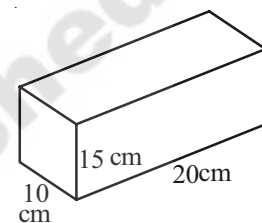# Surface Areas And Volume
(Cube and Cuboid)

14.0   Introduction

Suresh wants to wrap up his gift box. One of his friends suggested to buy 100 cm^2 paper another friend suggested to buy 200 cm^2. Whose suggestion is correct? How would he know that how much paper he has to buy?

15 cm 20cm 10 cm

It is obvious that size of the paper required depends on the surface area of the gift box .

In order to help ourselves in such situations, let us find the ways of calculating the surface areas of different solid objects.

14.1   Cuboid

Take a cuboid shaped box made up of thick paper or cardboard for example toothpaste box.

Cut and open it as shown in figure. Observe its shape of the faces. How many sets of identical faces are found?
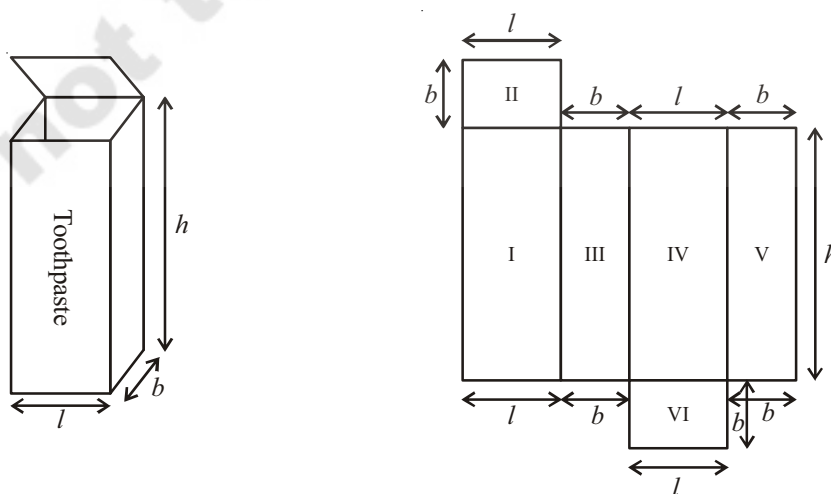
Look at the figure, if length 'l', breadth 'b', height 'h' are its dimensions, then you can find three pairs of identical faces.

Now we can see that the total surface area of a cuboid is

Area I + Area II + Area III + Area IV + Area V + Area VI

$$= h \times l + l \times b + b \times h + l \times h + b \times h + l \times b$$

So total surface area $\quad = 2\,(h \times l + b \times h + b \times l)$

$$= 2\,(lb + bh + hl)$$

The height, length and the breadth of the gift box are 20cm, 10cm and 15cm respectively.

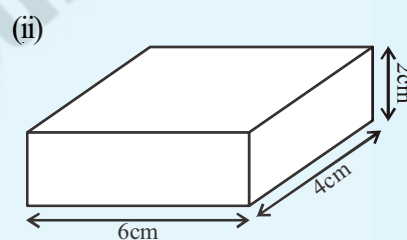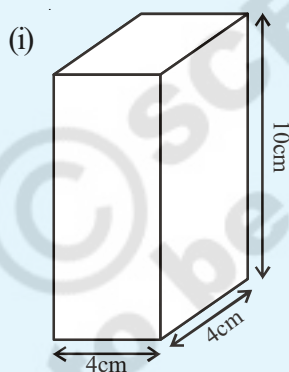Then the Total Surface Area(T.S.A) $= 2(20 \times 10 + 10 \times 15 + 15 \times 20)$

of the box $\qquad\qquad\qquad = 2\,(200 + 150 + 300)$

$$= 2\,(650) = 1300 \text{ cm}^2$$

Do This

1.   Find the total surface area of the following cuboid.

(i)

10cm

4cm

4cm

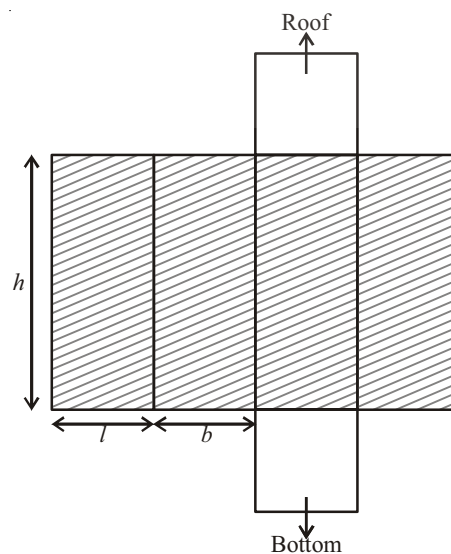(ii)

2cm

4cm

6cm

14.1.2 Lateral Surface Area:

•   The lateral faces (the faces excluding the top and bottom) make the lateral surface area of the cuboid. For example, the total area of all the four walls of the cuboidal room in which you are sitting is the lateral surface area of the room.

Hence, the Lateral Surface Area of a cuboid

$$\text{(L.S.A.)} = (l \times h) + (b \times h) + (l \times h) + (b \times h)$$

$$= 2lh + 2bh$$

$$= 2h\,(l + b)$$

Roof

h

l   b

Bottom

Try These

(i)   Take cuboid shaped duster (which your teacher uses in the class room). Measure its sides with scale and find out its surface area.

(ii)   Cover this duster with a graph paper, such that it just fits around the surface. Count the squares and verify the area you have calculatead.

(ii)   Measure length,width and height of your classroom and find

     (a)   The total surface area of the room, ignoring the area of windows and doors

     (b)   The lateral surface area of the room

     (c)   The total area of the room which is to be white washed.

Think, Discuss and Write

1.   Can we say that the total surface area of cuboid

     = lateral surface area + 2 × area of base.

2.   If we change the position of cuboid from (Fig. (i) to Fig. (ii) do the lateral surface areas become equal?

Fig (i)   Fig (ii)
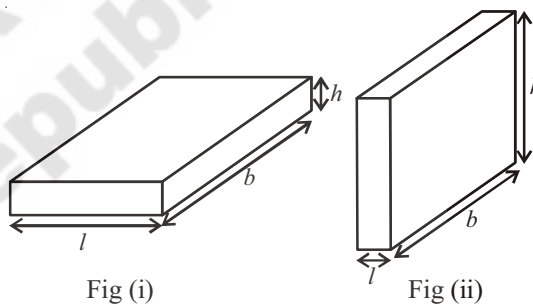
3.   Draw a figure of cuboid whose dimensions are l, b, h are equal. Derive the formula for LSA and TSA.

14.2   Cube

Draw the net Fig. (i) given below, on a graph paper and cut it out. Fold it along the lines as shown in Fig. (i) and joined the edges as shown in Fig(ii) and Fig. (iii). What is the shape of it? Examine its faces and its dimensions.

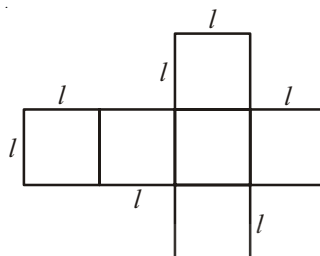
Fig (i)   Fig (ii)   Fig (iii)
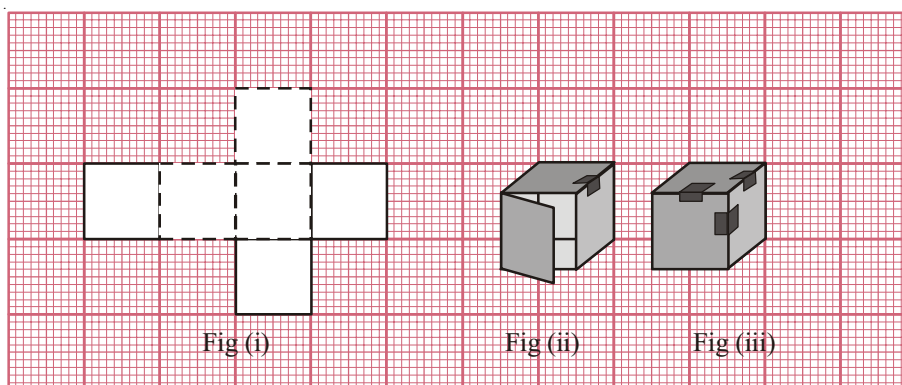
Observe the cube and its net diagram

In the figures (i) and (ii). Do all the faces of a cube are square in shape? Do the length, height and width of a cube are equal?
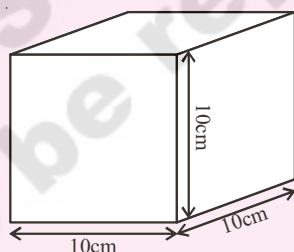
(i)   (ii)
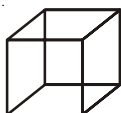
(a)   How many faces does a cube have ? Are all faces equal?

(b)   If each side of the cube is *l*, what will be the area of each face?

(c)   What is the total surface area of the cube.
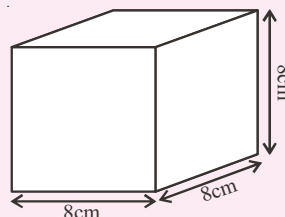
(d)   What is the lateral surface area of cube?

Try These

(i)   Find the surface area of cube 'A' and lateral surface area of cube 'B'
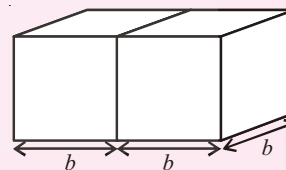
10cm

10cm

10cm

A

8cm

8cm

8cm

B

(ii)   Two cubes each with side 'b' are joined to form a cuboid as shown in the adjacent fig. What is the total surface area of this cuboid?

b   *b*   *b*

(iii)   How will you arrange 12 cubes of equal lengths to form a cuboid of smallest surface area?
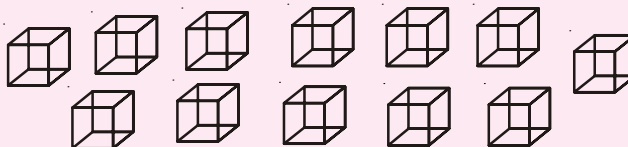
(iv)   The surface area of a cube of $4 \times 4 \times 4$ dimensions is painted. The cube is cut into 64 equal cubes. How many cubes have (a) 1 face painted? (b) 2 faces painted? (c) 3 faces painted? (d) no face painted ?

Example 1:   Find the surface area of a cuboid whose length, breadth and height are 15cm, 12cm and 10cm respectively.

Solution: Length of the cuboid (l)   =   15cm

Breadth of the cuboid (b)   =   12cm

Height of the cuboid (h)   =   10cm

Surface area of a cuboid   = $2(lb + bh + hl)$

$= 2(15 \times 12 + 12 \times 10 + 10 \times 15) \text{ cm}^2$

$= 2(180 + 120 + 150) \text{ cm}^2$

$= 2(450) \text{ cm}^2$

$= 900 \text{ cm}^2$

Example 2 :   If each edge of a cube is doubled. How many times will its surface area increase?

Solution: Let the edge of the cube be 'x'

Then edge of the new cube formed $= 2x$

Surface area of the original cube $= 6x^2$

Surface area of the new cube $= 6(2x)^2 = 6(4x^2) = 4(6x^2)$
when edge is doubled

Surface area of the new cube $= 4 \times$ Surface area of the original cube

Hence, the surface area of the new cube becomes 4 times that of the original cube.

Example 3:   Two cubes each of edge 6 cm are joined face to face. Find the surface area of the cuboid thus formed.

Solution: Look at the adjacent figure. Cube has six faces normally when two equal cubes are placed together, two side faces are not visible (Why?).
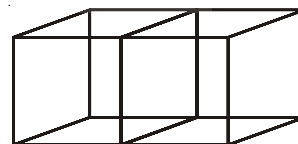
We are left with $12 - 2 = 10$ squared faces $= 10 \times l^2 \text{ cm}^2$

So, the total surface area of the cuboid   = $10 \times (6)^2 \text{ cm}^2$

$= 10 \times 36 \text{ cm}^2 = 360 \text{ cm}^2$

Alternate Method:

If two cubes of edges 6cm are joined face to face it will take the shape of a cuboid whose length, breadth and height are (6 + 6) cm, 6cm and 6cm i.e. 12 cm, 6cm and 6cm respectively. Thus, total surface area of the cuboid

$$= \quad 2\,(lb + bh + lh)$$

$$= \quad 2\,(12 \times 6 + 6 \times 6 + 12 \times 6)\ \text{cm}^2$$

$$= \quad 2\,(72 + 36 + 72)\ \text{cm}^2$$

$$= \quad 2 \times 180\ \text{cm}^2$$

$$= \quad 360\ \text{cm}^2$$

Example 4: Find the cost of painting of the outer surface of a closed box which is 60 cm long, 40 cm broad and 30 cm high at the rate of 50 paise per 20cm^2

Solution:

Length of the box (l)	=	60 cm
Breadth of the box (b)	=	40 cm
Hieght of the box (h)	=	30 cm
Total surface area of the box	=	$2\,(lb + bh + hl)$
	=	$2\,(60 \times 40 + 40 \times 30 + 60 \times 30)\ \text{cm}^2$
	=	$2(2400 + 1200 + 1800)\ \text{cm}^2$
	=	$2 \times 5400\ \text{cm}^2$
	=	$10800\ \text{cm}^2$

Cost of painting 20 cm^2 = 50 paise = ₹ $\dfrac{50}{100}$

∴ Cost of painting 1 cm^2 = ₹ $\dfrac{50}{100} \times \dfrac{1}{20}$

∴ Cost of painting 10800 cm^2 = ₹ $\dfrac{50}{100} \times \dfrac{1}{20} \times 10{,}800$ = ₹ 270

Exercise -14.1

1. There are two cuboidal boxes as shown in the given figure. Which box requires the less amount of material to make?
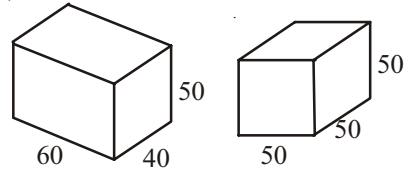
2. Find the side of a cube whose surface area is 600 cm^2.

3. Prameela painted the outer surface of a cabinet of measures 1m × 2m × 1.5m. Find the surface area she cover if she painted all except the bottom of the cabinet?

4. Find the cost of painting a cuboid of dimensions 20cm × 15 cm × 12 cm at the rate of 5 paisa per square centimeter.

14.3   Volume of Cube and Cuboid

Amount of space occupied by a three dimensional object is called its volume. Try to compare the volume of objects arround you. For example, volume of a room is greater than the volume of an almirah kept in the room. Similarly, volume of your pencil box is greater than the volume of the pen and the eraser kept inside it. Do you measure volume of either of these objects?

Remember, we use square units to find the area of a region. How will we find the volume. Here we will use cubic units to find the volume of a solid, as cube is the most convenient solid shape (just as square is the most convenient shape to measure this area).
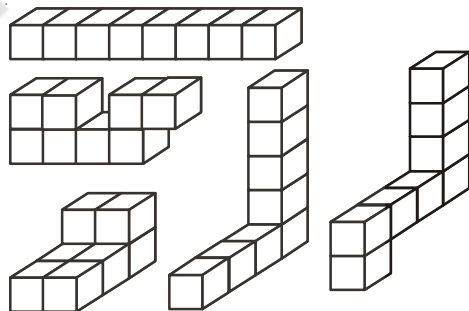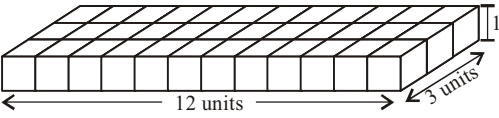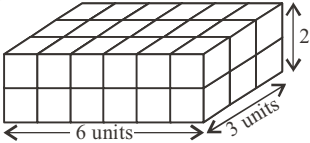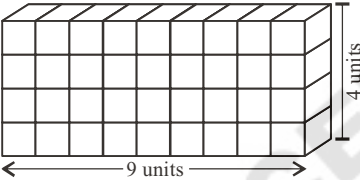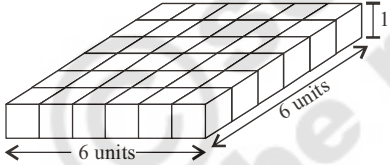
To measure the area we divide the area into square units, similarly, to find the volume of a solid we need to divide the space into cubical units. Unit cube is a cube of unit length . Observe that the volume of each of the solids which are arranged in different forms are of 8 cubic units (as in Fig above).

We can say that the volume of a solid is measured by counting the number of unit cubes it contains. Cubic units which we generally use to measure the volume are

1 cubic cm   =   1 cm × 1 cm × 1 cm = 1 cm^3

                =   10 mm × 10 mm × 10 mm = _____ mm^3

1 cubic m   =   1 m × 1 m × 1 m = 1 m^3

                =   100 cm × 100cm × 100cm = _____ cm^3

1 cubic mm   =   1 mm × 1 mm × 1 mm = 1 mm^3

                =   0.1 cm × 0.1 cm × 0.1 cm = _____ cm^3

14.3.1 Volume of a Cuboid:

Take 36 cubes of equal size (i.e.,side of each cube is same). Arrange them to form a cuboid. You can arrange them in many ways. Observe the following table and fill in the blanks.

	cuboid	length (*l*)	breadth (*b*)	height (*h*)	Total no. of unit cubes $l \times b \times h = V$
(i)	12 units, 3 units, 1	12	3	1	$12 \times 3 \times 1 = 36$
(ii)	6 units, 3 units, 2	...	...	...	...
(iii)	9 units, 4 units	...	...	...	...
(iv)	6 units, 6 units, 1	...	...	...	...

What do you observe? Do you find any relation between the dimensions of the cuboid and its volume?

Since we have used 36 cubes to form these cuboids, thus volume of each cuboid is 36 cubic units. This is equal to the product of length, breadth and height of the cuboid. From the above example we can say volume of cuboid = $l \times b \times h$. Since $l \times b$ is the area of its base we can also say that,

Volume of cuboid = Area of the base × height
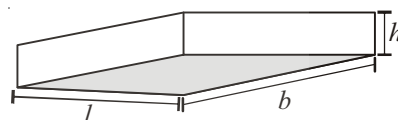
Activity

Take a sheet of paper. Measure its area. Pile up such sheets of paper of same size to make a cuboid (as in adjacent figure). Measure the height of this pile. Find the volume of the cuboid by finding the product of the area of the sheet and the height of this pile of sheets. Can you find the volume of paper sheet?
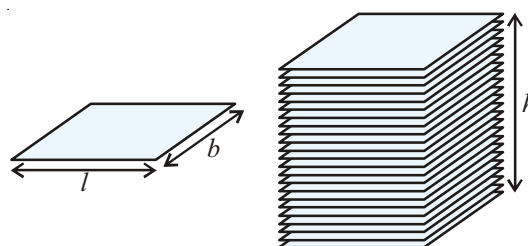
Do This

Let us find the volume of a cuboid whose length, breadth and height are 6cm., 4cm and 5cm respectively.

Let place 1 cubic centimeter blocks along the length of the cuboid . How many blocks can we place along the length? 6 blocks, as the length of the cuboid is 6 cm.
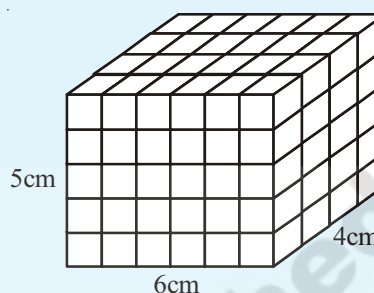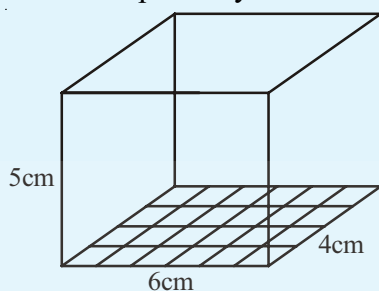
How many blocks can we place along its breadth? 4 blocks, as the breadth of the cuboid is 4cm. So there are 6 × 4 blocks can be placed in a layer.

How many layers of blocks can be placed in the cuboid? 5 layers, as the height of the cuboid is 5 cm. Each layer has 6 × 4 blocks. So, all the 5 layers will have 6 × 4 × 5 blocks i.e. length × breadth × height.

This discussion leads us to the formula for the volume of a cuboid:

Volume of a Cuboid = length × breadth × height

14.3.2 Vollume of a Cube

A Cube is a cuboid whose length, breadth and height are same,

$$\text{So Volume of a cube} = \text{side} \times \text{side} \times \text{side}$$
$$= (\text{side})^3 = a^3$$
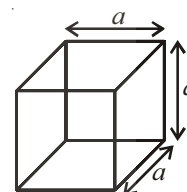
Where a is the side of the cube.

Length of Cube	Volume of the Cube
10mm = 1cm	$1000\,mm^3$ = $1cm^3$
10cm = 1dm	$1000\,c\,m^3$ = $1dm^3$
10dm = 1m	$1000\,d\,m^3$ = $1\,m^3$
100cm = 1m	$1000000\,c\,m^3$ = m^3
1000m = 1km	$1000000000\,m^3$ = $1km^3$

Generally, we measure the volumes of liquids in millilitres (ml) or litres (l)

Further $1 \, cm^3$ $= \quad 1 \, ml$

$\qquad\qquad 1000 \, cm^3 \quad = \quad 1 \, l$

$\qquad\qquad 1 \, m^3 \qquad = \quad 1000000 \, cm^3 = 1000 \, l$

$\qquad\qquad\qquad\qquad\quad = \quad 1 \, kl \, (kilolitre)$

Example 5: Find the volume of a block of wood whose length is 20cm, breadth is 10 cm and height is 8 cm.

Solution: The block of wood is a cuboid and the volume of a cuboid $= l \times b \times h$

Here, length $(l) = 20$ cm, breath $(b) = 10$ cm, and height $(h) = 8$ cm

Volume of the block $= 20$ cm $\times 10$ cm $\times 8$ cm $= 1600 \, cm^3$

Example 6: A water tank is 1.4 m long, 1m wide and 0.7m deep. Find the volume of the tank in litres.

Solution: Length of the tank $(l) = 1.4$ m $= 140$ cm

Breadth of the tank $(b) = 1$ m $= 100$ cm

Depth of the tank $(h) = 0.7 = 70$ cm

Volume of the tank $= l \times b \times h$

$= (140 \times 100 \times 70) \, cm^3$

$= \dfrac{140 \times 100 \times 70}{1000}$ litres.

$= 980$ litres

Do This

Arrange 64 unit cubes in as many ways as you can to form a cuboid. Find the surface area of each arrangement. Can solid cuboid of same volume have same surface area?

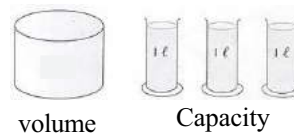**Do you know**

Capacity:

There is not much difference between the two words.i.e volume and capacity

(a) Volume refers to the amount of space occupied by an object.

(b) Capacity refers to the quantity that a container holds.

volume   Capacity

If a water tin holds $100 \, cm^3$ of water then the capacity of the water tin is $100 cm^3$. Capacity can also measure in terms of litres.

Example 7: Find the volume of a cuboid whose breadth is half of its length and height is double the length.

Solution: Let the length of the cuboid be x units

Then breadth of the cuboid $= \dfrac{x}{2}$ units

And height of the cuboid $= 2x$ units

Volume of the cuboid $=$ length $\times$ breadth $\times$ height

$$= (x \times \dfrac{x}{2} \times 2x) \text{ cubic units}$$

$$= x^3 \text{ cubic units.}$$

Example 8: A box is 1.8 m long, 90 cm wide, 60 cm height. Soap cakes of measurements 6 cm $\times$ 4.5 cm $\times$ 40 mm are to be packed in the box, so that no space is left. Find how many cakes can be packed in each box?

Solution:

Length of the box (l) $= 1.8$ m $= 180$ cm

Breadth of the box (b) $= 90$ cm

Height of the box (h) $= 60$ cm

Volume of the box $= l \times b \times h$

$= 180 \times 90 \times 60 \text{ cm}^3$

$= 972000 \text{ cm}^3$

Length of a soap cake $= 6$ cm

Breadth of a soap cake $= 4.5$ cm

Height of a soap cake $= 40$ mm $= 4$ cm

Volume of one soap cake $= 6 \times 4.5 \times 4 \text{ cm}^3$

$= 108.0 \text{ cm}^3$

$\therefore$ Required number of soap cakes

$$= \dfrac{\text{Volume of the box}}{\text{volume of one soapcake}}$$

$$= \dfrac{972000}{108}$$

$$= 9000$$

Hence, 9000 soap cakes can be packed in the box.

Example 9: How many cubes of side 3 cms each can be cut from wooden block in the form of a cuboid whose length, breadth and height are 21 cm, 9 cm and 8cm respectively. How much volume of wood is wasted?

Solution: Length of the cuboid (l) = 21 cm

Breadth of the cuboid (b) = 9 cm

Height of the cuboid (h) = 8 cm

Volume of cuboid = $21 \times 9 \times 8 = 1512$ cu cm.

No. of cubes that can be cut along the length $= \dfrac{21}{3} = 7$

No. of cubes that can be cut along the breadth $= \dfrac{9}{3} = 3$

No. of cubes that can be cut along the height $= \dfrac{8}{3} = 2.6$

Along the height we can cut only 2 pieces and remaining is waste.

$\therefore$ Total number of cubes cut $= 7 \times 3 \times 2$

$= 42$

Volume of each cube $= 3 \times 3 \times 3$ $= 27$ cm^3

Volume of all cubes $= 27 \times 42$

$= 1124$ cm^3

$\therefore$ Volume of the waste wood $= 1512 - 1124$ $= 388$ cm^3

Example 10: Water is pouring into a cuboidal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m^3. Find the number of hours it will take to fill the reservoir.

Solution: Volume of the reservoir $= 108$ m$^3 = 108 \times 1000$ litres

$(\because 1\text{m}^3 = 1000$ litres$)$

The reservoir is filling at the rate of 60 litres per minute.

$\therefore$ Required time $= \dfrac{108 \times 1000}{60}$ min.

$= \dfrac{108 \times 1000}{60 \times 60}$ hours $= 30$ hours.

Example 11 :   A village having a population of 4000, requires 150 litres water per head per day. It has a tank measuring 20 m , 15 m , 6 m. How many days for the water is sufficient enough once the tank is made full.

Solution: Volume of the tank $= 20 \text{ m} \times 15 \text{ m} \times 6 \text{ m}$

$$= 1800 \text{ m}^3 = 1800000 \ l$$

Volume of water consumed by 1 person in 1 day $= 150 \ l$.

Total volume of water consumed in a day by total population $= 150 \times 4000$

$$\text{Required number of days} = \frac{\text{Volume of the tank}}{\text{volume of water consumed in 1 day}}$$

$$= \frac{1800000}{150 \times 4000} = 3 \text{ days}$$

Exercise - 14.2

1.   Find the volume of the cuboid whose dimensions are given below.

	Length	Breadth	Height
(i)	8.2 m	5.3 m	2.6 m
(ii)	5.0 m	4.0 m	3.5 m
(iii)	4.5 m	2.0 m	2.5 m

2.   Find the capacity of the tanks with the following internal dimensions. Express the capacity in cubic meters and litres for each tank.

	Length	Breadth	Depth
(i)	3 m 20 cm	2 m 90 cm	1 m 50 cm
(ii)	2 m 50 cm	1 m 60 cm	1 m 30 cm
(iii)	7 m 30 cm	3 m 60 cm	1 m 40 cm

3.   What will happen to the volume of a cube if the length of its edge is reduced to half ? Is the volume get reduced ? If yes, how much ?

4. Find the volume of each of the cube whose sides are.

   (i) 6.4 cm   (ii) 1.3 m   (iii) 1.6 m.

5. How many bricks will be required to build a wall of 8 m long, 6m height and 22.5 cm thick, if each brick measures 25 cm by 11.25 cm by 6 cm?

6. A cuboid is 25 cm long , 15 cm broad, and 8 cm high . How much of its volume will differ from that of a cube with the edge of 16 cm?

7. A closed box is made up of wood which is 1cm thick .The outer dimensions of the box is 5 cm × 4 cm × 7 cm. Find the volume of the wood used.

8. How many cubes of edge 4cm, each can be cut out from cuboid whose length, breadth and height are 20 cm, 18 cm and 16 cm respectively

9. How many cuboids of size 4 cm × 3 cm × 2 cm can be made from a cuboid of size 12 cm × 9 cm × 6 cm?

10. A vessel in the shape of a cuboid is 30 cm long and 25 cm wide. What should be its height to hold 4.5 litres of water?

What we have discussed

1. If l, b, h are the dimensions of cuboid, then:

   (i) its lateral surface area is $2h(l+b)$

   (ii) its total surface area is $2(lb+bh+hl)$

2. Lateral Surface area of a cube is $4a^2$

3. Total Surface area of a cube is $6a^2$

4. Volume of a cuboid is $l \times b \times h$

5. Volume of a cube is $side \times side \times side = a^3$

6. $1\,cm^3 = 1\,ml$

   $1\,l = 1000\,cm^3$

   $1\,m^3 = 1000000\,cm^3 = 1000\,l$

   $= 1\,kl\,(\text{kilolitre})$