Chapter 5

Comparing quantities using Proportion

5.1 Introduction

In our day-to-day activities, some times we need to compare quantities. We learnt that ratio and percentages are used to compare quantities. Let us consider the following example.

Voting was conducted for class mentor, in a class of 40 students. Snigdha became first mentor by getting 24 votes and Siri became second mentor by getting 16 votes. So the ratio of votes polled to Snigdha and Siri is 24 : 16. After simplification, what is its ratio? It is 3:2.

Inversely the ratio of votes polled to Siri and Snigdha is 2:3. Can you say what a ratio is?

A **Ratio** is an **ordered** comparison of two quantities.

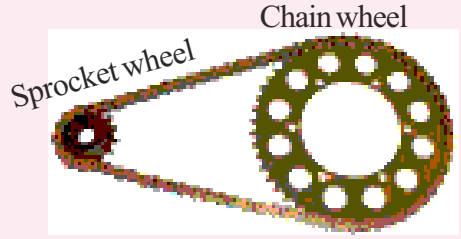
Try These

1. Find the ratio of gear your bicycle.

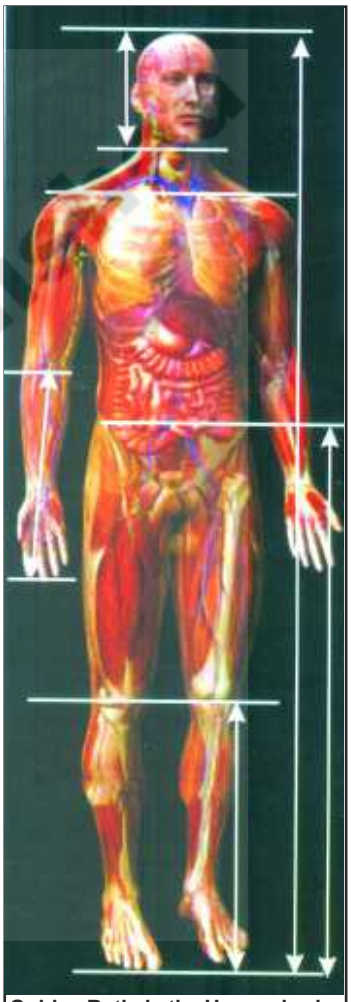
   Count the number of teeth on the chain wheel and the number of teeth for the sprocket wheel .

   $\{$*number of teeth on the chain wheel*$\}$: $\{$*number of teeth of Sprocket wheel*$\}$.

   This is called gear ratio. Write how many times Sprocket wheel turns for every time the chain wheel rotates once.



2. Collect News Paper cuttings related to percentages of any five different situations.



Golden Ratio in the Human body
Human beings are no exception to the golden ratio. In fact, our body architecture is one of the most perfect examples of this 'Divine proportion'.
Consider the following:

- Height : length between naval point and foot
- Length between shoulder line: length of the head.
- Length between finger tip to elbow: length between wrist and elbow
- Length between naval point to knee:length between knee and foot. **1.615:1 is Golden ratio.**

Compound Ratio

Some times we have to express two ratios as a single ratio. Why? Let us consider the following example to understand this.

Ramayya and Gopalam started a business with an investment of ₹ 2000 and ₹ 3000. At the end of the year in what ratio would they have to divide the annual profit obtained?

Ratio of investments = 2000: 3000

= 2: 3

Investments throughout the year are given below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total shares
Ramayya's shares	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	24
Gopalam.s shares	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	36

Ratio of their shares = 24: 36

= 2: 3 and ratio of time period = 1:1

What do you observe? Ratio of investments is equal to ratio of shares when time period is the same. So they will divide the profit in the ratio of their shares. So annual profit is to be divided in the ratio of 2:3

In the above example,

Case 1 : Suppose they both started the business with the same amount of ₹ 5000, but Ramayya did business for a period of 12months and Gopalam for a period of 9 months. How do they share the same profit? Do you say that because they started the business with the same amount, they have to divide the profit in the same ratio at the year ending?

Ratio of their investments = 5000: 5000 = 1:1

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total shares
Ramayya's shares	₹	₹	₹	₹	₹	₹	₹	₹	₹	₹	₹	₹	12
Gopalam.s shares	₹	₹	₹	₹	₹	₹	₹	₹	₹	-	-	-	9

Ratio of shares = 12: 9 = 4:3 and Ratio of time periods = 12: 9 = 4: 3

Their investment is the same, so they share the profit in the ratio of their shares i.e. ratio of their time period.

Case 2 : Further suppose Ramayya invested an amount of ₹ 2000 for 12 months and Gopalam invested an amount of ₹ 3000 for 9 months. In what ratio they have to divide the annual profit? Is it the ratio of investments or ratio of time period? Ramayya invested less amount but for more period. Gopalam invested more amount but for less period. Here we have to give importance for their investments as well as their investment periods. How to do that?

Ratio of investments = 2000: 3000 = 2:3

Ratio of time periods = 12: 9 = 4:3

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total shares
Ramayya's shares	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	₹ ₹	24
Gopalam's shares	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	₹ ₹ ₹	-	-	-	27

Ratio of shares = 24 : 27 = 8 : 9

= (2 × 12) : (3 × 9) = 8 : 9 (observe above table)

Here the ratio of investments is 2:3 and the ratio of time period is 4 : 3. So the ratio of shares is (2 × 12): (3 × 9) = 8 : 9. Hence they have to divide the annual profit in the ratio of 8 : 9. Do you find any relation between ratio of investment and time period and ratio of shares?

The ratio of shares can be written as $8 : 9 = \underbrace{2 : 3}_{\text{Product of antecedents}} :: \underbrace{4 : 3}_{\text{Product of consequents}}$ of ratios 2 : 3 and 4 : 3.

Two simple ratios are expressed like a single ratio as the ratio of product of antecedents to product of consequents and we call it **Compound ratio** of the given two simple ratios i.e. ratios are compounded by multiplying together the fractions which denote them.

a : b and c : d are any two ratios, then their compound ratio is $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ i.e. ac: bd.

Try These

1. Find the compound ratios of the following.

   (a) 3 : 4 and 2 : 3 (b) 4 : 5 and 4 : 5 (c) 5 : 7 and 2 : 9

2. Give examples for compound ratio from daily life.

Percentage:

Consider the following example.

M. K. Nagar high school students decided to sell tickets to a charity show. Class VIII students have 300 tickets to sell and class VII students have 250 tickets to sell. One hour before the show, eighth class students sold 225 tickets and seventh class students sold 200 tickets.

Which class students were closer to the goal of selling all their tickets?

To figure out which class students were closer their goal, you may try to compare the ratios 225:300 and 200:250. For eighth class students the ratio is 3:4 and for seventh class students the ratio is 4:5. Do you compare and say? It is difficult to have a meaningful comparison, hence we can't say directly, we need to have equivalent ratios of both which can be compared.

One way to compare quantities is to change them into percentages.

A **percentage (%)** compares a number to 100. The word **percent** means "per **every hundred**" or "**out of every hundred**". $100\% = \frac{100}{100}$. It is also a fraction with denominator 100.

Percentage of tickets sold by eighth class students is $= \frac{3}{4} \times \frac{100}{100} = \frac{75}{100} = 75\%$

Percentage of tickets sold by seventh class students $= \frac{4}{5} \times \frac{100}{100} = \frac{80}{100} = 80\%$

From this we understand that seventh class students were closer to the goal of selling all their tickets.

Percentage is number of parts out of 100. So the denominator is to be made 100 for which we are multiplying both numerator and denominator with 100.

We can use percentage as a common scale.

In the introductory part, we compared the number of votes polled to Snigdha and Siri by ratio. We can compare the same by percentages also.

The votes polled to Snigdha are 24 out of 40 or 3 out of 5 in the simplified form.

So percentage of votes are $\frac{3}{5} \times 100\% = 60\%$

By other method

Out of 40 votes, number of votes for Snigdha are 24.

So out of 100 votes, number of votes polled to Snigdha $= \frac{24}{40} \times 100 = 60$

Out of 100 votes 60 are for her, so percentage of her votes = 60%

Since all the students voted,

Percentage of votes for Snigdha + percentage of votes for Siri = 100%

60% + percentage of votes for Siri = 100%

Thus percentage of votes for Siri = 100% – 60% = 40%

5.2 Finding the increase or decrease percent

Consider the following situation.

- Class sizes have increased by 10%.
- House prices have dropped by 12%.
- CO_2 emissions need to fall by 25% by the year 2020.

Changes in quantities are often expressed as a percentage of the original quantity.

There are two different methods which can be used to solve increase or decrease in percentage problems. Let us see the following examples to understand this.

(1) A sales manager asked his team to increase the sales by 35% over previous month's sales which was ₹ 98,700. What is the target sales ?

Sales in the previous month = ₹ 98,700.

35% of ₹ 98,700 $= \frac{35}{100} \times 98,700$

= ₹ 34545

Target sales for the month

= ₹ 98,700 + 34,545

= ₹ 1, 33, 245.

<u>Unitary method.</u>

35% increase means,

₹ 100 increased to ₹ 135.

How much ₹ 98,700 will increase to?

Increased sales = ₹ $\frac{135}{100} \times 98,700$

= ₹ 1,33,245.

Decrease percentage in price would imply the actual decrease followed by its subtraction from the original price. Let us consider one example to understand this.

The original price of shoes is ₹ 550. They are for sale with a reduction of 10%. What is the new sale price of the shoes?

Price of shoes = ₹ 550.

Reduction = 10% of ₹ 550

$= \frac{10}{100} \times 550$ = ₹ 55.

New price = original price – reduction

= ₹ 550 – ₹ 55 = ₹ 495.

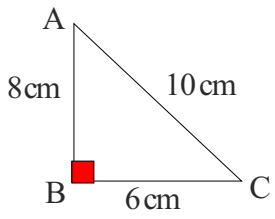
Think, Discuss and Write

1. Two times a number is 100% increase in the number. If we take half the number what would be the decrease in percent?
2. By what percent is ₹ 2000 less than ₹ 2400? Is it the same as the percent by which ₹ 2400 is more than ₹ 2000?

Exercise - 5.1

1. Find the ratio of the following
   (i) Smita works in office for 6 hours and Kajal works for 8 hours in her office. Find the ratio of their working hours.
   (ii) One pot contains 8 litre of milk while other contains 750 milliliter.
   (iii) speed of a cycle is 15km/h and speed of the scooter is 30km/h.
2. If the compound ratio of 5:8 and 3:7 is 45:x. Find the value of x.
3. If the compound ratio of 7:5 and 8:x is 84:60. Find x.
4. The compound ratio of 3:4 and the inverse ratio of 4:5 is 45:x. Find x.
5. In a primary school there shall be 3 teachers to 60 students. If there are 400 students enrolled in the school, how many teachers should be there in the school in the same ratio?
6. In the given figure, ABC is a triangle. Write all possible ratios by taking measures of sides pair wise.

   (Hint : Ratio of AB : BC = 8 : 6)



7. If 9 out of 24 students scored below 75% marks in a test. Find the ratio of student scored below 75% marks to the student scored 75% and above marks.

8. Find the ratio of number of vowels in the word' MISSISSIPPI' to the number of consonants in the simplest form.

9. Rajendra and Rehana own a business. Rehana receives 25% of the profit in each month. If Rehana received ₹ 2080 in particular month , what is the total profit in that month ?

10. In triangle ABC, AB = 2.2 cm, BC = 1.5 cm and AC = 2.3 cm. In triangle XYZ, XY = 4.4 cm, YZ = 3 cm and XZ = 4.6 cm. Find the ratio AB : XY, BC : YZ, AC : XZ. Are the lengths of corresponding sides of ΔABC and ΔXYZ are in proportion?

    [Hint : Any two triangles are said to be in proportion, if their corresponding sides are in the same ratio]

11. Madhuri went to a super market. The price changes are as follows. The price of rice reduced by 5% jam and fruits reduced by 8% and oil and dal increased by 10%. Help Madhuri to find the changed prices in the given table.

Item	Original price/kg	Changed price
Rice	₹ 30	
Jam	₹ 100	
Apples	₹ 280	
Oil	₹ 120	
Dal	₹ 80	

12. There were 2075 members enrolled in the club during last year. This year enrolment is decreased by 4% .
    (a) Find the decrease in enrolment.
    (b) How many members are enrolled during this year?

13. A farmer obtained a yielding of 1720 bags of cotton last year. This year she expects her crop to be 20% more. How many bags of cotton does she expect this year?

14. Points P and Q are both in the line segment AB and on the same side of its midpoint. P divides AB in the ratio 2 : 3, and Q divides AB in the ratio 3 : 4. If PQ = 2, then find the length of the line segment AB.

5.3 Finding discounts

In big shops and super markets we see price tags of the articles. Do you know what do we call them? This is called **marked price (M.P.)** of the article. Prices of the items are marked according to the price list quoted by the factory which is called **List price or catalogue price. or marked price**

Ravi went to shop to buy a book. Printed price of the book is ₹ 80. But shop owner gave him a discount of 15%. How much amount Ravi has paid to buy the book?

In our daily life we come across so many a situations where we are given a price discount on the articles.

Price discount is also called Rebate. It is given on marked price or List Price.

Now in the above example Ravi was given 15% discount. Printed price is ₹ 80. Then the discount will be $\frac{15}{100} \times 80 = ₹\ 12$. So the amount he has to pay is ₹ 80 – ₹ 12 = ₹ 68.

Let us see few more examples.

Example:1 A cycle is marked at ₹ 3600 and sold for ₹ 3312. What is the discount and discount percentage ?

Solution: Discount = marked price – sale price

= ₹ 3600 – ₹ 3312 = ₹ 288

Since discount is calculated on marked price. For calculating the discount percentage we use M.P. as the base.

On marked price of ₹ 3600, the discount is ₹ 288

On marked price of ₹ 100, how much will the discount be?

Discount percent = $\frac{288}{3600} \times 100 = 8\%$

We can also find discount when discount percent is given.

Example:2 The marked price of a ceiling fan is ₹ 1600 and the shop keeper allows a discount of 6% on it. Find its selling price.

Solution:

Raju solved it like this.

Discount = 6% of ₹ 1600

$= \frac{6}{100} \times 1600 = ₹ 96$

Selling Price = Marked price – discount

= ₹ 1600 – ₹ 96

= ₹ 1504.

Latha solved it in a different way.

6% decrease means

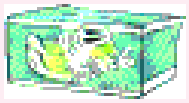
₹ 100 decreased to ₹ 94

So ₹ 1600 decreased to?

Selling price = $\frac{94}{100} \times 1600 = ₹ 1504$

Try These

1. Fill the selling price for each.

Item	Marked price in ₹	Discount %	Selling price in ₹
	450	7%	
	560	9%	
	250	5%	
	15000	15%	

Example: 3 Neelima went to shop to buy a dress. Marked price of the dress is ₹ 1000. Shop owner gave a discount of 20% and then 5%. Find the single discount equivalent to these two successive discounts.

Solution: Marked price of the article = ₹ 1000.

Percentage of first discount = 20%

First Discount = 20% of 1000

$$= \frac{20}{100} \times 1000 = ₹\ 200$$

Price after first discount = ₹ 1000 – ₹ 200

= ₹ 800.

Percentage of second discount = 5%

Second discount = 5% of ₹ 800

$$= \frac{5}{100} \times 800 = ₹\ 40$$

Price after second discount = ₹ 800 – ₹ 40 = ₹ 760.

Net selling price = ₹ 760.

20% discount means ₹ 100 is decreased to ₹ 80.

5% discount means ₹ 100 is decreased to ₹ 95.

∴ Net selling price

$$= 1000 \times \frac{80}{100} \times \frac{95}{100}$$

= ₹ 760

Single discount equivalent to given discounts = ₹ 1000 – ₹ 760 = ₹ 240.

For ₹ 1000 the discount amount is ₹ 240

Percentage of discount given at a time = ₹ 240 $\times \frac{24\cancel{0}}{1\cancel{0}\,\cancel{00}} \times \cancel{100}$ = 24%

What do you observe? Is the given single discount percentage is equivalent to the percentage of two given successive discounts.

Think, Discuss and Write

Preeti went to a shop to buy a dress. Its marked price is ₹ 2500. Shop owner gave 5% discount on it. On further insistence, he gave 3% more discount. What will be the final discount she obtained? Will it be equal to a single discount of 8%? Think, discuss with your friends and write it in your note book.

5.4 Estimation in percentages

Your bill in a shop is ₹ 477.80 and the shop keeper gives a discount of 15%. How would you estimate the amount to be paid?

Round off the bill to the nearest tens.₹ 477.80 are rounded off to ₹ 480. Then find 10% of this amount. It is ₹ 48. Take half of this. It is ₹ 24. So discount amount is ₹ 48 + ₹ 24 = ₹ 72. Amount to be paid approximately around ₹ 410.

Try These

(i) Estimate 20% of ₹ 357.30 (ii) Estimate 15% of ₹ 375.50

5.5 Profit and Loss

Prices related to buying and selling (Profit and Loss)

Observe the following situations.

- Sita bought a chair for ₹ 750 and sold it for ₹ 900.
- Mary bought 10g of gold for ₹ 25000 in last year and sold it for ₹ 30,000 in this year.
- Rahim bought a bicycle for ₹ 1600 and next year he sold it for ₹ 1400.
- Anitha purchased a car for ₹ 4.8 lakh and sold it for ₹ 4.1 lakh after 2 years.
- Hari purchased a house for ₹ 9 lakh and incurred an expenditure of ₹ 1 lakh for its repairs. He sold it for ₹ 10.7 lakh.

In the first four examples profit or loss is known by finding the difference between cost price and selling price.

But in the last example, what is the profit obtained by Hari? Is it ₹ 1.7 lakh? Obviously not. He incurred some additional expenditure on it before selling. What do we call such expenditures?

Some times the shop keeper has to spend on additional expenses like transportation, maintenance, labour, repair, commission, rent of godown etc. in addition to the price paid to buy an article. Such additional expenses are called **overhead expenses** and are to be added to the Cost price. Profit or loss is always calculated on this resultant cost price.

Think, Discuss and Write

What happens if cost price = selling price. Do we get any such situations in our daily life?

It is easy to find profit % or loss % in the above situations. But it will be more meaningful if we express them in percentages. Profit % is an example of **increase percent** of cost price and loss % is an example of **decrease percent of cost price**.

Let us see some examples.

Example:4 Radhika deals with second-hand goods. She bought a second hand refrigerator for ₹ 5000. She spends ₹ 100 on transportation and ₹ 500 on its repair. She sells the refrigerator for ₹ 7000.

Find (i) the total cost price of the refrigerator (ii) profit or loss percent.

Solution: (i) Total cost price = purchasing price+ transportation charges+ repair charges

= ₹ (5000 + 100 + 500) = ₹ 5600

So the total cost price is ₹ 5600.

(ii) Selling price is ₹ 7000. Here Selling price > cost price, so there is a profit.

Profit = selling price – cost price = ₹ 7000 – ₹ 5600 = ₹ 1400.

On cost price of ₹ 5600 profit is ₹ 1400

If cost price is ₹ 100, profit will be?

$$\text{Profit percent} = \frac{1400}{5600} \times 100 = 25\%$$

Example:5 Vinay bought a flat for ₹ 4,50,000. He spent ₹ 10,000 on its paintings and repair. Then he sold it for ₹ 4,25,500. Find his gain or loss and also its percent.

Solution: Total cost price = purchasing price+ repair charges.

= ₹ (4,50,000 + 10,000) = ₹ 4,60,000.

Selling price is ₹ 4,25,500. Here we can observe Selling price < cost price. So there is a loss.

Loss = cost price – selling price

= ₹ 4,60,000 – ₹ 4,25,500 = ₹ 34,500.

For cost price of ₹ 4, 60, 000 loss is ₹ 34,500 if its cost price is ₹ 100 what will the loss percentage be?

$$\text{Loss percent} = \frac{34,500}{4,60,000} \times 100 = 7.5\%$$

Example:6 Venkanna purchased 50 dozen bananas for ₹ 1250. He incurred transportation charges of ₹ 250. He could not sell five dozen bananas as they were spoiled. He sold the remaining bananas at ₹ 35 for each dozen. Will he get a profit or a loss? Find profit or loss percent.

Solution: Total cost price = Cost price of bananas + Transportation charges

= ₹ 1250 + ₹ 250 = ₹ 1500.

Number of dozens of bananas sold = Number of dozens purchased – number of dozens rotten

= 50 – 5 = 45

Selling price = ₹ 35 × 45 = ₹ 1575

Clearly selling price > cost price so it is a profit.

Profit = selling price – cost price = ₹ 1575 – ₹ 1500 = ₹ 75

On cost price of ₹ 1500 profit is ₹ 75

On cost price of ₹ 100 profit will be?

$$\text{Profit percent} = \frac{75}{1500} \times 100 = 5\%$$

Example:7 Malik sells two tables for ₹ 3000 each. He gains 20% on one table and on the other he loses 20%. Find his gain or loss percent on the whole transaction.

Solution:

For first table	For second table
Selling Price = ₹ 3000	Selling price = ₹ 3000
Profit percent = 20%	Loss percent = 20%
profit percent means increased percent on cost price	Loss percent means decreased percent on cost price
Selling price is ₹ 120 when Cost price is ₹ 100	Selling price is ₹ 80 when cost price is ₹ 100
When selling price is ₹ 3000 what will be the cost price?	When selling price is ₹ 3000 what will be the cost price?
Cost price = $₹ 100 \times \frac{3000}{120} = ₹ 2500$	Cost price = $₹ 100 \times \frac{3000}{80} = ₹ 3750$

Total cost price on two tables = ₹ 2500 + ₹ 3750 = ₹ 6250

Total selling price on two tables = ₹ 3000 + ₹ 3000 = ₹ 6000.

Since cost price > selling price. So there is a loss

Loss = cost price – selling price = ₹ 6250 – ₹ 6000 = ₹ 250

On cost price of ₹ 6250 loss is ₹ 250

On cost price of ₹ 100 what will be the loss?

Loss percent = $250 \times \frac{100}{6250} = 4\%$

So there is a loss of 4% on the whole transaction.

Think, Discuss and Write

A shop keeper sold two TV sets at ₹ 9, 900 each. He sold one at a profit of 10% and the other at a loss of 10%. On the whole whether he gets profit or loss. If so what is its percentage?

5.6 Sales Tax / Value Added Tax (VAT)

Government collects taxes on every sale. This is called VAT. Shop keepers collect this from the customers and pay it to the Government. Why does the government charge taxes like this? Do you know? With the taxes collected, government does several welfare activities.

Sales tax is levied on the sale of movable goods. VAT is imposed on goods only and not services and it has replaced sales tax. The percent of VAT is different for different items. In general, on the essential commodities, there is an exemption from VAT, 1% on bullion and precious stones, 5% on industrial inputs and capital goods and items of mass consumption for all other items it is 14.5%. (Rates in 2012 fixed by Government of India).

VAT is charged on the Selling Price of an item and will be included in the bill. VAT is an increase percent of selling price. Observe the following VAT added bill.

Ganapati went to a medical shop to buy medicines for his mother. The shop keeper gave the bill which appears like this. Bill amount was ₹ 372.18. It contains 5% VAT.

(i) Find the bill amount before VAT was added.

Tax Invoice No. : 2012?3015490072l4 Date : 15-09-2012 20:48:31

Name : Ganpathi Age : 35 Gender : Doc:dr Do.Reg. No. :

Cus.ID:20121301549000617 Add: Sainathpura)

S.	Product	Mfgr	Sch	Batch	Exp.	MRP.	Rate	Qty	Amount
1	BETATROP TAB	SUN	H	BSK4198	12-14	5.9	5.9	60	318.60
2.	ECOSPRIN 150 MG TAB	USV	H	04004652	05-14	0.4242857	0.38	42	16.04
3.	LASIX 40 MG TAB	AVENTIS	H	0212016	03-16	0.44733334	0.40	15	6.04
4.	ELDERVIT PLUS CAD	ELDER	C	SE0022008	08-13	2.3333333	2.10	15	31.5

Amount saved : 41.35 VAT ON ₹ 354.45 @ 5% = 17.72 Total : 372.18

Rounded Total : 372.00

From the bill copy it is clear that the actual bill amount = ₹ 354.45 ,Vat @ 5% = ₹17.72

Example:8 The cost of a pair of shoes is ₹ 450. The sales tax charged was 6%. Find the bill amount.

Solution: On ₹ 100 the sales tax paid is ₹ 6.

On ₹ 450 the tax to be paid is?

Sales tax paid = ₹ $\frac{6}{100} \times 450$ = ₹ 27.

Bill amount = Cost of item + sales tax = ₹ 450 + ₹ 27 = ₹ 477.

Exercise - 5.2

1. In the year 2012, it was estimated that there were 36.4 crore Internet users worldwide. In the next ten years, that number will be increased by 125%. Estimate the number of Internet users worldwide in 2022.

2. A owner increases the rent of his house by 5% at the end of each year. If currently its rent is ₹ 2500 per month, how much will be the rent after 2 years?

3. On Monday, the value of a company's shares was ₹ 7.50. The price increased by 6% on Tuesday, decreased by 1.5% on Wednesday, and decreased by 2% on Thursday. Find the value of each share when trade opened on Friday.

4. With most of the Xerox machines, you can reduce or enlarge your original by entering a percentage for the copy. Reshma wanted to enlarge a 2 cm by 4 cm drawing. She set the Xerox machine for 150% and copied her drawing. What will be the dimensions of the copy of the drawing be?

5. The printed price of a book is ₹ 150. And discount is 15%. Find the actual amount to be paid.

6. The marked price of an gift item is ₹ 176 and sold it for ₹ 165. Find the discount percent.

7. A shop keeper purchased 200 bulbs for ₹ 10 each. However 5 bulbs were fused and put them into scrap. The remaining were sold at ₹ 12 each. Find the gain or loss percent.

8. Complete the following table with appropriate entries (Wherever possible)

S. No.	Cost Price (C.P.)	Expenses	Selling Price(S.P.)	Profit	Loss	Profit Percentage	Loss Percentage
1	₹ 750	₹ 50		₹ 80			
2	₹ 4500	₹ 500			₹ 1,000		
3	₹ 46,000	₹ 4000	₹ 60,000				
4	₹ 300	₹ 50				12%	
5	₹ 330	₹ 20					10%

9. A table was sold for ₹ 2,142 at a gain of 5%. At what price should it be sold to gain 10%.

10. Gopi sold a watch to Ibrahim at 12% gain and Ibrahim sold it to John at a loss of 5%. If John paid ₹ 1,330, then find how much did Gopi sold it?

11. Madhu and Kavitha purchased a new house for ₹ 3,20,000. Due to some economic problems they sold the house for ₹ 2, 80,000.
Find (a) The loss incurred (b) the loss percentage.

12. A pre-owned car show-room owner bought a second hand car for ₹ 1,50,000. He spent ₹ 20,000 on repairs and painting, then sold it for ₹ 2,00,000. Find whether he gets profit or loss. If so, what percent?

13. Lalitha took a parcel from a hotel to celebrate her birthday with her friends. It was billed with ₹ 1,450 including 5% VAT. Lalitha asked for some discount, the hotel owner gave 8% discount on the bill amount. Now find the actual amount that lalitha has to pay to the hotel owner

14. If VAT is included in the price, find the original price of each of the following.

S. No.	Item	VAT %	Bill amount(in ₹)	Original Price(in ₹)
(i)	Diamond	1%	₹ 10,100	
(ii)	Pressure cooker	5%	₹ 2,940	
(iii)	Face powder	14.5%	₹ 229	

15. Find the buying price of each of the following items when a sales tax of 5% is added on them.
(i) a towel of ₹ 50 (ii) Two bars of soap at ₹ 35 each.

16. A Super-Bazar prices an item in rupees and paise so that when 4% sales tax is added, no rounding is necessary because the result is exactly in 'n' rupees, where 'n' is a positive integer. Find the smallest value of 'n'.

5.7 Compound interest

Interest is the money paid by bank or post office when money is deposited with them. Also it is paid by the borrower to the person or organisation that lent money. Interest is the extra amount paid on principal amount with a year marked percent.

But how do we calculate this interest? When the interest is calculated uniformly on the original principal throughout the loan period, what do you call such interest calculation? Yes! It is called simple interest. It is also an increase percent on the Principal. Let us see an example to understand this.

Example:9 A sum of ₹ 2500 is borrowed at a rate of 12% per annum for 3 years. Find the simple interest on this sum and also the amount to be paid at the end of 3 years.

Solution: Here P = ₹ 2500, T = 3 years, R = 12%

$$\text{As} \quad I = \frac{PTR}{100}$$

$$= \frac{2500 \times 3 \times 12}{100}$$

Interest for 3 years = ₹ 900.

Amount to be paid at the end of 3 years = Principal + Interest

= ₹ 2500 + ₹ 900 = ₹ 3400.

We see that Amount = Principal + Interest = $P + \frac{P \times T \times R}{100} = P\left(1 + \frac{T \times R}{100}\right)$

When T = 1year, Amount $A = P\left(1 + \frac{R}{100}\right)$

Try These :

Complete the table

S. No.	Principal (P) in ₹	Time (T) in years	Rate of interest p.a. (R) in %	Interest (I) = $\frac{P \times T \times R}{100}$ in ₹
1	3000	3	6	
2		2	5	50
3	1875		12	675
4	1080	2.5		90

Ramesh borrowed an amount of ₹100 at the rate of 10% p.a. (per annum) from Sreenu as a hand loan. After 2 years he went to Sreenu to repay his debt. Ramesh gave an amount of ₹ 120 and Sreenu said he has to pay ₹ 1 more. To find out the difference in their calculations, both of them did their calculations on a paper as shown below.

Ramesh's method			Sreenu's method		
1st year	Principal amount Interest @ 10% Total amount	₹ 100 ₹ 10 ₹ 110	1st year	Principal amount Interest @ 10% Total amount	₹ 100 ₹ 10 ₹ 110
2nd year	Principal Interest @ 10% Amount to be paid at the end of 2nd year	₹ 100 ₹ 10 = Principal + Interest on 1st year + Interest on 2nd year = 100+10+10 = ₹120	2nd year	Principal Interest @ 10% Amount to be paid at the end of 2nd year	₹ 110 ₹ 11 ₹ 121

The difference in the two methods is ₹1. Why is there a difference in both the methods? You can easily observe that while doing the calculation of interest for 2nd year Ramesh took principal amount as ₹ 100 whereas for doing the same Sreenu took ₹ 110. We call the interest calculated by Ramesh as Simple interest. Do you know what we call the interest calculated by Sreenu? In case of Sreenu, the interest is calculated on amount accumulated till then. It is called compound interest.. So Compound interest allows you to earn interest on interest. Which type of interest would you prefer and why ?

5.8 Deducing a formula for Compound interest

In the above example we discussed earlier, we observed that Sreenu calculated compound interest. If it is a year or two, it is easy to do the calculations like that. But if we have more than two of years, should we calculate in the same way? Is there a shorter way for finding compound interest? Let us consider an example and try to find it out.

When t = 1year Amount (A) = $P\left(1+\frac{R}{100}\right)$ with simple interest

Let P_1 = ₹ 10,000 and R = 12% per annum

Sreenu's method			Generalisation of same method.		
1st year	Principal P_1	₹ 10,000		Principal	P_1
	Amount A_1	$10000\left(1+\frac{12}{100}\right)$ $=10000\left(\frac{112}{100}\right)$ = ₹ 11,200	1st year	Amount A_1	$A_1 = P_1\left(1+\frac{R}{100}\right)$
2nd year	Principal P_2	₹ 11,200	2nd year	Principal	$P_2 = P_1\left(1+\frac{R}{100}\right)$
	Amount A_2	$11200\left(1+\frac{12}{100}\right)$ $=11200\left(\frac{112}{100}\right)$ = ₹ 12,544		Amount A_2	$A_2 = P_2\left(1+\frac{R}{100}\right)$ $= P_1\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)$ $= P_1\left(1+\frac{R}{100}\right)^2$

Proceeding in this way the amount at the end of 'n' years will be $A_n = P_1\left(1+\frac{R}{100}\right)^n$

Thus the amount on compound interest

Simply we can say that $A = P\left(1+\frac{R}{100}\right)^n$

But by using this we get only the amount to be paid at the end of 'n' years. How do we get compound interest? Yes it is so simple. From the final amount subtract principal to get compound interest.

$$\therefore C.I = P\left(1+\frac{R}{100}\right)^n - P$$

So what is the difference between simple interest and compound interest? Simple interest remains the same every year . But compound interest increases over time.

Example:10 What will be the amount and compound interest, if ₹ 5000 is invested at 8% per annum 2 years?

Solution: P = ₹ 5000; R = 8% and n = 2 years

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$= 5000\left(1+\frac{8}{100}\right)^2$$

$$= 5000 \times \frac{108}{100} \times \frac{108}{100} = ₹\ 5832.$$

Interest earned = Amount – Principal

= ₹ 5832 – ₹ 5000

= ₹ 832

Do These

1. How much compound interest is earned by investing ₹ 20 000 for 6 years at 5% per annum compounded annually. ?
2. Find compound interest on ₹ 12600 for 2 years at 10% per annum compounded annually.

5.9 Interest compounded annually or Half yearly (Semi Annually)

You may observe that in the previous problems we are using the word compounded annually. Does it have any significance? Yes, it has. Because we can also have interest rates compounded half yearly or quarterly.

When interest is not compounded annually what do we call the time period after which interest is added to principal? It is called **Conversion period**. When interest is compounded half yearly, there are two conversion periods in a year each after 6 months. In such a case, the interest will be half of the annual rate and the number of times that interest is compounded is twice the number of years.

Example:11 Calculate Compound interest on ₹ 1000 over a period of 1 year at 10% per annum if interest is compounded half yearly.

Solution: As interest is compounded half yearly, so there will be 2 conversion periods in a year.

So n=2

Rate of interest for 6 months rate $= \frac{1}{2} \times 10\% = 5\%$

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$A = 1000\left(1+\frac{5}{100}\right)^2$$

$$= 1000\left(\frac{105}{100}\right)^2$$

= ₹ 1102.50

Compound interest = A – P = 1102.50 – 1000 = ₹ 102.50

Do These

Find the number of conversion times the interest is compounded and rate for each

1. A sum taken for $1\frac{1}{2}$ years at 8% per annum is compounded half yearly.
2. A sum taken for 2 years at 4% per annum compounded half yearly.

Think, Discuss and Write

What will happen if interest is compounded quarterly? How many conversion periods will be there? What about the quarter year rate- how much will it be of the annual rate? Discuss with your friends.

Example:12 What amount is to be repaid on a loan of ₹ 12000 for $1\frac{1}{2}$ year at 10% per annum if interest is compounded half yearly.

Solution: As interest is compounded half yearly, so number of conversion periods in $1\frac{1}{2}$ years is 3, So $n = 3$

rate for half year $= \frac{1}{2} \times 10\% = 5\%$

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$A = 12000\left(1+\frac{5}{100}\right)^3$$

$$= 12000\left(\frac{105}{100}\right)^3$$

$$= ₹\ 13891.50$$

Compound interest $= A - P$

$= 13891.50 - 12000$

$= ₹\ 1891.50$

Example:13 Yadaiah for his family needs borrowed ₹ 5120 at $12\frac{1}{2}$ % per annum compounded annually. How much amount he has to pay to clear the debt at the end of two year nine months? Also find total how much interest he has paid?

Solution: Reshma tried to solve this problem like this

She first converted the time in years. 2 year 9 months $= 2\frac{9}{12}$ year $= 2\frac{3}{4}$ years

She tried to substitute this in the known formula $A = 5120\left(1+\frac{25}{200}\right)^{2\frac{3}{4}}$

Now she was stuck. She asked her teacher, how would she find a power which is fractional?

The teacher gave her a hint. First find the amount for the whole part. Then use this as principal to get simple interest for $\frac{3}{4}$ year

So $A = P\left(1+\frac{R}{100}\right)^n$

$A = 5120\left(1+\frac{25}{200}\right)^2$

$= 5120\left(\frac{225}{200}\right)^2$

$= ₹\ 6480$

Interest for remaining 9 months $= 6480 \times \frac{25}{2} \times \frac{3}{4} \times \frac{1}{100} = ₹\ 607.50.$

So Yadaiah has to pay at the end of 2 year 9 months

$= 6480 + 607.50 = ₹\ 7087.50$

So total compound interest $= 7087.50 - 5120 = ₹\ 1967.50$

5.10 Application of Compound Interest formula

Where do we use this compound interest formula? Not only for calculating interest, but it can also be used in different cases . For example,

- Increase (or decrease) in population
- The growth of bacteria if the rate of growth is known
- The value of an item, if its price increases (or decreases) in the intermediate years.

Example:14 The population of a village is 6250. It is found that the rate of increase in population is 8% per annum. Find the population after 2 years.

Solution: Here P = 6250 R = 8% T = 2 years

Population after 2 years $A = P\left(1+\frac{R}{100}\right)^n$

$A = 6250\left(1+\frac{8}{100}\right)^2$

$= 6250\left(\frac{108}{100}\right)^2$

$= 7290$

Example:15 A rubber ball is dropped from a certain height. It is found to rebounce only 90% of its previous height. If it is dropped from the top of a 25m tall building, to what height would it raise after bouncing on the ground two times.

Solution: The ball rises to a height of 90% at the first bounce. So at each bounce the loss in height is 10%

So taking R = -10% the problem can be solved.

P = 25 m and n = 2

The height to which it raises after bouncing two times on the ground

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$A = 25\left(1-\frac{10}{100}\right)^2$$

$$= 25\left(\frac{90}{100}\right)^2$$

$$= 20.25 \text{ m}$$

Exercise - 5.3

1. Sudhakar borrows ₹ 15000 from a bank to renovate his house. He borrows the money at 9% p.a. simple interest over 8 years. What are his monthly repayments?
2. A TV was bought at a price of ₹ 21000. After 1 year the value of the TV was depreciated by 5% (Depreciation means reduction of the value due to use and age of the item). Find the value of the TV after 1 year.
3. Find the amount and the compound interest on ₹ 8000 at 5% per annum, for 2 years compounded annually.
4. Find the amount and the compound interest on ₹ 6500 for 2 years, compounded annually, the rate of interest being 5% per annum during the first year and 6% per annum during the second year.

5. Prathibha borrows ₹ 47000 from a finance company to buy her first car. The rate of simple interest is 17% and she borrows the money over a 5 year period. Find: (a) How much amount Prathibha should repay the finance company at the end of five years. (b) her equal monthly repayments.

6. The population of Hyderabad was 68,09,000 in the year 2011. If it increases at the rate of 4.7% per annum. What will be the population at the end of the year 2015.

7. Find Compound interest paid when a sum of ₹ 10000 is invested for 1 year and 3 months at $8\frac{1}{2}$ % per annum compounded annually.

8. Arif took a loan of ₹ 80,000 from a bank. If the rate of interest is 10% per annum, find the difference in amounts he would be paying after $1\frac{1}{2}$ years, if the interest is (i) compounded annually (ii) compounded half yearly.

9. I borrowed ₹ 12000 from Prasad at 6% per annum simple interest for 2 years. Had I borrowed this sum at 6% per annum compounded annually, what extra amount would I have to pay?

10. In a laboratory the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5, 06,000

11. Kamala borrowed ₹ 26400 from a bank to buy a scooter at a rate of 15% per annum compounded yearly. What amount will she pay at the end of 2 years and 4 months to clear the loan?

12. Bharathi borrows an amount of ₹ 12500 at 12% per annum for 3 years at a simple interest and Madhuri borrows the same amount for the same time period at 10% per annum, compounded annually. Who pays more interest and by how much?

13. Machinery worth ₹ 10000 depreciated by 5%. Find its value after 1 year.

14.. Find the population of a city after 2 years which is at present 12 lakh, if the rate of increase is 4%.

15. Calculate compound interest on ₹ 1000 over a period of 1 year at 10% per annum, if interest is compounded quarterly?

What we have discussed

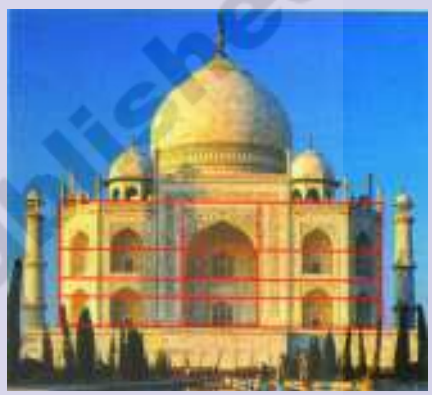
1. Two simple ratios are expressed like a single ratio as the ratio of product of antecedents to product of consequents and we call it Compund ratio of the given two simple ratios. a:b and c:d are any two ratios, then their compound ratio is $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ i.e. ac:bd.
2. A percentage (%) compares a number to 100. The word percent means "per every hundred" or "out of every hundred". $100\% = \frac{100}{100}$ it is also a fraction with denominator 100.
3. Discount is a decrease percent of marked price.

   Price reduction is called Rebate or discount. It is calculated on marked price or List Price.
4. Profit or loss is always calculated on cost price. Profit is an example of increase percent of cost price and loss is an example of decrease percent of cost price.
5. VAT will be charged on the Selling Price of an item and will be included in the bill.

   VAT is an increase percent on Selling Price.
6. Simple interest is an increase percent on the Principal
7. Simple interest $(1) = \frac{P \times T \times R}{100}$ where P = principal T = Time in years R = Rate of interest.
8. Amount = Principal + Interest $= P + \frac{P \times T \times R}{100} = P\left(1 + \frac{T \times R}{100}\right)$
9. Compount interest allows you to earn interest on interest.
10. Amount at the end of 'n' years using compound interest is $A = P\left(1 + \frac{R}{100}\right)^n$
11. The time period after which interest is added to principal is called conversion period. When interest is compounded half yearly, there are two conversion periods in a year, each after 6 months. In such a case, half year rate will be half of the annual rate.

Do you Know?

In ancient Greece, artists and architects believed there was a particular rectangular shape that looked very pleasing to the eye. For rectangles of this shape, the ratio of long side to the short side is roughly **1.615:1**. This ratio is very close to what is known as Golden ratio. The famous Greek temple the Parthenon, made entirely of white marble in the 5th century B.C. was built according to the Golden Ratio. The Taj Mahal in India is also an example of architecure for Golden ratio.



Addition of Equal Ratios

1. What is the sum of $\frac{1}{2},\frac{2}{4},\frac{3}{6},\frac{4}{8},.....,\frac{100}{200}$?

   can we add like this?

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=.....=\frac{100}{200} = \frac{1+2+3+4+.....+100}{2+4+6+8+.....+200}$$

$$= \frac{5050}{2\times 5050} = \frac{1}{2}$$

If $\frac{p_1}{q_1}=\frac{p_2}{q_2}=\frac{p_3}{q_3}=.....=\frac{p_n}{q_n}$ then $\frac{p_1+p_2+p_3+....+p_n}{q_1+q_2+q_3+.....+q_n}=\frac{p_1}{q_1}$

2. $\frac{a}{b}=\frac{c}{d}$ iff $\frac{a+b}{b}=\frac{c+d}{d}$ $\quad (b,d>0)$

   $\frac{1}{2}=\frac{3}{6}$ iff $\frac{1+2}{2}=\frac{3+6}{6}$

   $\frac{3}{2}=\frac{9}{6}$ again this can be written as $\frac{5}{2}=\frac{15}{6}$.....