CONSTRUCTION OF TRIANGLES 9

9.0 Introduction

You will learn how to construct triangles in this chapter. You do not need all the six elements i.e. the three angles and three sides of a triangle to construct a triangle. A triangle can be drawn if you know the elements that are required for two triangles to be congurent. Thus, a triangle can be drawn in any of the situations given below i.e., if we know the-

(i) Three sides of the triangle.

(ii) Two sides and the angle included between them.

(iii) Two angles and the side included between them

(iv) Hypotenuse and one adjacent side of a right-angled triangle.

A triangle can also be drawn if two of its sides and a non-included angle are given. However, it is important to remember that this condition is not suffiecient to make two triangles, congruent.

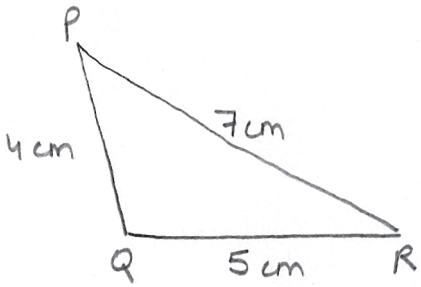
Let us learn to construct triangles in each of the above cases.

9.1 Construction of a triangle when measurements of the three sides are given

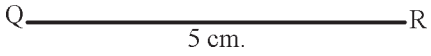
In the construction of any geometrical figure, drawing a rough sketch first, helps in indentifying the sides. So we should first draw a rough sketch of the triangle we want to construct and label it with the given measurements.

Example 1: Construct a Δ PQR with sides PQ = 4 cm, QR = 5 cm and RP = 7 cm.

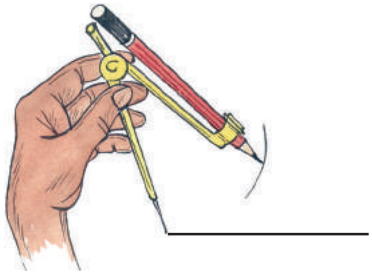
STEP 1 : Draw a rough sketch of the triangle and label it with the given measureents.

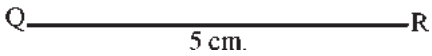


STEP 2 : Draw a line segment QR of length 5 cm.

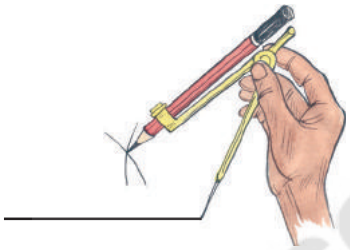


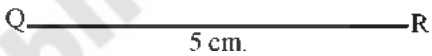
STEP 3 : With centre Q, draw an arc of radius 4 cm.



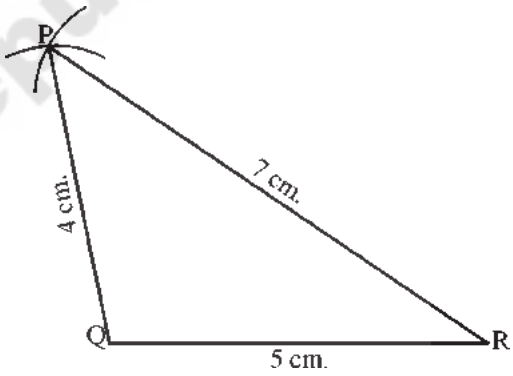


STEP 4 : Since P is at a distance of 7 cm from R, draw another arc from R with radius 7 cm such that it intersects first arc at P.





STEP 5 : Join Q,P and P,R. The required Δ PQR is constructed.



Try This

1. Construct a triangle with the same measurements given in above example, taking PQ as base. Are the triangles congurent?

2. Construct a Δ PET, PE = 4.5 cm, ET = 5.4 cm and TP = 6.5 cm in your notebook. Now construct Δ ABC, AB = 5.4 cm, BC = 4.5 cm and CA = 6.5 cm on a piece of paper. Cut it out and place it on the figure you have constructed in your note book. Are the triangles congruent? Write your answer using mathematical notation.

Exercise - 1

1. Construct Δ ABC in which AB = 5.5 cm, BC = 6.5 cm and CA = 7.5 cm.

2. Construct Δ NIB in which NI = 5.6 cm, IB = 6 cm and BN = 6 cm. What type of triangle is this?

3. Construct an equilateral Δ APE with side 6.5 cm.

4. Construct a Δ XYZ in which XY = 6 cm, YZ = 8 cm and ZX = 10 cm. Using protractor find the angle at X. What type of triangle is this?

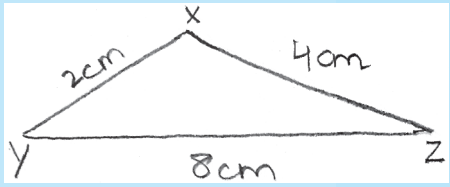
5. Construct Δ ABC in which AB = 4 cm, BC = 7 cm and CA= 3 cm. Which type of triangle is this?

6. Construct Δ PEN with PE = 4 cm, EN = 5 cm and NP = 3 cm. If you draw circles instead of arcs how many points of intersection do you get? How many triangles with given measurements are possible? Is this true in case of every triangle?

Try This

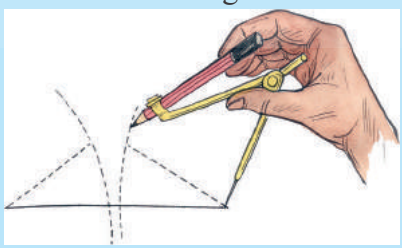
Sushanth prepared a problem: Construct Δ XYZ in which XY = 2 cm, YZ = 8 cm and XZ = 4 cm.

He also drew the rough sketch as shown in Figure 1.



Reading the problem, Srija told Sushanth that it would not be possible to draw a triangle with the given measurements.

However, Sushanth started to draw the diagram as shown in Figure 2.

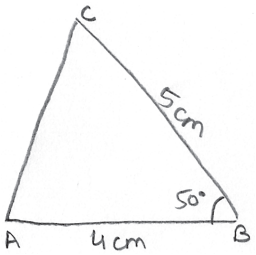


Check whether Sushanth can draw the triangle. If not why? Discuss with your friends. What property of triangles supports Srija's idea?

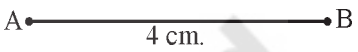
9.2 Construction of a triangle with two given sides and the included angle

Example 2 : Construct Δ ABC in which AB = 4 cm, BC = 5cm and $\angle$ B = 50°.

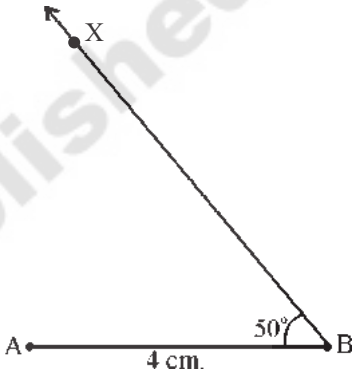
STEP 1 : Draw a rough sketch of a triangle and label it with the given measurements.



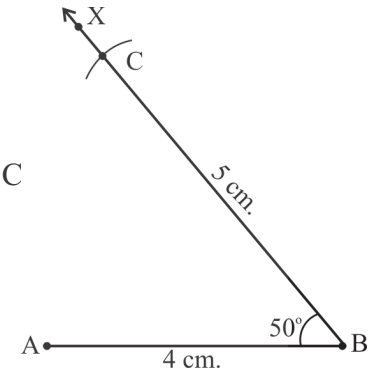
STEP 2 : Draw a line segment AB of length 4 cm.



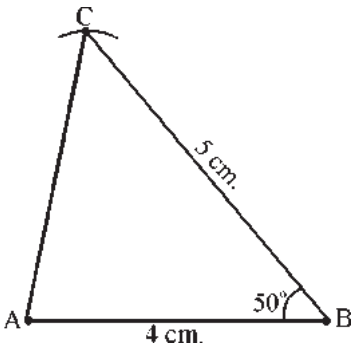
STEP 3 : Draw a ray $\overrightarrow{BX}$ making an angle 50° with AB.



STEP 4 : Draw an arc of radius 5 cm from B, which cuts $\overrightarrow{BX}$ at C



STEP 5 : Join C, A to get the required Δ ABC.



Exercise - 2

1. Draw ΔCAR in which CA = 8 cm, $\angle$A = 60° and AR = 8 cm. Measure CR, $\angle$R and $\angle$C. What kind of triangle is this?

2. Construct ΔABC in which AB = 5 cm, $\angle$B = 45° and BC = 6 cm.

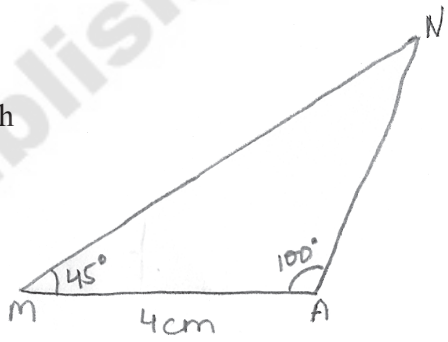
3. Construct ΔPQR such that $\angle$R=100°, QR = RP = 5.4 cm.

4. Construct ΔTEN such that TE = 3 cm, $\angle$E = 90° and NE = 4 cm.

9.3 Construction of a triangle when two angles and the side between the angles is given

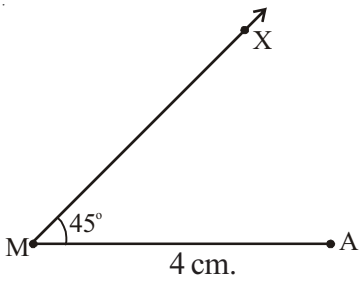
Example 3 : Construct ΔMAN with MA = 4 cm, $\angle$M = 45° and $\angle$A = 100°.

STEP 1 : Draw rough sketch of a triangle and label it with the given measurements.



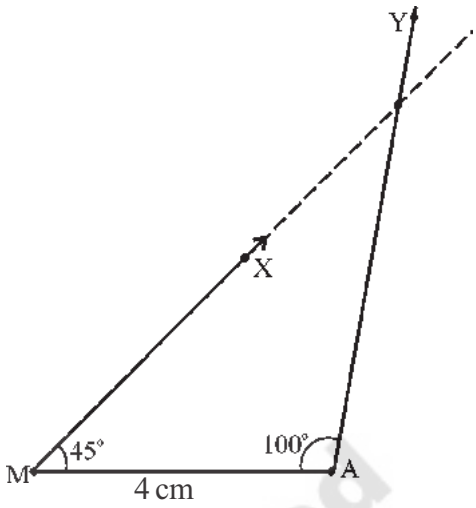
STEP 2 : Draw line segment MA of length 4 cm.

STEP 3 : Draw a ray $\overrightarrow{MX}$, making an angle 45° at M.

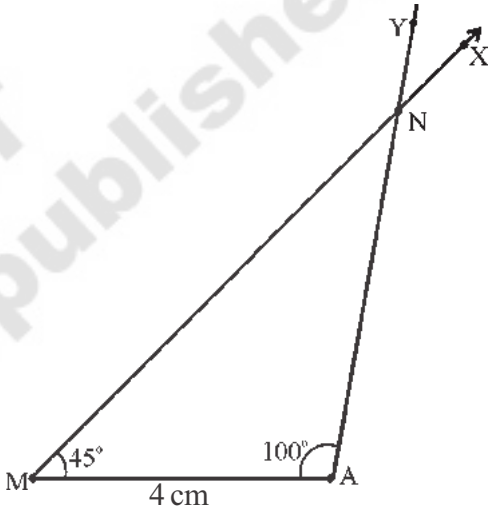


STEP 4 : Draw a ray $\overrightarrow{AY}$, making an angle 100° at A.

Extend the ray $\overrightarrow{MX}$ if necessary to intersect ray $\overrightarrow{AY}$.



STEP 5 : Mark the intersecting point of the two rays as N. You have the required Δ MAN



Try This

Construct a triangle with angles 105° and 95° and a side of length of your choice. Could you construct the triangle? Discuss and justify.

Exercise - 3

1. Construct Δ NET with measurement NE = 6.4 cm, ∠N = 50° and ∠E = 100°.

2. Construct Δ PQR such that QR = 6 cm, ∠Q = ∠R = 60°. Measure the other two sides of the triangle and name the triangle.

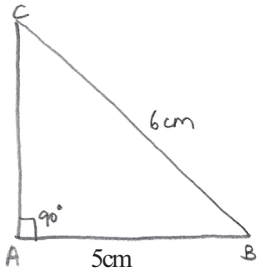
3. Construct Δ RUN in which RN = 5cm, ∠R = ∠N = 45°. Measure the other angle and other sides. Name the triangle.

9.4 Construction of right-angled triangle when the hypotenuse and a side are given.

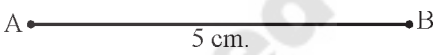
Example 4 : Construct Δ ABC, right-angled at A, and BC = 6 cm; AB = 5 cm.

STEP 1 : Draw a rough sketch of right-angled triangle and label it with given information.

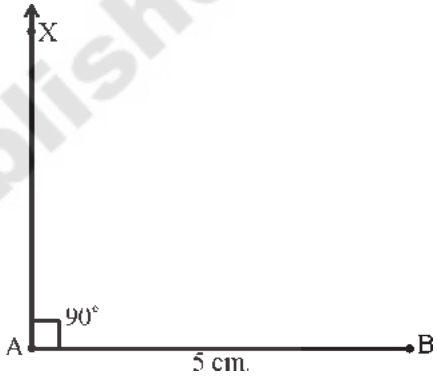
Note: side opposite to the right angle is called hypotenuse.



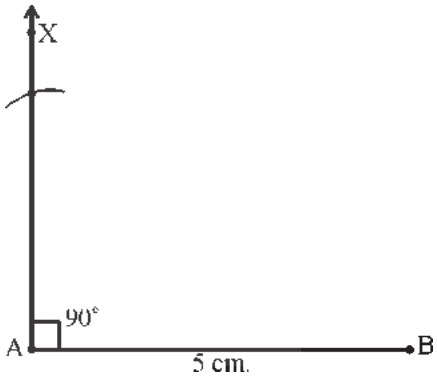
STEP 2 : Draw a line segment AB of length 5 cm.



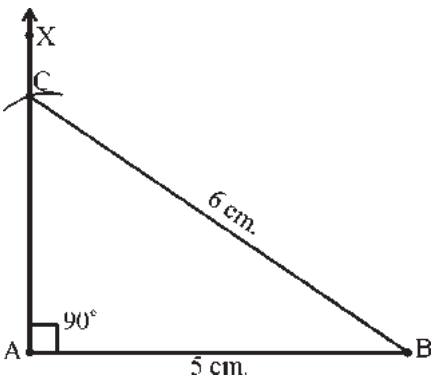
STEP 3 : Construct a ray $\overrightarrow{AX}$ perpendicular to AB at A.



STEP 4 : Draw an arc from B with radius 6cm to intersect $\overrightarrow{AX}$ at 'C'.



STEP 5 : Join B,C to get the required Δ ABC.



Exercise - 4

1. Construct a right-angled ΔABC such that $\angle B = 90°$, $AB = 8$ cm and $AC = 10$ cm.

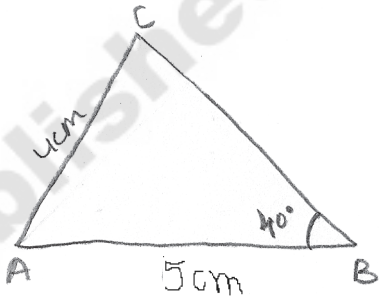
2. Construct a ΔPQR, right-angled at R, hypotenuse is 5 cm and one of its adjacent sides is 4 cm.

3. Construct an isosceles right-angled ΔXYZ in which $\angle Y = 90°$ and the two sides are 5 cm each.

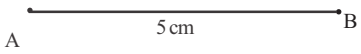
9.5 Construction of triangle when two sides and the non-included angle are given

Example 5 : Construct ΔABC such that $AB = 5$ cm, $AC = 4$ cm, $\angle B = 40°$.

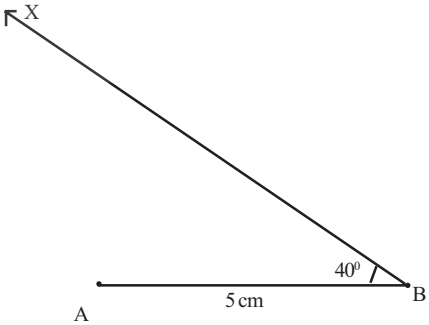
STEP 1 : Draw rough sketch of ΔABC and label it with the given measurements.



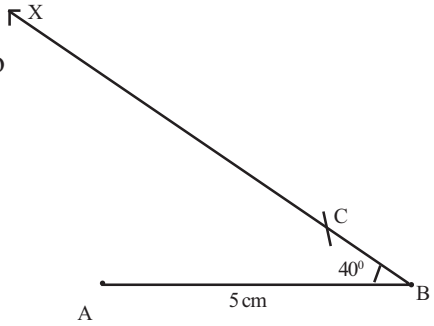
STEP 2 : Draw a line segment AB of length 5 cm.



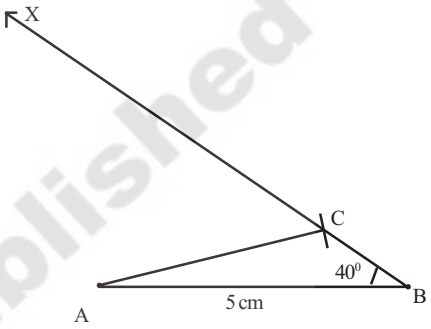
STEP 3 : Draw a ray $\overrightarrow{BX}$ making an angle 40° at B.



STEP 4 : With A as centre and radius 4 cm, draw an arc to cut ray $\overrightarrow{BX}$.

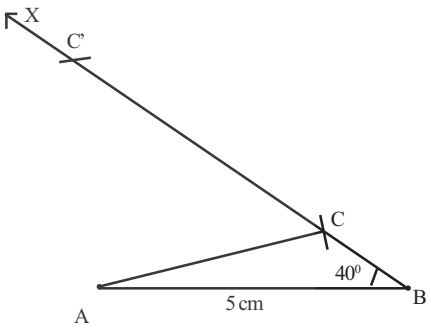


STEP 5 : Mark the intersecting point as C and join C, A to get the required Δ ABC.

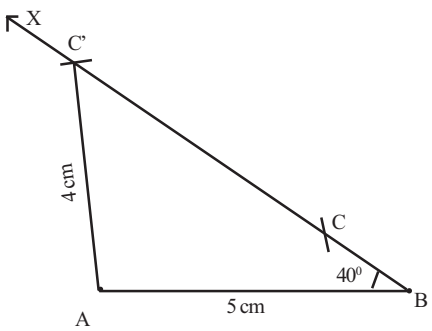


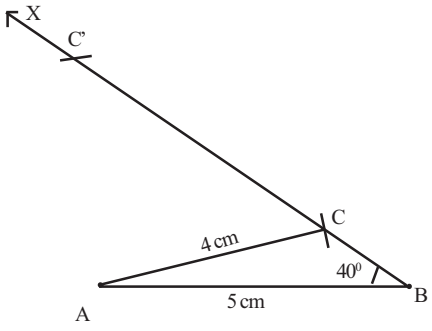
Can you cut the ray $\overrightarrow{BX}$ at any other point?

You will see that as $\angle$ B is acute, the arc from A of radius 4 cm cuts the ray $\overrightarrow{BX}$ twice.



So we may have two triangles as given below:





Try This

Construct a triangle with two sides of length of your choice and the non-included angle as an obtuse angle. Can you draw two triangles in this solution?

Exercise - 5

1. Construct ΔABC in which $AB = 4.5$ cm, $AC = 4.5$ cm and $\angle B = 50°$. Check whether you get two triangles.

2. Construct ΔXYZ such that $XY = 4.5$ cm, $XZ = 3.5$ cm and $\angle Y = 70°$. Check whether you get two triangles.

3. Construct ΔANR with the sides AN and AR of lengths 5 cm and 6 cm respectively and $\angle N$ is 100°. Check whether you get two triangles.

4. Construct ΔPQR in which $QR = 5.5$ cm, $QP = 5.5$ cm and $\angle Q = 60°$. Measure RP. What kind of triangle is this?

5. Construct the triangles with the measurement given in the following table.

Triangle	Measurements
ΔABC	$BC = 6.5$ cm, $CA = 6.3$ cm, $AB = 4.8$ cm.
ΔPQR	$PQ = 8$ cm, $QR = 7.5$ cm, $\angle PQR = 85°$
ΔXYZ	$XY = 6.2$ cm, $\angle Y = 130°$, $\angle Z = 70°$
ΔABC	$AB = 4.8$ cm, $AC = 4.8$ cm, $\angle B = 35°$
ΔMNP	$\angle N = 90°$, $MP = 11.4$ cm., $MN = 7.3$ cm.
ΔRKS	$RK = KS = SR = 6.6$ cm.
ΔPTR	$\angle P = 65°$, $PT = PR = 5.7$ cm.

Looking Back

A triangle can be constructed when.

(i) The three sides of the triangle are given.

(ii) Two sides and the angle included between them is given.

(iii) Two angles and their included side is given.

(iv) The hypotenuse and one adjacent side of a right angle triangle are given.

(v) Two sides and the not included angle are given.