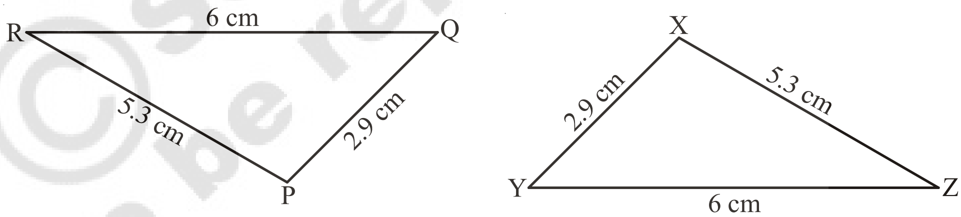
Side-Side-Side (SSS) criterion for congruence of triangles: If three sides of a triangle are equal to the corresponding three sides of another triangle, then the triangles are congruent.

Try This

Measure the lengths of ΔLMN. Now, construct a triangle with these measurements on a sheet of paper. Place this triangle over ΔLMN. Are the triangles congruent? What criterion of congruency applies over here?



Example 1 : Is $\Delta PQR \cong \Delta XYZ$? Also, write the corresponding angles of the two triangles.



Solution : According to the given figure of ΔPQR and ΔXYZ, we have

$PQ = XY = 2.9$ cm

$QR = YZ = 6$ cm

$RP = ZX = 5.3$ cm

Therefore, by Side-Side-Side congruence criterion, $\Delta PQR \cong \Delta XYZ$

Clearly, the point P corresponds to point X, point Q corresponds to point Y and the point R corresponds to point Z.

So, $\angle P, \angle X$; $\angle Q, \angle Y$; $\angle R, \angle Z$ are pairs of corresponding angles.

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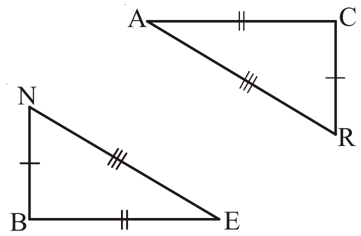
Exercise - 1

1. Decide whether the SSS congruence is true with the following figures. Give reasons

(i)



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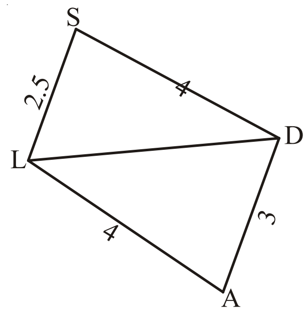
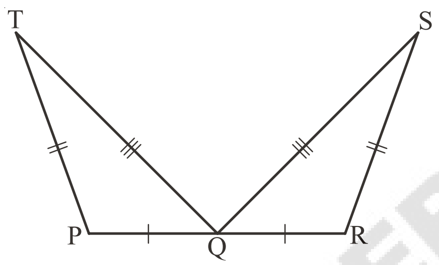


2. For the following congruent triangles, find the pairs of corresponding angles.

(i)



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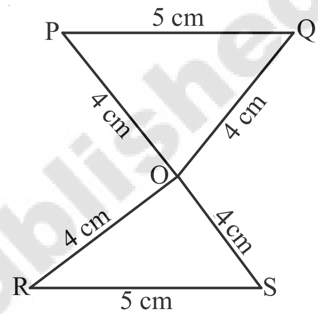


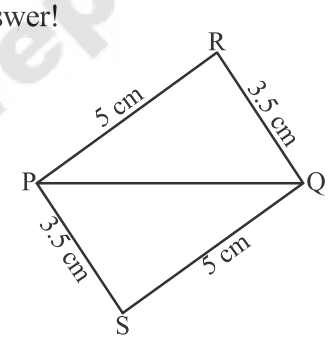
3. In adjacent figure, choose the correct answer!

(i) $\Delta PQR \cong \Delta PQS$

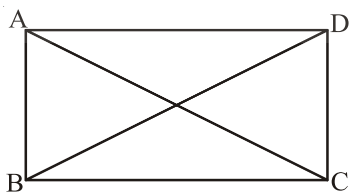
(ii) $\Delta PQR \cong \Delta QPS$

(iii) $\Delta PQR \cong \Delta SQP$

(iv) $\Delta PQR \cong \Delta SPQ$

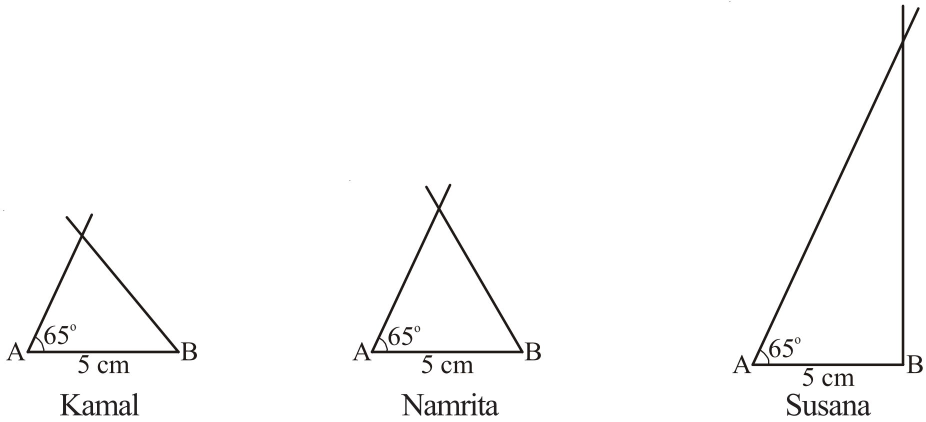


4. In the figure given below, AB = DC and AC = DB. Is $\Delta ABC \cong \Delta DCB$.



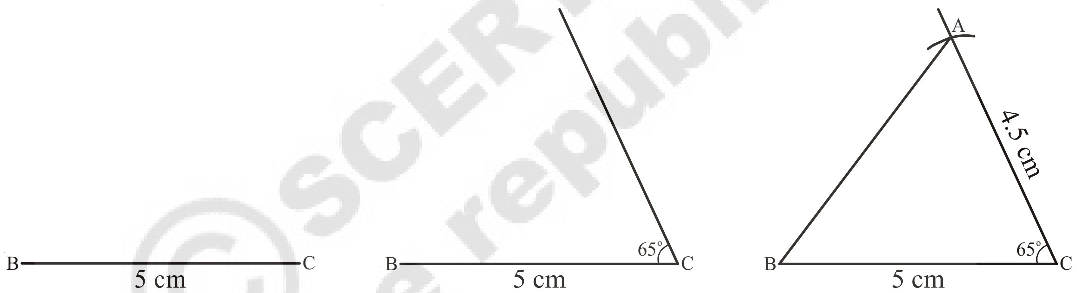
8.3.2 Side-Angle-Side Congruence

We have seen that it is not possible to draw congruent triangles, if we are given only the measurements of one side. Now, what if you were given one angle and one side? Kamal, Namrita and Susana were told to draw triangles with one side equal to 5 cm and one angle equal to 65°. They drew the following dissimilar triangles.



Now, what if the three of them knew the two sides of the triangle and the angle included between these sides. The three children decided to draw triangles with sides 5 cm and 4.5 cm and the included angle of $\angle 65$.

Kamal drew ΔABC. He drew BC as the base= 5 cm. He then made $\angle C = 65^o$ using a protractor and then marked point A at a length of 4.5 cm on the angular line. He then joined points A and B.



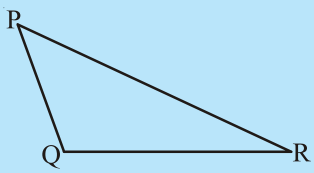
Can you draw the 65° angle at point B with side AB = 4.5 cm. Will the triangle that is formed be congruent to Kamal's triangle? Can you take the base to be 4.5 cm, side = 5 cm and included angle = 65°? Will the triangle that is formed be congruent to Kamal's triangle? You will find that the triangles formed in all these situations are congruent triangles.

Therefore, if we want to make a copy of ΔABC or a triangle congruent to ΔABC, we need the lengths of the two sides and the measure of the angle between the two sides. This is referred to as the Side-Angle-Side(SAS) criterion for congruence of triangles.

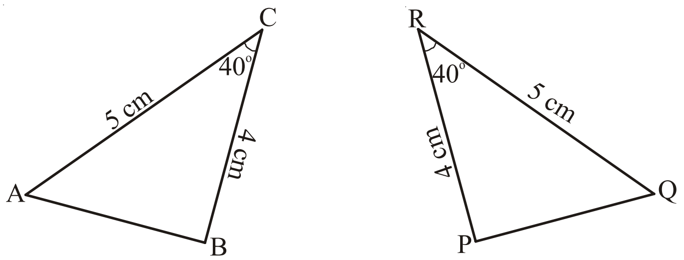
Side-Angle-Side(SAS) criterion for congruence of triangles: If two sides and the angle included between the two sides of a triangle are congruent to the corresponding two sides and the included angle of another triangle, then the triangles are congruent.

Try This

In ΔPQR measure the lengths PQ and QR as well as $\angle Q$. Now, construct a triangle with these three measurements on a sheet of paper. Place this triangle over ΔPQR. Are the triangles congruent? What criterion of congruency applies over here?



Example 2 : See the measurements of the triangles given below. Are the triangles congruent? Which are the corresponding vertices and angles?



Solution : In ΔABC and ΔPQR $AC = QR$ and $BC = PR$ and included angle $\angle C \cong \angle R$

So, $\Delta ABC \cong \Delta QPR$.

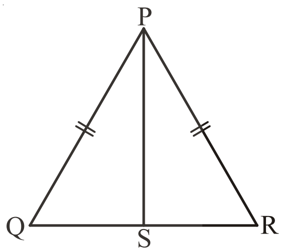
The correspondence is as follows:

$A \leftrightarrow Q$, $B \leftrightarrow P$ and $C \leftrightarrow R$

Therefore, $\angle A \cong \angle Q$, $\angle B \cong \angle P$ and $\angle C \cong \angle R$

Example 3 : In ΔPQR, $PQ = PR$ and PS is angle bisector of $\angle P$.

Are ΔPQS and ΔPRS congruent? If yes, give reason.



Solution : In ΔPQS and ΔPRS

$PQ = PR$ (given)

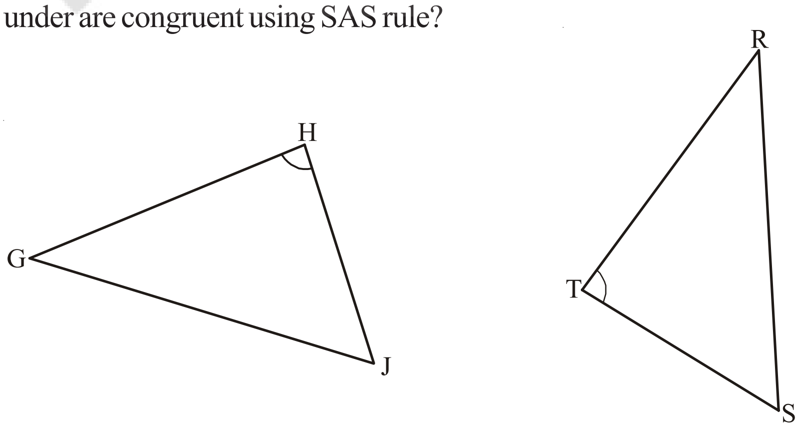
$PS = PS$ (common side in both the triangles)

and included angle $\angle QPS \cong \angle RPS$ (PS is the angle bisector)

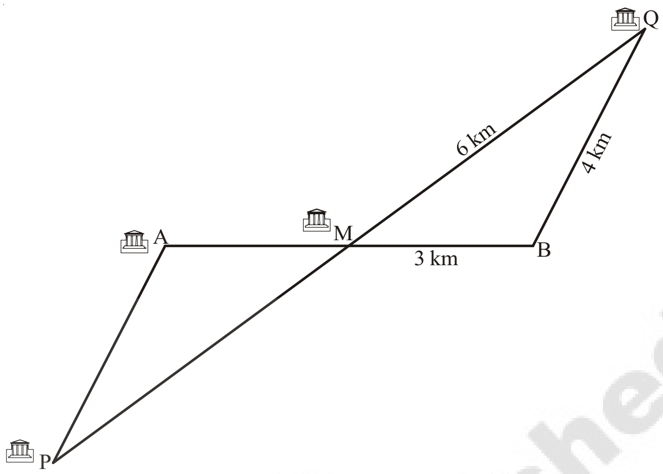
Therefore, $\Delta PQS \cong \Delta PRS$ (by SAS rule)

Exercise - 2

1. What additional information do you need to conclude that the two triangles given here under are congruent using SAS rule?

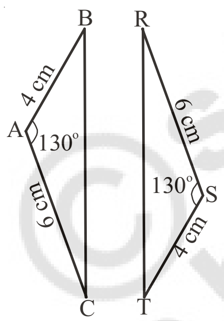


2. The map given below shows five different villages. Village M lies exactly halfway between the two pairs of villages A and B as well as and P and Q. What is the distance between village A and village P. (Hint: check if $\Delta PAM \cong \Delta QBM$)



3. Look at the pairs of triangles given below. Are they congruent? If congruent write the corresponding parts.

(i)



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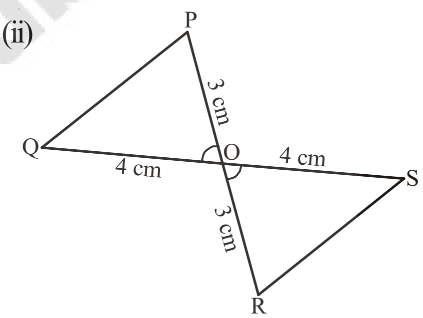

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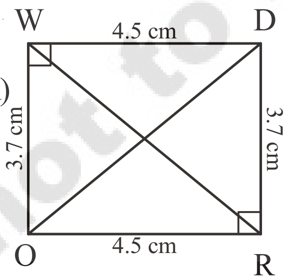

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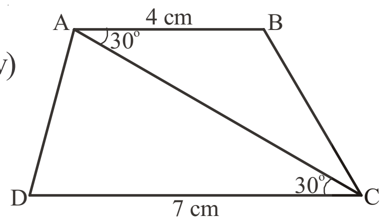
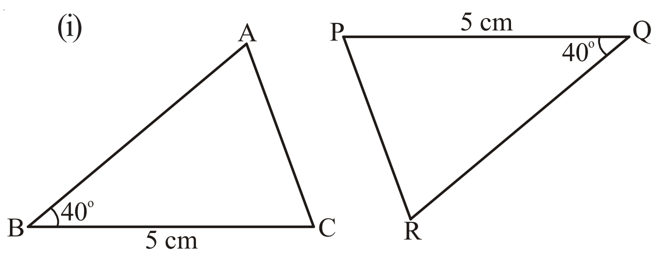


4. Which corresponding sides do we need to know to prove that the triangles are congruent using the SAS criterion?

(i)



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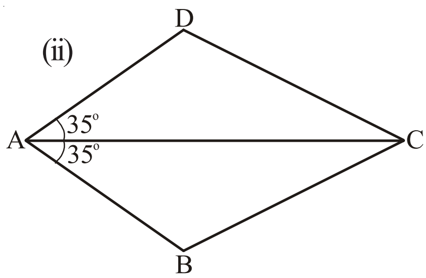

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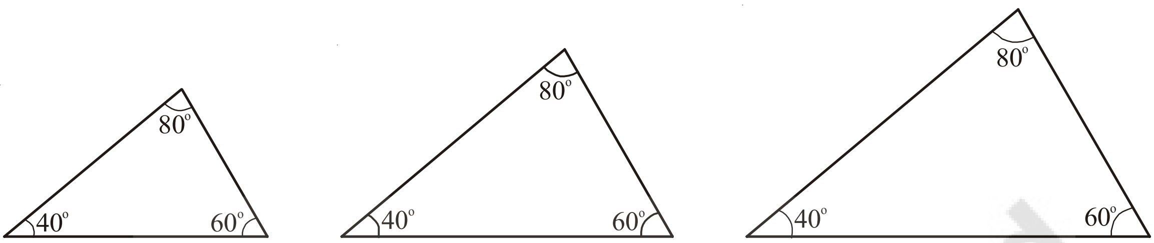
8.3.3 Angle-Side-Angle congruency (ASA)

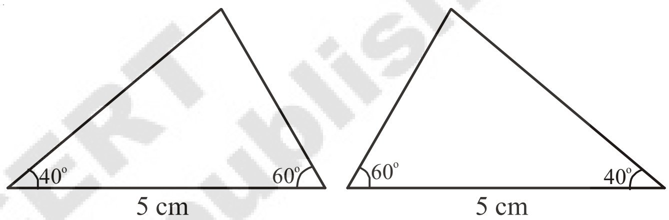
Can the children construct a triangle if they know only one angle of the triangle? What if they know two angles? Will children be able to draw congruent triangles if they know all the angles of the triangle?

Kamal, Namrita and Susana drew the following triangles of angles- 40°, 60° and 80°.



Kamal Namrita Susana

Therefore, though the angles of all the triangles are congruent, the lengths of their sides is not and they are not congruent.



Kamal Susana

Thus, we need to know the length of the sides, to draw congruent triangles. What if we have two angles and one side? Kamal and Namrita drew the following triangles with angles 60° and 40° and side 5 cm. When both the children constructed their triangles they made the given side, the included side.

We can conclude that if we want to make a copy of a triangle or a triangle congruent to another triangle, then we need to know two angles and the length of the side included between the two angles. This is referred to as the Angle-Side-Angle criterion of congruence.

Angle-Side-Angle criterion of congruence: If two angles and the included side of a triangle are congruent to the two corresponding angles and included side of another triangle then the triangles are congruent.

Try This

Teacher has asked the children to construct a triangle with angles 60°, 40° and with a side 5 cm. Sushma calculated the third angle of the triangle as 80° using angle - sum property of triangle. Then Kamal, Sushma and Namratha constructed triangles differently using the following measurements.

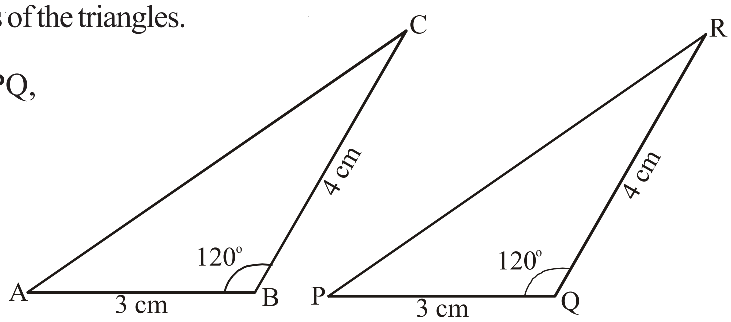
Kamal: 60°, 40° and 5cm side (as teacher said)

Sushma: 80°, 40°, and 5 cm side

Namratha: 60°, 80° and 5 cm side.

They cut these triangles and place them one upon the other. Are all of them congruent? You also try this.

Example 4 : Two triangles ΔCAB and ΔRPQ are given below. Check whether the two are congruent? If they are congruent, what can you say about the measures of the remaining elements of the triangles.



Solution : In ΔCAB and ΔRPQ,

$BC = QR = 4$ cm

$\angle B = \angle Q = 120°$

$AB = PQ = 3$ cm

Thus, two sides and included angle of ΔCAB are equal to the corresponding sides and included angle of ΔRPQ.

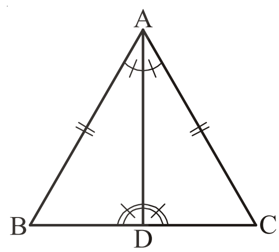
Therefore, by Side-Angle-side criterion of congruency $\Delta CAB \cong \Delta RPQ$

Thus, in the two triangles

$AC \cong PR$

$\angle C \cong \angle R$ and $\angle A \cong \angle P$

Example-5 : In the following picture, the equal angles in the two triangles are shown. Are the triangles congruent?



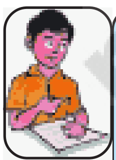
Solution : In ΔABD and ΔACD

$\angle BAD \cong \angle CAD$ (given in the question)

$\angle ADB \cong \angle ADC$ (given in the question)

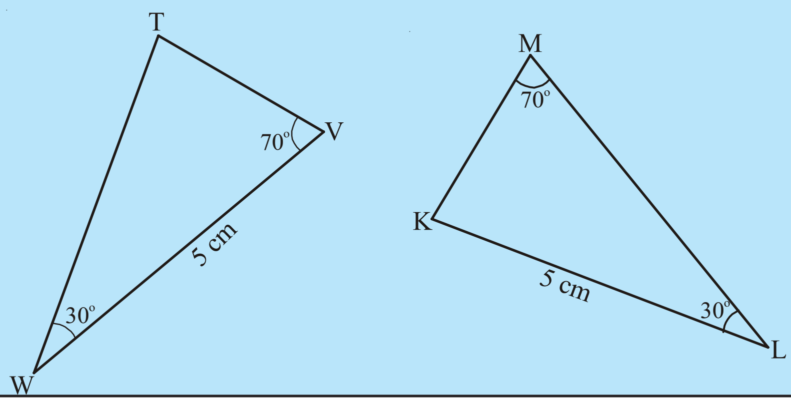
$AD \cong AD$ (common side, seen in the figure)

Thus, by Angle-side-Angle congruence criterion $\Delta ABD \cong \Delta ACD$



Try This

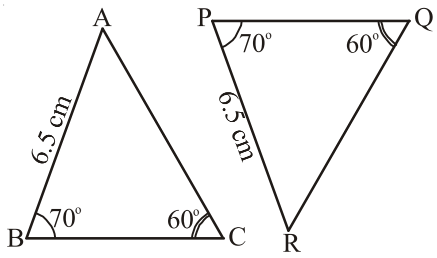
Is the following pair of triangles congruent? Give reason to support your answer.



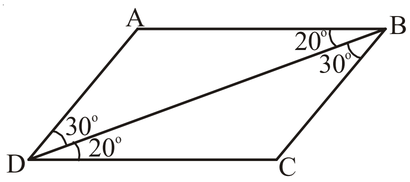
Exerciese - 3

1. In following pairs of triangles, find the pairs which are congruent? Also, write the criterion of congruence.

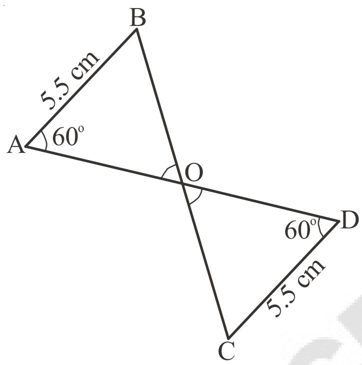
(i)



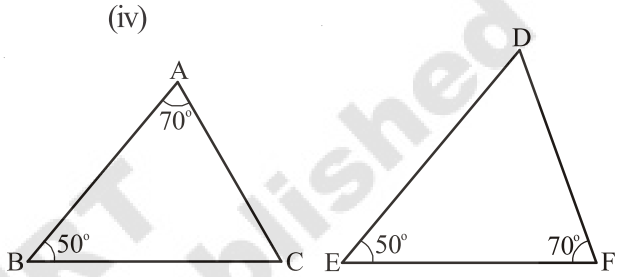
(ii)



(iii)



(iv)

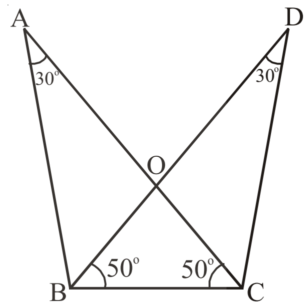


2. In the adjacent figure.

(i) Are ΔABC and ΔDCB congruent?

(ii) Are ΔAOB congruent to ΔDOC?

Also identify the relation between corresponding elements and give reason for your answer.



8.3.4 Right-Angle Hypotenuse Side congruence

In right-angled triangles we already know that one of the angles is a right angle. So what else do we need to prove that the two triangles are congruent?

Let us take the example of ΔABC with $\angle B = 90$. Can we draw a triangle congruent to this triangle, if,

(i) only BC is known

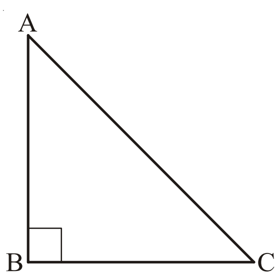
(ii) only $\angle C$ is known

(iii) $\angle A$ and $\angle C$ are known

(iv) AB and BC are known

(v) $\angle C$ and BC are known

(vi) BC and the hypotenuse AC are known



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When you try to draw the rough sketches of these triangles, you will find it is possible only in cases (iv), (v) and (vi).

The last of the situations is new to us and it is called the Right-Angle Hypotenuse Congruence Criterion.

Right-Angle Hypotenuse Congruence Criterion: If the hypotenuse and one side of a right angled triangle are equal to the corresponding hypotenuse and side of the other right angled triangle, then the triangles are congruent.

Example 6 : Given below are measurements of some parts of two triangles. Examine whether the two triangles are congruent or not, using RHS congruence rule. In case of congruent triangles, write the result in symbolic form :

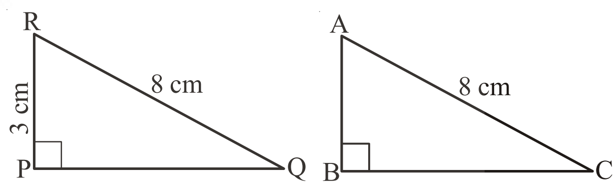
ΔABC	ΔPQR
(i) $\angle B = 90^o$, AC = 8 cm, AB = 3 cm	$\angle P = 90^o$, PR = 3 cm, QR = 8 cm
(ii) $\angle A = 90^o$, AC = 5 cm, BC= 9 cm	$\angle Q = 90^o$, PR = 8 cm, PQ = 5 cm

Solution :

(i) Here, $\angle B = \angle P = 90^o$

hypotenuse, AC = hypotenuse, RQ (= 8 cm) and

side AB = side RP (= 3 cm)



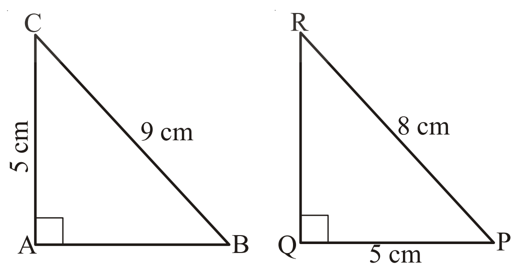
So, $\Delta ABC \cong \Delta RPQ$ (By RHS Congruence rule).

(ii) Here, $\angle A = \angle Q = 90^o$ and

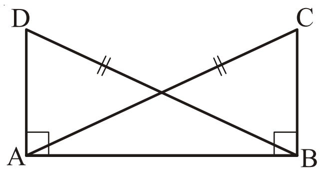
side AC = side PQ (= 5 cm).

hypotenuse, BC $\neq$ hypotenuse, PR

So, the triangles are not congruent.



Example 7 : In the adjacent figure, $\overline{DA} \perp \overline{AB}$, $\overline{CB} \perp \overline{AB}$ and AC = BD.



State the three pairs of equal parts in ΔABC and ΔDAB.

Which of the following statements is meaningful?

(i) $\Delta ABC \cong \Delta BAD$ (ii) $\Delta ABC \cong \Delta ABD$

Solution : The three pairs of equal parts are :

$\angle ABC = \angle BAD$ ($= 90^{\circ}$)

AC = BD (Given)

AB = BA (Common side)

$\Delta ABC \cong \Delta BAD$ (By RHS congruence rule).

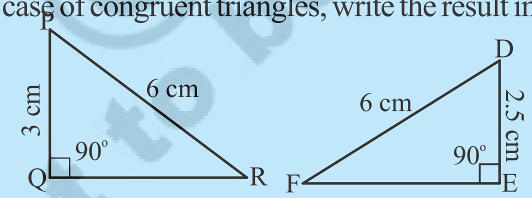
From the above,

statement (i) is true;

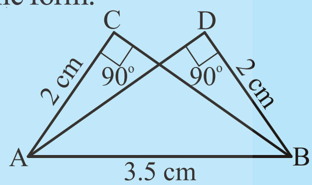
statement (ii) is not meaningful, in the sense that the correspondence among the vertices is not staisfied.

Try This

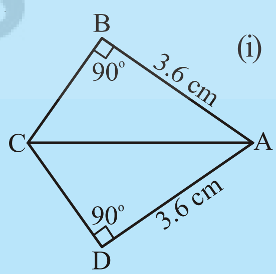
1. In the figures given below, measures of some parts of triangles are given. By applying RHS congruence rule, state which pairs of triangles are congruent. In case of congruent triangles, write the result in symbolic form.



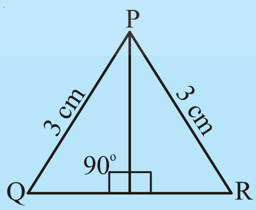
(i)



(ii)



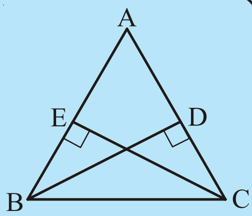
(iii)



(iv)

2. It is to be established by RHS congruence rule that $\Delta ABC \cong \Delta RPQ$. What additional information is needed, if it is given that $\angle B = \angle P = 90^{\circ}$ and AB = RP?

3. In the adjacent figure, $\overline{BD}$ and $\overline{CE}$ are altitudes of ΔABC such that $BD = CE$.

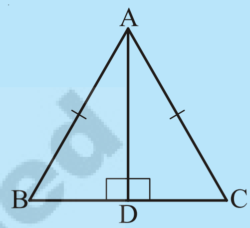


(i) State the three pairs of equal parts in ΔCBD and ΔBCE.

(ii) Is $\Delta CBD \cong \Delta BCE$? Why or why not?

(iii) Is $\angle DBC = \angle EBC$? Why or why not?

4. ABC is an isosceles triangle with $\overline{AB} = \overline{AC}$ and $\overline{AD}$ is one of its altitudes (fig ...).



(i) State the three pairs of equal parts in ΔADB and ΔADC.

(ii) Is $\Delta ADB \cong \Delta ADC$? Why or why not?

(iii) Is $\angle B \cong \angle C$? Why or why not?

(iv) Is $BD \cong CD$? Why or why not?

Exercise - 4

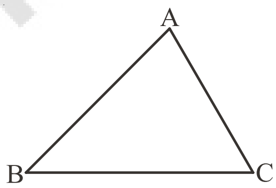
1. Which congruence criterion do you use in the following?

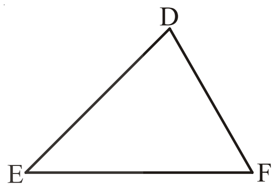
(i) Given : $AC = DF$

$AB = DE$

$BC = EF$

So, $\Delta ABC \cong \Delta DEF$



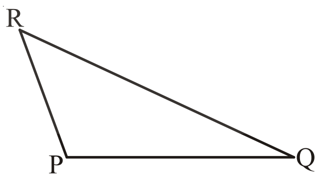


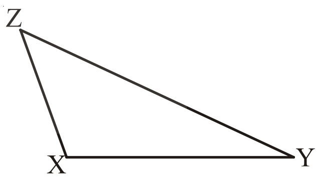
(ii) Given : $ZX = RP$

$RQ = ZY$

$\angle PRQ \cong \angle XZY$

So, $\Delta PQR \cong \Delta XYZ$



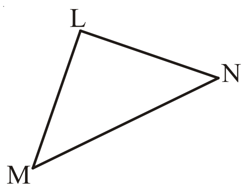


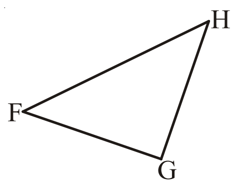
(iii) Given : $\angle MLN \cong \angle FGH$

$\angle NML \cong \angle GFH$

$ML = FG$

So, $\Delta LMN \cong \Delta GFH$





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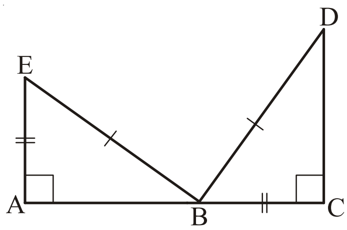
(iv) Given : $EB = DB$

$AE = BC$

$\angle A = \angle C = 90^{\circ}$

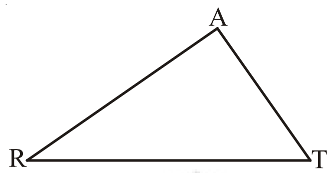
So, $\Delta ABE \cong \Delta CDB$



2. You want to show that $\Delta ART \cong \Delta PEN$,

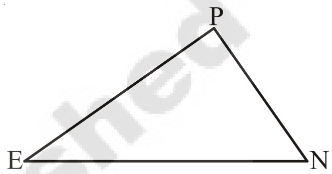
(i) If you have to use SSS criterion, then you need to show

(a) AR = (b) RT = (c) AT =



(ii) If it is given that $\angle T = \angle N$ and you are to use SAS criterion, you need to have

(a) RT = and (ii) PN =



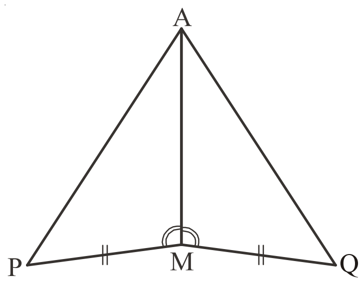
(iii) If it is given that AT = PN and you are to use ASA criterion, you need to have

(a) ? (b) ?

3. You have to show that $\Delta AMP \cong \Delta AMQ$.

In the following proof, supply the missing reasons.

Steps	Reasons
(i) $PM = QM$	(i) ....
(ii) $\angle PMA \cong \angle QMA$	(ii) ....
(iii) $AM = AM$	(iii) ....
(iv) $\Delta AMP \cong \Delta AMQ$	(iv) ....

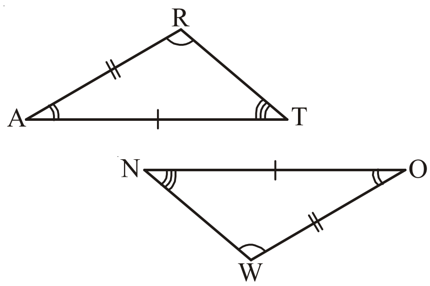


4. In ΔABC, $\angle A = 30^{\circ}$, $\angle B = 40^{\circ}$ and $\angle C = 110^{\circ}$

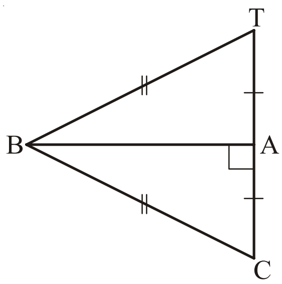
In ΔPQR, $\angle P = 30^{\circ}$, $\angle Q = 40^{\circ}$ and $\angle R = 110^{\circ}$

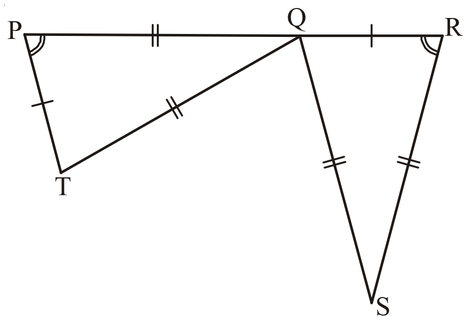
A student says that $\Delta ABC \cong \Delta PQR$ by AAA congruence criterion. Is he justified? Why or why not?

5. In the figure, the two triangles are congruent. The corresponding parts are marked. We can write $\Delta RAT \cong$?



6. Complete the congruence statement.





$\Delta ABC \cong$?

$\Delta QRS \cong$?

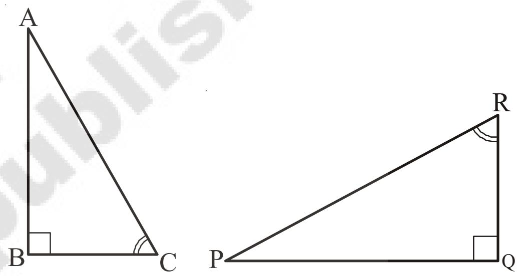
7. In a squared sheet, draw two triangles of equal areas such that

(i) the triangles are congruent.

(ii) the triangles are not congruent.

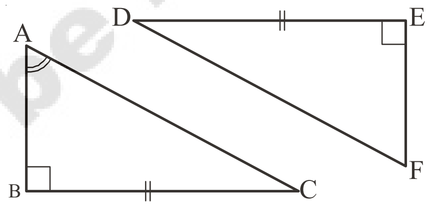
What can you say about their perimeters?

8. If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



9. Explain, why

$\Delta ABC \cong \Delta FED$.



Looking Back

1. Congruent objects are objects having the same shape and size.
2. The method of superimposition examines the congruence of plane figures.
3. Two line segments say, AB and CD are congruent if they have equal lengths. We write this as $AB \cong CD$. However, it is common to write it as $AB = CD$.
4. If all the parts of one triangle are equal to the corresponding parts of other triangle, then the triangles are congruent.

5. The necessary and sufficient conditions for two triangles to be congruent are as follows:

(i) Side-Side-Side (SSS) criterion for congruence: If three sides of a triangle are equal to the corresponding three sides of another triangle, then the triangles are congruent.

(ii) Side-Angle-Side(SAS) criterion for congruence: If two sides and the angle included between the two sides of a triangle are equal to the corresponding two sides and the included angle of another triangle, then the triangles are congruent.

(iii) Angle-Side-Angle criterion of congruence: If two angles and the included side of a triangle are equal to the corresponding two angles and included side of another triangle then the triangles are congruent.

(iv) Right-Angle Hypotenuse criterion of congruence: If the hypotenuse and one side of a right-angled triangle are equal to the corresponding hypotenuse and side of the other right-angled triangle, then the triangles are congruent.