2. Verify the identity $(a-b)^2 \equiv a^2 - 2ab + b^2$ geometrically by taking

(i) $a = 3$ units, $b = 1$ unit

(ii) $a = 5$ units, $b = 2$ units

3. Verify the identity $(a+b)(a-b) \equiv a^2 - b^2$ geometrically by taking

(i) $a = 3$ units, $b = 2$ units

(ii) $a = 2$ units, $b = 1$ unit

What we have discussed

1. There are number of situations in which we need to multiply algebraic expressions.
2. A monomial multiplied by a monomial always gives a monomial.
3. While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.
4. In carrying out the multiplication of an algebraic expression with another algebraic expression (monomial / binomial / trianomial etc.) we multiply term by term i.e. every term of the expression is multiplied by every term in the another expression.
5. An **identity** is an equation, which is true for all values of the variables in the equation. On the other hand, an equation is true only for certain values of its variables. An equation is not an identity.
6. The following are identities:

    I. $(a+b)^2 \equiv a^2 + 2ab + b^2$

    II. $(a-b)^2 \equiv a^2 - 2ab + b^2$

    III. $(a+b)(a-b) \equiv a^2 - b^2$

    IV. $(x+a)(x+b) \equiv x^2 + (a+b)x + ab$
7. The above four identities are useful in carrying out squares and products of algebraic expressions. They also allow easy alternative methods to calculate products of numbers and so on.

Chapter **12**

Factorisation

12.0 Introduction

Let us consider the number 42 . Try to write the '42' as product of any two numbers.

$$\begin{aligned} 42 &= 1 \times 42 \\ &= 2 \times 21 \\ &= 3 \times 14 \\ &= 6 \times 7 \end{aligned}$$

Thus 1, 2, 3, 6, 7, 14, 21 and 42 are the factors of 42. Among the above factors, which are prime numbers?

Do you write 42 as product of prime numbers? Try.

Rafi did like this	Sirisha did like this	Akbar did like this
$42 = 2 \times 21$ $= 2 \times 3 \times 7$	$42 = 3 \times 14$ $= 2 \times 3 \times 7$	$42 = 6 \times 7$ $= 2 \times 3 \times 7$

What have you observe? We observe that $2 \times 3 \times 7$ is the product of prime factors in every case.

Now consider another number say '70'

The factors of 70 are 1, 2, 5, 7, 10, 14, 35 and 70

70 can be written as $2 \times 5 \times 7$ as the product of prime factors.

The form of factorisation where all factors are primes is called product of **prime factor form**.

$$\begin{aligned} 70 &= 1 \times 70 \\ &= 2 \times 35 \\ &= 5 \times 14 \\ &= 7 \times 10 \end{aligned}$$

Do This:

Express the given numbers in the form of product of primes

(i) 48 (ii) 72 (ii) 96

As we did for numbers we can also express algebraic expressions as the product of their factors. We shall learn about factorisation of various algebraic expressions in this chapter.

12.1 Factors of algebraic expressions:

Consider the following example :

$7yz \quad = 7(yz) \qquad$ (7 and yz are the factors)

$\quad\;\; = 7y(z) \qquad$ ($7y$ and z are the factors)

$\quad\;\; = 7z(y) \qquad$ ($7z$ and y are the factors)

$\quad\;\; = 7 \times y \times z \qquad$ ($7, y$ and z are the factors)

Among the above factors $7, y, z$ are irreducible factors. The phrase '*irreducible*' is used in the place of '*prime*' in algebraic expressions. Thus we say that $7\times y\times z$ is the irreducible form of $7yz$. Note that $7\times(yz)$ or $7y(z)$ or $7z(y)$ are not an irreducible form.

> '1' is the factor of $7yz$, since $7yz = 1\times 7\times y\times z$. In fact '1' is the factor of every term. But unless required, '1' need not be shown separately.

Let us now consider the expression $7y(z+3)$. It can be written as $7y\,(z + 3) = 7 \times y \times (z+3)$. Here $7, y, (z + 3)$ are the irreducible factors.

Similarly $5x\,(y+2)\,(z+3) = 5 \times x \times (y + 2) \times (z + 3)$ Here 5 , x , $(y + 2)$, $(z + 3)$ are irreducible factors.

Do This

1. Find the factors of following :

   (i) $8x^2yz$ (ii) $2xy\,(x+y)$ (iii) $3x+y^3z$

12.2 Need of factorisation:

When an algebraic expression is factorised, it is written as the product of its factors. These factors may be numerals, algebraic variables, or terms of algebraic expressions.

Consider the algebraic expression $23a+23b+23c$. This can be written as $23(a+b+c)$, here the irreducible factors are 23 and $(a+b+c)$. 23 is a numerical factor and $(a+b+c)$ is algebraic factor.

Consider the algebraic expressions (i) x^2y+y^2x+xy (ii) $(4x^2-1) \div (2x-1)$.

The first expression $x^2y+y^2x+xy = xy(x+y+1)$ thus the above algebraic expression is written in simpler form.

The second case $(4x^2 - 1) \div (2x - 1)$

$$\frac{4x^2-1}{2x-1} = \frac{(2x)^2-(1)^2}{2x-1}$$

$$= \frac{(2x+1)(2x-1)}{(2x-1)}$$

$$= (2x + 1)$$

From above illustrations it is noticed that the factorisation has helped to write the algebraic expression in simpler form and it also helps in simplifying the algebraic expression

Let us now discuss some methods of factorisation of algebraic expressions.

12.3 Method of common factors:

Let us factorise $3x + 12$

On writing each term as the product of irreducible factors we get :

$3x + 12 = (3 \times x) + (2 \times 2 \times 3)$

What is the common factors of both terms ?

By taking the common factor 3, we get

$3 \times [x + (2\times2)] = 3 \times (x + 4) = 3\ (x + 4)$

Thus the expression $3x + 12$ is the same as $3\ (x + 4)$.

Now we say that 3 *and* $(x + 4)$ are the factors of $3x + 12$ .Also note that these factors are irreducible.

Now let us factorise another expression $6ab+12b$

$6ab+12b = (\underline{2 \times 3} \times a \times \underline{b}) + (2 \times \underline{2 \times 3 \times b})$

$= \underline{2 \times 3 \times b} \times (a + 2) \quad = 6b\ (a + 2)$

Note that $6b$ is the HCF of $6ab$ and $12b$

$\therefore\ 6ab + 12b = 6b\ (a + 2)$

Example 1: Factorize (i) $6xy + 9y^2$ (ii) $25\,a^2b + 35ab^2$

Solution: (i) $6xy + 9y^2$

We have $6\,x\,y = 2 \times \underline{3} \times x \times \underline{y}$ and $9y^2 = 3 \times \underline{3 \times y} \times y$

3 and 'y' are the common factors of the two terms

Hence, $6xy + 9y^2$

$$= (2 \times \underline{3} \times x \times \underline{y}) + (3 \times \underline{3 \times y} \times y) \quad \textit{(Combining the terms)}$$

$$= \underline{3 \times y} \times [(2 \times x) + (3 \times y)]$$

$$\therefore\ 6xy + 9y^2 = 3y(2x + 3y)$$

(ii) $25a^2b + 35ab^2 = (5 \times \underline{5} \times a \times \underline{a \times b}) + (\underline{5} \times 7 \times \underline{a \times b} \times b)$

$$= \underline{5 \times a \times b} \times [(5 \times a) + (7 \times b)]$$

$$= 5ab(5a + 7b)$$

$$\therefore\ 25a^2b + 35ab^2 = 5ab(5a + 7b)$$

Example 2: Factorise $3x^2 + 6x^2y + 9xy^2$

$$3x^2 + 6x^2y + 9xy^2 = (\underline{3 \times x} \times x) + (2 \times \underline{3 \times x} \times x \times y) + (3 \times \underline{3 \times x} \times y \times y)$$

$$= \underline{3 \times x}\,[x + (2 \times x \times y) + (3 \times y \times y)]$$

$$= 3x(x + 2xy + 3y^2) \quad \text{(taking } 3 \times x \text{ as common factor)}$$

$$\therefore\ 3x^2 + 6x^2y + 9xy^2 = 3x(x + 2xy + 3y^2)$$

Do This

Factorise (i) $9a^2 - 6a$ (ii) $15a^3b - 35ab^3$ (iii) $7lm - 21lmn$

12.4 Factorisation by grouping the terms

Observe the expression $\boldsymbol{ax + bx + ay + by}$. You will find that there is no single common factor to all the terms. But the first two terms have the common factor 'x' and the last two terms have the common factor 'y'. Let us see how we can factorise such an expression.

On grouping the terms we get $(ax + bx) + (ay + by)$

$(ax + bx) + (ay + by) = x(a + b) + y(a + b)$ (By taking out common factors from each group)

$= (a + b)(x + y)$ (By taking out common factors from the groups)

The expression $\boldsymbol{ax + bx + ay + by}$ is now expressed as the product of its factors .The factors are $(a+b)$ and $(x + y)$, which are irreducible.

The above expression can be factorised by another way of grouping, as follows :

$$ax + ay + bx + by = (ax + ay) + (bx + by)$$

$$= a(x + y) + b(x + y)$$

$$= (x + y)(a + b)$$

Note that the factors are the same except the order.

Do This

Factorise (i) $5xy + 5x + 4y + 4$ (ii) $3ab + 3b + 2b + 2$

Example 3: Factorise $6ab - b^2 - 2bc + 12ac$

Solution: Step 1: Check whether there are any common factors for all terms. Obviously none.

Step 2: On regrouping the first two terms we have

$6ab - b^2 = b(6a - b)$ ——— I

Note that you need to change order of the last two terms in the expression as $12ac - 2bc$.

Thus $12ac - 2bc = 2c(6a - b)$ ——— II

Step 3: Combining I and II together

$6ab - b^2 - 2bc + 12ac = b(6a - b) + 2c(6a - b)$

$= (6a - b)(b + 2c)$ By taking out common factor $(6a - b)$

Hence the factors of $6ab - b^2 - 2bc + 12ac$ are $(6a - b)$ and $(b + 2c)$

Exercise - 12.1

1. Find the common factors of the given terms in each.

(i) $8x, 24$ (ii) $3a, 21ab$ (iii) $7xy, 35x^2y^3$ (iv) $4m^2, 6m^2, 8m^3$

(v) $15p, 20qr, 25rp$ (vi) $4x^2, 6xy, 8y^2x$ (vii) $12x^2y, 18xy^2$

2. Factorise the following expressions

(i) $5x^2 - 25xy$ (ii) $9a^2 - 6ax$ (iii) $7p^2 + 49pq$

(iv) $36a^2b - 60a^2bc$ (v) $3a^2bc + 6ab^2c + 9abc^2$

(vi) $4p^2 + 5pq - 6pq^2$ (vii) $ut + at^2$

3. Factorise the following :

(i) $3ax - 6xy + 8by - 4bx$ (ii) $x^3 + 2x^2 + 5x + 10$

(iii) $m^2 - mn + 4m - 4n$ (iv) $a^3 - a^2b^2 - ab + b^3$ (v) $p^2q - pr^2 - pq + r^2$

12.5 Factorisation using identities:

We know that $(a+b)^2 \equiv a^2 + 2ab + b^2$

$(a-b)^2 \equiv a^2 - 2ab + b^2$

$(a+b)(a-b) \equiv a^2 - b^2$ are algebraic identities.

We can use these identities for factorisation, if the given expression is in the form of RHS (Right Hand Side) of the particular identity. Let us see some examples.

Example 4: Factorise $x^2 + 10x + 25$

Solution: The given expression contains three terms and the first and third terms are perfect squares. That is x^2 and 25 (5^2). Also the middle term contains the positive sign. This suggests that it can be written in the form of $\boldsymbol{a^2 + 2ab + b^2}$,

so $x^2 + 10x + 25 = (x)^2 + 2\,(x)\,(5) + (5)^2$

We can compare it with $a^2 + 2ab + b^2$ which in turn is equal to the LHS of the identity i.e. $(a+b)^2$. Here $a = x$ and $b = 5$

We have $x^2 + 10x + 25 = (x+5)^2 = (x+5)\,(x+5)$

Example 5: Factorise $16z^2 - 48z + 36$

Solution: Taking common numerical factor from the given expression we get

$16z^2 - 48z + 36 = (4 \times 4z^2) - (4 \times 12z) + (4 \times 9) = 4(4z^2 - 12z + 9)$

Note that $4z^2 = (2z)^2$; $9 = (3)^2$ and $12z = 2\,(2z)\,(3)$

$4z^2 - 12z + 9 = (2z)^2 - 2\,(2z)\,(3) + (3)^2$ since $a^2 - 2ab + b^2 = (a-b)^2$

$= (2z-3)^2$

By comparison , $16z^2 - 48z + 36 = 4(4z^2 - 12z + 9) = 4\,(2z-3)^2$

$= 4(2z-3)(2z-3)$

Example 6: Factorise $25p^2 - 49q^2$

Solution: We notice that the expression is a difference of two perfect squares.

i.e., the expression is of the form $a^2 - b^2$.

Hence Identity $a^2 - b^2 = (a+b)\,(a-b)$ can be applied

$25p^2 - 49q^2 = (5p)^2 - (7q)^2$

$= (5p + 7q)\,(5p - 7q)$ [$\because a^2 - b^2 = (a+b)\,(a-b)$]

Therefore, $25p^2 - 49q^2 = (5p + 7q)\,(5p - 7q)$

Example 7: Factorise $48a^2 - 243b^2$

Solution: We see that the two terms are not perfect squares. But both has '3' as common factor.

That is $48a^2 - 243b^2 = 3\,[16a^2 - 81b^2]$

$= 3\,[(4a)^2 - (9b)^2]$ Again $a^2 - b^2 = (a+b)(a-b)$

$= 3\,[\,(4a + 9b)\,(4a - 9b)]$

$= 3\,(4a + 9b)\,(4a - 9b)$

Example 8: Factorise $x^2 + 2xy + y^2 - 4z^2$

Solution: The first three terms of the expression is in the form $(x+y)^2$ and the fourth term is a perfect square.

Hence $x^2 + 2xy + y^2 - 4z^2 = (x + y)^2 - (2z)^2$

$= [(x + y) + 2z]\,[(x + y) - 2z]$

$= (x + y + 2z)\,(\,x + y - 2z)$

$\boxed{a^2 - b^2 = (a+b)\,(a-b)}$

Example 9: Factorise $p^4 - 256$

Solution: $p^4 = (p^2)^2$ and $256 = (16)^2$

Thus $p^4 - 256 = (p^2)^2 - (16)^2$

$= (p^2 - 16)\,(\,p^2 + 16\,)$

$= (p+4)\,(p-4)\,(\,p^2 + 16\,)$

$\boxed{\because\ p^2 - 16 = (p+4)\,(p-4)}$

12.6 Factors of the form $(x + a)\,(x + b) = x^2 + (a + b)x + ab$

Observe the expressions $x^2 + 12x + 35$, $x^2 + 6x - 27$, $a^2 - 6a + 8$, $3y^2 + 9y + 6$.... etc. These expression can not be factorised by using earlier used identities, as the constant terms are not perfect squares.

Consider $x^2 + 12x + 35$.

All these terms cannot be grouped for factorisation. Let us look for two factors of 35 whose sum is 12 so that it is in the form of identity $x^2 + (a + b)x + ab$

Consider all the possible ways of writing the constant as a product of two factors.

35 =	1×35	$1 + 35 = 36$
	$(-1) \times (-35)$	$-1 - 35 = -36$
	5×7	$5 + 7 = 12$
	$(-5) \times (-7)$	$-5 - 7 = -12$

Sum of which pair is equal to the coefficient of the middle terms ? Obviously it is $5 + 7 = 12$

$\therefore\ x^2 + 12x + 35 \ = \ x^2 + (5+7)\,x + 35$

$= \ x^2 + 5x + 7x + 35$ $\quad (\because\ 12x = 5x + 7x)$

$= \ x\,(x+5) + 7\,(x+5)$ (By taking out common factors)

$= \ (x+5)\,(x+7)$ (By taking out $(x+5)$ as common factor)

From the above discussion we may conclude that the expression which can be written in the form of $x^2 + (a+b)\,x + ab$ can be factorised as $(x+a)\,(x+b)$

Example 10: Factorise $m^2 - 4m - 21$

Solution: Comparing $m^2 - 4m - 21$ with the identity $x^2 + (a+b)\,x + ab$ we note that

$ab = -21$, and $a+b = -4$. So, $(-7) + 3 = -4$ and $(-7)(3) = -21$

Hence $\quad m^2 - 4m - 21 = m^2 - 7m + 3m - 21$

$= m\,(m-7) + 3\,(m-7)$

$= (m-7)\,(m+3)$

Therefore $\quad m^2 - 4m - 21 = (m-7)\,(m+3)$

Factors of –21 and	their sum
$-1 \times 21 = -21$	$-1 + 21 = 20$
$1 \times (-21) = -21$	$1 - 21 = -20$
$\boxed{-7 \times 3 = -21}$	$\boxed{-7 + 3 = -4}$
$-3 \times 7 = -21$	$-3 + 7 = 4$

Example 11: Factorise $4x^2 + 20x - 96$

Solution: We notice that 4 is the common factor of all the terms.

Thus $4x^2 + 20x - 96 = 4\,[x^2 + 5x - 24]$

Now consider $\quad x^2 + 5x - 24$

$= x^2 + 8x - 3x - 24$

$= \ x\,(x+8) - 3\,(x+8)$

$= \ (x+8)\,(x-3)$

Therefore $4x^2 + 20x - 96 = 4\,(x+8)\,(x-3)$

Factors of –24 and	their sum
$-1 \times 24 = -24$	$-1 + 24 = 23$
$1 \times (-24) = -24$	$1 - 24 = -23$
$-8 \times 3 = -24$	$3 - 8 = -5$
$\boxed{-3 \times 8 = -24}$	$\boxed{-3 + 8 = 5}$

Exercise - 12.2

1. Factorise the following expression-

(i) $a^2 + 10a + 25$ (ii) $l^2 - 16l + 64$ (iii) $36x^2 + 96xy + 64y^2$

(iv) $25x^2 + 9y^2 - 30xy$ (v) $25m^2 - 40mn + 16n^2$

(vi) $81x^2 - 198\,xy + 121y^2$ (vii) $(x+y)^2 - 4xy$ *(Hint : first expand* $(x+y)^2$*)*

(viii) $l^4 + 4l^2m^2 + 4m^4$

2. Factorise the following

(i) $x^2 - 36$ (ii) $49x^2 - 25y^2$ (iii) $m^2 - 121$

(iv) $81 - 64x^2$ (v) $x^2y^2 - 64$ (vi) $6x^2 - 54$

(vii) $x^2 - 81$ (viii) $2x - 32x^5$ (ix) $81x^4 - 121x^2$

(x) $(p^2 - 2pq + q^2) - r^2$ (xi) $(x + y)^2 - (x - y)^2$

3. Factorise the expressions-

(i) $lx^2 + mx$ (ii) $7y^2 + 35Z^2$ (iii) $3x^4 + 6x^3y + 9x^2Z$

(iv) $x^2 - ax - bx + ab$ (v) $3ax - 6ay - 8by + 4bx$ (vi) $mn + m + n + 1$

(vii) $6ab - b^2 + 12ac - 2bc$ (viii) $p^2q - pr^2 - pq + r^2$ (ix) $x(y+z) - 5(y+z)$

4. Factorise the following

(i) $x^4 - y^4$ (ii) $a^4 - (b+c)^4$ (iii) $l^2 - (m-n)^2$

(iv) $49x^2 - \frac{16}{25}$ (v) $x^4 - 2x^2y^2 + y^4$ (vi) $4(a+b)^2 - 9(a-b)^2$

5. Factorise the following expressions

(i) $a^2 + 10a + 24$ (ii) $x^2 + 9x + 18$ (iii) $p^2 - 10q + 21$ (iv) $x^2 - 4x - 32$

6. The lengths of the sides of a triangle are integrals, and its area is also integer. One side is 21 and the perimeter is 48. Find the shortest side.

7. Find the values of 'm' for which $x^2 + 3xy + x + my - m$ has two linear factors in x and y, with integer coefficients.

12.7 Division of algebraic expressions

We know that division is the inverse operation of multiplication.

Let us consider $3x \times 5x^3 = 15x^4$

Then $15x^4 \div 5x^3 = 3x$ and $15x^4 \div 3x = 5x^3$

Similarly consider $6a(a+5) = (6a^2 + 30)$

Therefore $(6a^2 + 30) \div 6a = a + 5$

and also $(6a^2 + 30) \div (a+5) = 6a.$

12.8 Division of a monomial by another monomial

Consider $24x^3 \div 3x$

$$\therefore\ 24x^3 \div 3x$$

$$= \frac{2\times2\times2\times3\times x\times x\times x}{3\times x}$$

$$= \frac{(3\times x)(2\times2\times2\times x\times x)}{(3\times x)} = 8x^2$$

Example 12: Do the following Division

(i) $70x^4 \div 14x^2$ (ii) $4x^3y^3z^3 \div 12xyz$

Solution: (i) $70x^4 \div 14x^2$

$$= \frac{2\times5\times7\times x\times x\times x\times x}{2\times7\times x\times x}$$

$$= \frac{5\times x\times x}{1}$$

$$= 5x^2$$

(ii) $4x^3y^3z^3 \div 12xyz$

$$= \frac{4\times x\times x\times x\times y\times y\times y\times z\times z\times z}{12\times x\times y\times z}$$

$$= \frac{1}{3}x^2y^2z^2$$

12.9 Division of an expression by a monomial:

Let us consider the division of the trinomial

$6x^4+10x^3+8x^2$ by a monomial $2x^2$

$$6x^4 + 10x^3 + 8x^2 = [2 \times 3 \times x \times x \times x \times x] + [2 \times 5 \times x \times x \times x] + [2 \times 2 \times 2 \times x \times x]$$

$$= (2x^2)(3x^2) + (2x^2)(5x) + 2x^2(4)$$

$$= 2x^2[3x^2 + 5x + 4]$$

Note that $2x^2$ is common factor

Thus $(6x^4 + 10x^3 + 8x^2) \div 2x^2$

$$= \frac{6x^4+10x^3+8x^2}{2x^2} = \frac{2x^2(3x^2+5x+4)}{2x^2}$$

$$= (3x^2 + 5x + 4)$$

Alternatively each term in the expression could be divided by the monomial (using the cancellation method)

$(6x^4 + 10x^3 + 8x^2) \div 2x^2$

$$= \frac{6x^4}{2x^2}+\frac{10x^3}{2x^2}+\frac{8x^2}{2x^2}$$

$$= 3x^2 + 5x + 4$$

Here we divide each term of the expression in the numerator by the monomial in the denominator

Example 13: Divide $30\,(a^2bc + ab^2c + abc^2)$ by $6abc$

Solution : $30\,(a^2bc + ab^2c + abc^2)$

$= 2 \times 3 \times 5\,[(a \times a \times b \times c) + (a \times b \times b \times c) + (a \times b \times c \times c)]$

$= 2 \times 3 \times 5 \times a \times b \times c\ \ (a + b + c)$

Thus $30\,(a^2bc + ab^2c + abc^2) \div 6abc$

$$= \frac{2 \times 3 \times 5 \times abc(a+b+c)}{2 \times 3 \times abc}$$

$= 5\,(a + b + c)$

Alternatively $30\,(a^2bc + ab^2c + abc^2) \div 6abc$

$$= \frac{30a^2bc}{6abc} + \frac{30ab^2c}{6abc} + \frac{30abc^2}{6abc}$$

$= 5a + 5b + 5c$

$= 5\,(a + b + c)$

12.10 Division of Expression by Expression:

Consider $(3a^2 + 21a) \div (a+7)$

Let us first factorize $3a^2 + 21a$ to check and match factors with the denominator

$$(3a^2 + 21a) \div (a+7) = \frac{3a^2 + 21a}{a+7}$$

$$= \frac{3a(a+7)}{a+7} = 3a$$

$= 3a$

Example 14: Divide $39y^3(50y^2 - 98)$ by $26y^2(5y+7)$

Solution : $39y^3(50y^2 - 98) = 3 \times 13 \times y \times y \times y \times [2\,(25y^2 - 49)]$

$= 2 \times 3 \times 13 \times y \times y \times y \times [(5y)^2 - (7)^2]$ $\boxed{a^2 - b^2 = (a+b)(a-b)}$

$= 2 \times 3 \times 13 \times y \times y \times y \times [(5y + 7)\ (5y - 7)]$

$= 2 \times 3 \times 13 \times y \times y \times y \times (5y + 7)\,(5y - 7)$

Also $26y^2(5y + 7) = 2 \times 13 \times y \times y \times ((5y + 7)$

$\therefore \quad [\,39y^3(50y^2 - 98)\,] \div [\,26y^2(5y + 7)\,]$

$$= \frac{[2 \times 3 \times 13 \times y \times y \times y(5y+7)(5y-7)]}{[2 \times 13 \times y \times y \times (5y+7)]}$$

$= 3y\,(5y - 7)$

Example 15: Divide $m^2 - 14m - 32$ by $m+2$

Solution : We have $m^2 - 14m - 32 = m^2 - 16m + 2m - 32$

$= m(m - 16) + 2(m - 16)$

$= (m - 16)(m+2)$

$(m^2 - 14m - 32) \div m+2 = (m - 16)(m+2) \div (m+2)$

$= (m - 16)$

Example 16: Divide $42(a^4 - 13a^3 + 36a^2)$ by $7a(a - 4)$

Solution : $42(a^4 - 13a^3 + 36a^2) = 2 \times 3 \times 7 \times a \times a \times (a^2 - 13a + 36)$

$= 2 \times 3 \times 7 \times a \times a \times (a^2 - 9a - 4a + 36)$

$= 2 \times 3 \times 7 \times a \times a \times [a(a - 9) - 4(a - 9)]$

$= 2 \times 3 \times 7 \times a \times a \times [(a - 9)(a - 4)]$

$= 2 \times 3 \times 7 \times a \times a \times (a - 9)(a - 4)$

$42(a^4 - 13a^3 + 36a^2) \div 7a(a - 4) = 2 \times 3 \times 7 \times a \times a \times (a - 9)(a - 4) \div 7a(a - 4)$

$= 6a(a - 9)$

Example 17: Divide $x(3x^2 - 108)$ by $3x(x - 6)$

Solution : $x(3x^2 - 108) = x \times [\,3(x^2 - 36)]$

$= x \times [\,3(x^2 - 6^2)]$

$= x \times [3(x + 6)(x - 6)]$

$= 3 \times x \times [\,(x + 6)(x - 6)]$

$x(3x^2 - 108) \div 3x(x - 6) = 3 \times x \times [\,(x + 6)(x - 6)] \div 3x(x - 6)$

$= (x + 6)$

Exercise - 12.3

1. Carry out the following divisions

   (i) $48a^3$ by $6a$ (ii) $14x^3$ by $42x^2$

   (iii) $72a^3b^4c^5$ by $8ab^2c^3$ (iv) $11xy^2z^3$ by $55xyz$ (v) $-54l^4m^3n^2$ by $9l^2m^2n^2$

2. Divide the given polynomial by the given monomial

   (i) $(3x^2 - 2x) \div x$ (ii) $(5a^3b - 7ab^3) \div ab$

   (iii) $(25x^5 - 15x^4) \div 5x^3$ (iv) $(4l^5 - 6l^4 + 8l^3) \div 2l^2$

   (v) $15\,(a^3b^2c^2 - a^2b^3c^2 + a^2b^2c^3) \div 3abc$ (vi) $(3p^3 - 9p^2q - 6pq^2) \div (-3p)$

   (vii) $(\frac{2}{3}a^2b^2c^2 + \frac{4}{3}ab^2c^2) \div \frac{1}{2}abc$

3. Workout the following divisions :

   (i) $(49x - 63) \div 7$ (ii) $12x\,(8x - 20) \div 4(2x - 5)$

   (iii) $11a^3b^3(7c - 35) \div 3a^2b^2(c - 5)$

   (iv) $54lmn\,(l + m)\,(m + n)\,(n + l) \div 81mn\,(l + m)\,(n + l)$

   (v) $36\,(x + 4)\,(x^2 + 7x + 10) \div 9\,(x + 4)$ (vi) $a\,(a + 1)\,(a + 2)\,(a + 3) \div a\,(a + 3)$

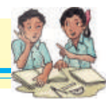
4. Factorize the expressions and divide them as directed :

   (i) $(x^2 + 7x + 12) \div (x + 3)$ (ii) $(x^2 - 8x + 12) \div (x - 6)$

   (iii) $(p^2 + 5p + 4) \div (p + 1)$ (iv) $15ab\,(a^2 - 7a + 10) \div 3b\,(a - 2)$

   (v) $15lm\,(2p^2 - 2q^2) \div 3l\,(p + q)$ (vi) $26z^3(32z^2 - 18) \div 13z^2(4z - 3)$

Think Discuss and Write



While solving some problems containing algebraic expressions in different operations, some students solved as given below. Can you identity the errors made by them? Write correct answers.

1. Srilekha solved the given equation as shown below-

$3x + 4x + x + 2x = 90$

$9x = 90$ Therefore $x = 10$

What could say about the correctness of the solution?

Can you identify where Srilekha has gone wrong?

2. Abraham did the following

For $x = -4,\ 7x = 7 - 4 = -3$

3. John and Reshma have done the multiplication of an algebraic expression by the following methods : verify whose multiplication is correct.

John	**Reshma**
(i) $3(x-4) = 3x - 4$	$3(x-4) = 3x - 12$
(ii) $(2x)^2 = 2x^2$	$(2x)^2 = 4x^2$
(iii) $(2a-3)(a+2) = 2a^2 - 6$	$(2a-3)(a+2) = 2a^2 + a - 6$
(iv) $(x+8)^2 = x^2 - 64$	$(x+8)^2 = x^2 + 16x + 64$

4. Harmeet does the division as $(a+5) \div 5 = a+1$

His friend Srikar done the same $(a+5) \div 5 = a/5 + 1$

and his friend Rosy did it this way $(a+5) \div 5 = a$

Can you guess who has done it correctly? Justify!

Exercise - 12.4

Find the errors and correct the following mathematical sentences

(i) $3(x-9) = 3x - 9$

(ii) $x(3x+2) = 3x^2 + 2$

(iii) $2x + 3x = 5x^2$

(iv) $2x + x + 3x = sx$

(v) $4p + 3p + 2p + p - 9p = 0$

(vi) $3x + 2y = 6xy$

(vii) $(3x)^2 + 4x + 7 = 3x^2 + 4x + 7$

(viii) $(2x)^2 + 5x = 4x + 5x = 9x$

(ix) $(2a+3)^2 = 2a^2 + 6a + 9$

(x) Substitute $x = -3$ in

(a) $x^2 + 7x + 12 = (-3)^2 + 7(-3) + 12 = 9 + 4 + 12 = 25$

(b) $x^2 - 5x + 6 = (-3)^2 - 5(-3) + 6 = 9 - 15 + 6 = 0$

(c) $x^2 + 5x = (-3)^2 + 5(-3) + 6 = -9 - 15 = -24$

(xi) $(x-4)^2 = x^2 - 16$

(xii) $(x+7)^2 = x^2 + 49$

(xiii) $(3a+4b)(a-b) = 3a^2 - 4a^2$

(xiv) $(x+4)(x+2) = x^2 + 8$

(xv) $(x-4)(x-2) = x^2 - 8$

(xvi) $5x^3 \div 5x^3 = 0$

(xvii) $2x^3 + 1 \div 2x^3 = 1$

(xviii) $3x + 2 \div 3x = \dfrac{2}{3x}$

(xix) $3x + 5 \div 3 = 5$

(xx) $\dfrac{4x+3}{3} = x+1$

What we have discussed

1. Factorisation is a process of writing the given expression as a product of its factors.
2. A factor which cannot be further expressed as product of factors is an irreducible factor.
3. Expressions which can be transformed into the form: $a^2 + 2ab + b^2$; $a^2 - 2ab + b^2$; $a^2 - b^2$ and $x^2 + (a+b)x + ab$ can be factorised by using identities.
4. If the given expression is of the form $x^2 + (a+b)x + ab$, then its factorisation is $(x+a)(x+b)$
5. Division is the inverse of multiplication. This concept is also applicable to the division of algebraic expressions.

Gold Bach Conjecture

Gold Bach found from observation that every odd number seems to be either a prime or the sum of a prime and twice a square.

Thus 21 = 19 + 2 or 13 + 8 or 3 + 18.

It is stated that up to 9000, the only exceptions to his statement are

$5777 = 53 \times 109$ and $5993 = 13 \times 641$,

which are neither prime nor the sum of a primes and twice a square.