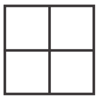
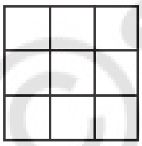
Chapter 6

Square Roots and Cube Roots

6.0 Introduction

Let us make square shapes using unit squares.
Observe the number of unit squares used.

A unit square is a square whose side is 1 unit

S.No.	Figure	Length of the side in units	No.of unit squares used
1		1	1
2		2	4
3		3	9

Similarly make next two squares.

Can you guess how many total unit squares are required for making a square whose side is 6 units?

From the above observations we could make square shapes with 1, 4, 9, 16, 25 … unit squares.

The numbers 1, 4, 9, 16, 25, … can be expressed as

$1 = 1 \times 1 = 1^2$

$4 = 2 \times 2 = 2^2$

$9 = 3 \times 3 = 3^2$

$16 = 4 \times 4 = 4^2$

$25 =$ …… $\times$ …… $=$ ……..

$36 =$ ……. $\times$ …… $=$ ……..

………………………………..

………………………………...

$m = n \times n = n^2$ where m, n are integers.

Observe the pattern of factors in each case

You might have observed in the given pattern that the numbers are expressed as the product of two equal factors. Such numbers are called perfect squares.

Observe the following perfect square numbers

Ex: (i) $9 = 3 \times 3$ (ii) $49 = 7 \times 7$ (iii) $1.44 = 1.2 \times 1.2$

(iv) $2.25 = 1.5 \times 1.5$ (v) $\frac{9}{16} = \frac{3}{4} \times \frac{3}{4}$ (vi) $\frac{4}{12.25} = \frac{2}{3.5} \times \frac{2}{3.5}$

In case (i) and (ii) we have noticed the perfect square numbers 9 and 49 are integers. The general form of such perfect square numbers is $m = n \times n$ (where m and n are integers).

In case (iii), (iv) and (v), (vi) the perfect square numbers are not integers. Hence, they are not square numbers.

If an integer 'm' is expressed as n^2 where n is an integer then 'm' is a square number or 'm' is a square of 'n'.

Perfect square : A rational number that is equal to the square of another rational number.

Square number : An integer that is a square of another integer. Thus

"All square numbers are perfect squares" but all perfect squares may not be square numbers.

Ex: 2.25 is a perfect square number because it can be expressed as $2.25 = (1.5)^2 = 1.5 \times 1.5$, it is not square of an integer. Therefore, it is not a square number.

Is 42 a square number?

We know that $6^2 = 36$ and $7^2 = 49$, if 42 is a square number it must be the square of an integer. Which should be between 6 and 7. But there is no such integer between 6 and 7.

Therefore 42 is not a square number.

Observe the perfect squares in the given table

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Are there any other square numbers that exist other than the numbers shown in the table.

Do This:

1. Find the perfect squares between (i) 100 and 150 (ii) 150 and 200
2. Is 56 a perfect square? Give reasons?

6.1 Properties of square numbers :

Observe and fill the following table.

Number	Square
1	1
2	4
3	9
4	16
5	25
6	....
7	49
8	64
....	81
10	100

Number	Square
11	121
12	144
13	....
14	196
15	225
16	....
17	289
18	324
19	361
20	400

Number	Square
21	441
22	....
23	529
....	576
25	625
....	....
....	....
....	....
....	....
....	....

Observe the digits in the units place of the square numbers in the above table. Do you observe all these numbers end with 0, 1, 4, 5, 6 or 9 at units place, none of these end with 2, 3, 7 or 8 at units place. "That is the numbers that have 2, 3, 7 or 8 in the units place are not perfect squares." Can we say that all numbers end with 0, 1, 4, 5, 6 or 9, at unit place are square numbers? Think about it.

Try These:

1. Guess and give reason which of the following numbers are perfect squares. Verify from the above table.

   (i) 84 (ii) 108 (iii) 271 (iv) 240 (v) 529

Write the squares of 1, 9, 11, 19, 21

Have you noticed any relationship between the units digit of numbers and their squares?
It is observed that if a number has 1 or 9 in the units place, then the units digit in its square number is only 1.

If a number has 4 or 6 in the units place, then the units digit in its square is always 6

Similarly, explore the units digit of squares of numbers ending with 0,2,3, 5, 7 and 8.

Try These:

1. Which of the following have one in its units place?

   (i) 126^2 (ii) 179^2 (iii) 281^2 (iv) 363^2

2. Which of the following have 6 in the units place?

   (i) 116^2 (ii) 228^2 (iii) 324^2 (iv) 363^2

Think, Discuss and Write:

Vaishnavi claims that the square of even numbers are even and that of odd are odd. Do you agree with her? Justify.

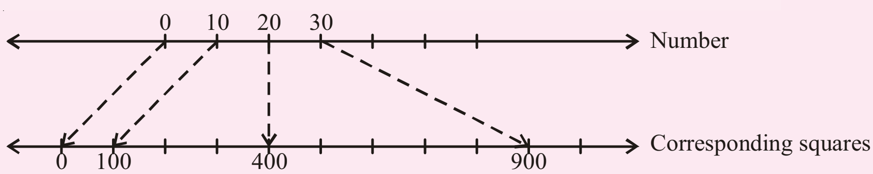
Observe and complete the table:

Numbers	**No.of digits in its square**	
	(Minimum)	(Maximum)
1-9	1	2
10-99	…….	4
100-999	5	…….
1009-9999	7	8
n digit	…….	…….

Try These:

1. Guess, How many digits are there in the squares of

   (i) 72 (ii) 103 (iii) 1000

2. 

   27 lies between 20 and 30

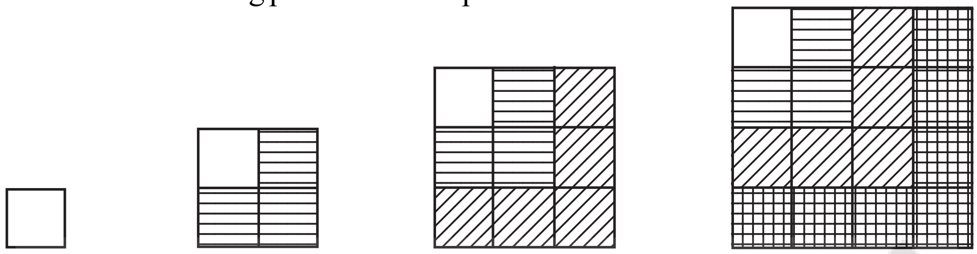
   27^2 lies between 20^2 and 30^2

   Now find what would be 27^2 from the following perfect squares.

   (i) 329 (ii) 525 (iii) 529 (iv) 729

6.2. Interesting patterns in square:

1. Observe the following pattern and complete.



$1 = 1^2$ $\quad$ $1+3 = 4 = 2^2$ $\quad$ $1+3+5 = 9 = 3^2$ $\quad$ $1+3+5+7 = 16 = 4^2$

$1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$

$1 + 3 + 5 + 7 + 9 + 11 =$ ………. $= (\ )^2$

$1 + 3 + 5 + 7 + 9 + 11 + 13 =$ ………. $= (\ )^2$

From this, we can generalize that the sum of first ‘n’ odd natural numbers is equal to ‘n^2’.

2. Observe the following pattern and supply the missing numbers

$(11)^2 = 121$

$(101)^2 = 10201$

$(1001)^2 = 1002001$

$(10001)^2 =$ …………..

$(1000001)^2 =$ ……………

3. Observe the pattern and complete it

1^2	=	1
11^2	=	1 2 1
111^2	=	1 2 3 2 1
1111^2	=	1 2 3 4 3 2 1
11111^2	=	……………………
111111^2	=	……………………

> A palindrome is a word; phrase, a sentence or a numerical that reads the same forward or backward.
> Ex. NOON, MALAYALAM, MADAM
> Rats live on no evil star.
> 15651

These numbers are called palindromic numbers or numerical palindrome

4. From the following pattern find the missing numbers

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + 12^2 = 13^2$

$4^2 + 5^2 + (\ )^2 = 21^2$

$5^2 + (\ )^2 + 30^2 = (\ )^2$

$6^2 + 7^2 + (\ )^2 = (\ )^2$

> Observe the sum of the squares:
> Do you find any relation between the bases of squares ?
> How the base of the third number is related to the base of first and second square numbers?
> How the base of the resultant square number is related to the base of the third square number?

5. Find the missing numbers using the given pattern

$3^2 = 9 = 4 + 5$ $\qquad \left(\frac{3^2-1}{2} + \frac{3^2+1}{2}\right)$

$5^2 = 25 = 12 + 13$ $\qquad \left(\frac{5^2-1}{2} + \frac{5^2+1}{2}\right)$

$7^2 = 49 = 24 + 25$ $\qquad (\qquad + \qquad)$

$11^2 = 121 = \ldots.. + \ldots..$ $\qquad \left(\frac{11^2-1}{2} + \frac{11^2+1}{2}\right)$

$15^2 = 225 = \ldots.. + \ldots..$ $\qquad (\qquad + \qquad)$

From this, we can conclude that the square of any odd number say n can be expressed as the sum of two consecutive numbers as $\left(\frac{n^2-1}{2} + \frac{n^2+1}{2}\right)$

6. Numbers between successive square numbers:

Observe and complete the following table

Successive squares	Numbers between the successive square numbers	Relation
$1^2 = 1;\ 2^2 = 4$	2, 3 (2 numbers lies between 1 and 4)	2×Base of first number 1, (2×1=2)
$2^2 = 4;\ 3^2 = 9$.	5, 6, 7, 8 (4 numbers lies between 4 and 9)	2×Base of first number 2, (2×2=4)
$3^2 = 9;\ 4^2 = 16$	10,11,12,13,14,15 (6 numbers lies) between 9 and 16)	2×Base of first number 3 (2×3=6)
$4^2 = 16;\ 5^2 = 25$	....	2×Base of first number 4, (2×4=8)
$5^2 = 25;\ 6^2 = 36$	....	....
....	....	....

From the above table have you observed any relation between the successive square numbers and numbers between them?

With the help of the above table, try to find the number of non square numbers between n^2 and $(n + 1)^2$. There are '2n' non square numbers between n^2 and $(n + 1)^2$.

Do This:

1. How many non perfect square numbers are there between 9^2 and 10^2 ?
2. How many non perfect square numbers are there between 15^2 and 16^2?

Try These:

Rehan says there are 37 non square numbers between 9^2 and 11^2. Is he right? Give your reason.

Exercise - 6.1

1. What will be the units digit of the square of the following numbers?
   (i) 39 (ii) 297 (iii) 5125 (iv) 7286 (v) 8742
2. Which of the following numbers are perfect squares?
   (i) 121 (ii) 136 (iii) 256 (iv) 321 (v) 600
3. The following numbers are not perfect squares. Give reasons?
   (i) 257 (ii) 4592 (iii) 2433 (iv) 5050 (v) 6098
4. Find whether the square of the following numbers are even or odd ?
   (i) 431 (ii) 2826 (iii) 8204 (iv) 17779 (v) 99998
5. How many numbers lie between the square of the following numbers
   (i) 25; 26 (ii) 56; 57 (iii) 107;108
6. Without adding, find the sum of the following numbers
   (i) $1 + 3 + 5 + 7 + 9 =$
   (ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 =$
   (iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 =$

6.3 Pythagorean triplets:

Consider the following

(i) $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

(ii) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

The numbers (3, 4, 5) and (5, 12, 13) are some examples for Pythagorean triplets.

Generally a, b, c are the positive integers. If $a^2 + b^2 = c^2$ then (a, b, c) are said to be pythagorean triplet.

If there are no common factors other than '1' among a,b,c then the triplet (a,b,c) is called primitive triplet.

Do This

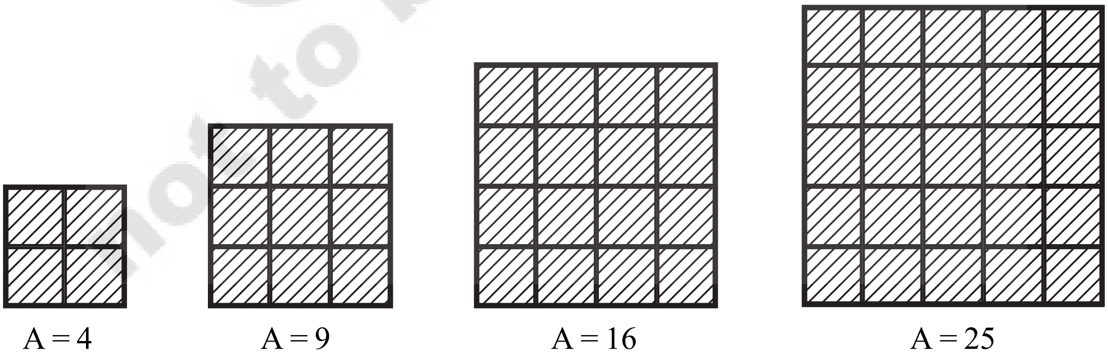
1. Check whether the following numbers form Pythagorean triplet

   (i) 2, 3, 4 (ii) 6, 8, 10 (iii) 9, 10, 11 (iv) 8, 15, 17

2. Take a Pythagorean triplet. Write their multiples. Check whether these multiples form a Pythagorean triplet.

6.4 Square Roots

Observe the following squares and complete the table.



Area of the square (in cm^2) (A)	Side of the square (in cm) (S)
$4 = 2 \times 2$	2
$9 = 3 \times 3$	3
$16 = 4 \times 4$	______
$25 = 5 \times 5$	______

The number of unit squares in a row / column represents the side of a square.

Do you find any relation between the area of the square and its side?

We know that the area of the square = side × side = side^2

If the area of a square is 169 cm^2. What could be the side of the square?

Let us assume that the length of the side be 'x' cm.

$\Rightarrow$ $169 = x^2$

To find the length of the side, it is necessary to find a number whose square is 169.

We know that $169 = 13^2$. Then the length of the side = 13 cm.

Therefore, if a square number is expressed, as the product of two equal factors, then one the factors is called the square root of that square number. Thus, the square root of 169 is 13. It can be expressed as $\sqrt{169} = 13$ (symbol used for square root is $\sqrt{\ }$). Thus it is the inverse operation of squaring.

> 25 is the square of both 5 and –5.
> Therefore, the square root of 25 is 5 or –5.
> But in this chapter we are confined to the positive square root which is also called principal square root. It is written as
> $\therefore\ \sqrt{25} = 5$.

Example 1: $3^2 = 9$ therefore square root of 9 is 3($\sqrt{9} = 3$)

$4^2 = 16$ therefore square root of 16 is 4($\sqrt{16} = 4$)

$5^2 = 25$ therefore square root of 25 is 5($\sqrt{25} = 5$)

If $y^2 = x$ then square root of x is y $\left(\sqrt{x} = y\right)$

Example 2: 1. $\sqrt{4} = 2$ because $2^2 = 4$

2. $\sqrt{16} = 4$ because $4^2 = 16$

3. $\sqrt{225} = 15$ because $15^2 = 225$ etc.

Complete the following table:

Square	Square roots
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25}$ = ……..
$6^2 = 36$	$\sqrt{36}$ = ……..
7^2 = ……...	$\sqrt{\quad}$ = ……...
8^2 = ……..	$\sqrt{\quad}$ = ………
9^2 = ……..	$\sqrt{\quad}$ = ………
10^2 = ……..	$\sqrt{\quad}$ = ………

6.5 Finding the Square root through subtraction of successive odd numbers:

We know that, every square number can be expressed as a sum of successive odd natural numbers starting from 1.

Consider,
$$1+3 = 4 = 2^2$$
$$1+3+5 = 9 = 3^2$$
$$1+3+5+7 = 16 = 4^2$$
$$1+3+5+7+9 = 25 = 5^2$$

Finding square root is the reverse order of this pattern.

For example, find $\sqrt{49}$

Step 1: $49 - 1 = 48$ (Subtracting of first odd number)

Step 2: $48 - 3 = 45$ (Subtracting of 2^{nd} odd number)

Step 3: $45 - 5 = 40$ (Subtracting of 3^{rd} odd number)

Step 4: $40 - 7 = 33$

Step 5: $33 - 9 = 24$

Step 6: $24 - 11 = 13$

Step 7: $13 - 13 = 0$

Observe we know
$1+3+5+7+9+11+13 = 7^2 = 49$
$49-[1+3+5+7+9+11+13] = 0$
Hence 49 is a perfect square.

From 49, we have subtracted seven successive odd numbers starting from 1 and obtained zero (0) at 7^{th} step.

$$\therefore \sqrt{49} = 7$$

Note: If the result of this process is not zero then the given number is not a perfect square.

Do This:

(i) By repeated subtraction, find whether the following numbers are perfect squares or not?

(i) 55 (ii) 90 (iii) 121

It is easy to find the square roots of any square numbers by the above repeated subtraction process. But in case of bigger numbers such as 625, 729.......... it is time taking process. So, Let us try to find simple ways to obtain the square roots.

There are two methods of finding the square root of the given numbers. They are

(i) Prime factorization method

(ii) Division method

6.6 Finding the Square Root Through Prime Factorisation Method:

Let us find the square root of 484 by prime factorization method.

2	484
2	242
11	121
11	11
	1

Step 1: Resolve the given number 484 into prime factors, we get

$484 = 2 \times 2 \times 11 \times 11$

Step 2: Make pairs of equal factors, we get

$484 = (2 \times 2) \times (11 \times 11)$

$484 = (2 \times 11) \times (2 \times 11) = (2 \times 11)^2$

$\sqrt{484} = \sqrt{(2\times 11)^2}$

$= 2 \times 11$

$= 22$

Step 3: Choosing one factor out of every pair

By doing so, we get

$\sqrt{484} = 2 \times 11 = 22$

Therefore, the square root of 484 is 22.

Now we will see some more examples

Example 3 : Find the square root of 1296 by Prime Factorization

Solution : Resolving 1296 into Prime factors, we get

$1296 = (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (3 \times 3)$

$\sqrt{1296} = 2 \times 2 \times 3 \times 3$

$\therefore \sqrt{1296} = 36$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

Example 4 : Find the square root of 2025

Solution : Resolving 2025 into Prime factors, we get

$2025 = (3 \times 3) \times (3 \times 3) \times (5 \times 5)$

$\sqrt{2025} = 3 \times 3 \times 5$

$\therefore \sqrt{2025} = 45$

5	2025
5	405
3	81
3	27
3	9
3	3
	1

Example 5: Find the smallest number by which 720 should be multiplied to get a perfect square.

Solution : Resolving 720 into Prime factors, we get

2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

$720 = (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times 5$

We see that 2, 2, 3 exist in pairs, while 5 is alone

So, we should multiply the given number by 5 to get a perfect square.

Therefore, the perfect square so obtained is

$720 \times 5 = 3600$

Example 6: Find the smallest number by which 6000 should be divided to get a perfect square and also find the square root of the resulting number.

Solution : Resolving 6000 into Prime factors, we get

2	6000
2	3000
2	1500
2	750
3	375
5	125
5	25
5	5
	1

$6000 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{5 \times 5} \times 5$

We can see that, 2, 2, and 5 exists in pairs while 3 and 5 do not exists in pairs

So, we must divide the given number by $3 \times 5 = 15$

Therefore perfect square obtained $= 6000 \div 15 = 400$

$400 \quad = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

The square root of 400 is

2	400
2	200
2	100
2	50
5	25
5	5
	1

$$\sqrt{400} = \sqrt{(2\times2)\times(2\times2)\times(5\times5)}$$
$$= 2 \times 2 \times 5$$
$$= 20$$

Exercise - 6.2

1. Find the square roots of the following numbers by Prime factorization method.

(i) 441 (ii) 784 (iii) 4096 (iv) 7056

2. Find the smallest number by which 3645 must be multiplied to get a perfect square.
3. Find the smallest number by which 2400 is to be multiplied to get a perfect square and also find the square root of the resulting number.
4. Find the smallest number by which 7776 is to be divided to get a perfect square.
5. 1521 trees are planted in a garden in such a way that there are as many trees in each row as there are rows in the garden. Find the number of rows and number of trees in each row.
6. A school collected ₹ 2601 as fees from its students. If fee paid by each student and number students in the school were equal, how many students were there in the school?
7. The product of two numbers is 1296. If one number is 16 times the other, find the two numbers?
8. 7921 soldiers sat in an auditorium in such a way that there are as many soldiers in a row as there are rows in the auditorium. How many rows are there in the auditorium?
9. The area of a square field is 5184 m^2. Find the area of a rectangular field, whose perimeter is equal to the perimeter of the square field and whose length is twice of its breadth.

6.7 Finding square root by division method :

We have already discussed the method of finding square root by prime factorisation method. For large numbers, it becomes lengthy and difficult. So, to overcome this problem we use division method.

Let us find the square root of 784 by division method.

$$\left|\, 7\overline{84} \,\right|$$

Step 1 : Pair the digits of the given number, starting from units place to the left. Place a bar on each pair.

$$\begin{array}{r|c|l} 2 & 7\overline{84} & 2 \end{array}$$

Step 2 : Find the largest number whose square is less than or equal to the first pair or single digit from left (i.e. 2). Take this number as the divisor and the quotient.

$$\begin{array}{r|l|l} 2 & 7\overline{84} & 2 \\ & 4 & \\ \hline & 3 & \end{array}$$

Step 3 : Subtract the product of the divisor and quotient ($2 \times 2 = 4$) from first pair or single digit (i.e. $7 - 4 = 3$)

$$\begin{array}{r|r|l} 2 & 7\overline{84} & 2 \\ & -4 & \\ \hline & 384 & \end{array}$$

Step 4 : Bring down the second pair (i.e. 84) to the right of the Remainder (i.e. 3). This becomes the new dividend (i.e. 384).

$$\begin{array}{r|r|l} 2 & 7\overline{84} & 2 \\ & -4 & \\ \hline 4\square & 384 & \end{array}$$

Step 5 : From the next possible divisor double the quotient (i.e $2 \times 2 = 4$) and write a box on its right.

2	$\overline{7}\,\overline{84}$	28
	−4	
4$\boxed{8}$	384	
	384	
	0	

Step 6 : Guess the largest possible digit to fill the box in such a way that the product of the new divisor and this digit is equal to or less than the new dividend (i.e. $48 \times 8 = 384$).

2	$\overline{7}\,\overline{84}$	28
	−4	
48	384	
	−384	
	0	

Step 7 : By subtracting, we get the remainder zero. The final quotient 28, is the square root

$\therefore \sqrt{784} = 28$

Think, Discuss and Write

Observe the following divisions, give reasons why $\boxed{8}$ in the divisor 48 is considered in the above example?

$$4)\,384\,(9 \quad 36 \quad 24 \quad 81 = 9^2$$

$$4)\,384\,(8 \quad 32 \quad 64 \quad 64 = 8^2$$

$$4)\,384\,(7 \quad 28 \quad 104 \quad 49 = 7^2$$

Now, we will see some more examples.

Example 7: Find the square root of 1296

Solution: Step 1

$\overline{12}\,\overline{96}$

Step 2

3	$\overline{12}\,\overline{96}$	3
	9	

Step 3

3	$\overline{12}\,\overline{96}$	3
	-9	

Step 4

3	$\overline{1296}$	3
	-9	
6	396	

Step 5

3	$\overline{12}\,\overline{96}$	36
	-9	
66	396	
	- 396	
	0	

Observe

$$6)\,396\,(6 \quad 36 \quad 36 \quad 36 = 6^2 \quad 0$$

$\therefore \sqrt{1296} = 36$

Example 8: Find the square root of 8281

Solution:

9	$\overline{82}\ \overline{81}$	91
	-81	
181	181	
	-181	
	0	

Therefore $\sqrt{8281} = 91$

Observe

$$\begin{array}{r|l}
18 & 181 \;(1 \\
 & 18 \\ \hline
 & 1 \\
 & 1 = 1^2 \\ \hline
 & 0
\end{array}$$

Example 9: Find the greatest four digit number which is a perfect square

Solution: Greatest four digit number is 9999

We find square root of 9999 by division method.

The remainder 198 shows that it is less than 9999 by 198

9	$\overline{99}\ \overline{99}$	99
	-81	
189	18 99	
	-17 01	
	1 98	

This means if we subtract 198 from 9999, we get a perfect square.

$\therefore$ 9999 – 198 = 9801 is the required perfect square.

Example 10: Find the least number which must be subtracted from 4215 to make it a perfect square?

Solution: We find by division method that

The remainder is 119

6	$\overline{42}\ \overline{15}$	64
	-36	
1	6 15	
124	-4 96	
	1 19	

This means, if we subtract 119 from 4215. We get a perfect square.

Hence, the required least number is 119.

6.8 Square roots of decimals using division method :

Let us begin with the example $\sqrt{17.64}$

Step 1: Place the bars on the integral part of the number i.e. 17 in the usual manner. Place the bars on every pair of decimal part from left to right

$\overline{17}.\overline{64}$

Step 2: Find the largest number (i.e. 4) whose square is less than or equal to the first pair of integral part (i.e. 17). Take this number 4 as a divisor and the first pair 17 as the dividend. Get the remainder as 1.

4	$\overline{17}\ .\ \overline{64}$	4
	-16	
	1	

Divide and get the remainder i.e. 1

Step 3: Write the next pair (i.e. 64) to the right of the remainder to get 164, which becomes the new dividend.

4	$\overline{17}.\overline{64}$	4
	-16	
	1.64	

Step 4: Double the quotient ($2 \times 4 = 8$) and write it as 8 in the box on its right. Since 64 is the decimal part so, put a decimal point in the quotient (i.e. 4)

4	$\overline{17}.\overline{64}$	4
	-16	
8□	- 164	

Step 5: Guess the digit to fill the box in such a way that the product of the new divisor and the digit is equal to or less than the new dividend 164. In this case the digit is 2. Divide and get the remainder.

4	$\overline{17}.\overline{64}$	4.2
	-16	
8$\boxed{2}$	164	
	-164	
	0	

Step 6: Since the remainder is zero and no pairs left.

$\sqrt{17.64} = 4.2$

Now, let us see some more examples.

Example 11: Find the square root of 42.25 using division method.

Solution: Step 1 : $\overline{42}\ .\ \overline{25}$

Step 2 :

6	$\overline{42}.\overline{25}$	6
	-36	
	6	

Step 3 :

6	$\overline{42}.\overline{25}$	6.5
6	-36	
125	625	
	-625	
	0	

$\therefore \sqrt{42.25} = 6.5$.

Example 12: Find $\sqrt{96.04}$

Solution:

9	$\overline{96}.\overline{04}$	9.8
9	-81	
188	1504	
	-1504	
	0	

Therefore $\sqrt{96.04} = 9.8$

6.9 Estimating square roots of non perfect square numbers :

So far we have learnt the method for finding the square roots of perfect squares. If the numbers are not perfect squares, then we will not be able to find the exact square roots. In all such cases we atleast need to estimate the square root.

Let us estimate the value of $\sqrt{300}$ to the nearest whole number.

300 lies between two perfect square numbers 100 and 400

$\therefore \quad 100 < 300 < 400$

$10^2 < 300 < 20^2$

i.e. $\quad 10 < \sqrt{300} < 20$

But still we are not very close to the square number. we know that $17^2 = 289,\ 18^2 = 324$

Therefore $\quad 289 < 300 < 324$

$17 < \sqrt{300} < 18$

As 289 is more closer to 300 than 324.

The approximate value of $\sqrt{300}$ is 17.

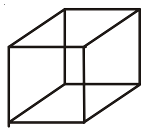
Exercise - 6.3

1. Find the square roots of the following numbers by division method.
   (i) 1089 (ii) 2304 (iii) 7744 (iv) 6084 (v) 9025
2. Find the square roots of the following decimal numbers.
   (i) 2.56 (ii) 18.49 (iii) 68.89 (iv) 84.64
3. Find the least number that is to be subtracted from 4000 to make it perfect square
4. Find the length of the side of a square whose area is 4489 sq.cm.
5. A gardener wishes to plant 8289 plants in the form of a square and found that there were 8 plants left. How many plants were planted in each row?
6. Find the least perfect square with four digits.
7. Find the least number which must be added to 6412 to make it a perfect square?
8. Estimate the value of the following numbers to the nearest whole number
   (i) $\sqrt{97}$ (ii) $\sqrt{250}$ (iii) $\sqrt{780}$

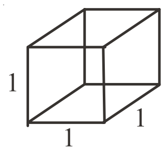
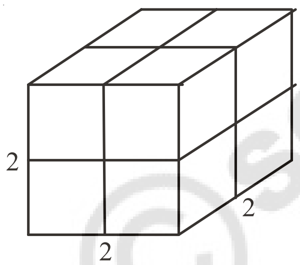
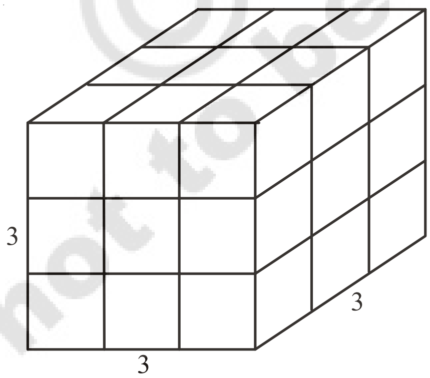
Cubes and Cube Roots

6.10 Cubic Numbers

We know that a cube is a solid figure with six identical squares.



Now let us make cubic shapes using these unit cubes

S.No.	Figure	Length of the side	No.of unit cubes used
1		1	1
2		2	8
3		3	27

Can you make next cube? Guess how many unit cubes are required to make a cube whose side is 5 units?

So, we require 1, 8, 27, 64 …….. unit cubes to make cubic shapes.

These numbers 1, 8, 27, 64 …… are called cubic numbers or perfect cubes.

As $1 = 1 \times 1 \times 1 = 1^3$

$8 = 2 \times 2 \times 2 = 2^3$

$27 = 3 \times 3 \times 3 = 3^3$

$64 = \ldots\ldots.. \times \ldots\ldots \times \ldots\ldots =$

So, a cube number is obtained when a number is multiplied by itself for three times.

That is, cube of a number 'x' is $x \times x \times x = x^3$

Is 49 a cube number ? No, as $49 = 7 \times 7$ and there is no natural number which when multiplied by itself three times gives 49. We can also see that $3 \times 3 \times 3 = 27$ and $4 \times 4 \times 4 = 64$. This shows that 49 is not a perfect cube.

Try These

1. Is 81 a perfect cube?
2. Is 125 a perfect cube?

Observe and complete the following table.

Number	Cube
1	$1^3 = 1 \times 1 \times 1 = 1$
2	$2^3 = 2 \times 2 \times 2 = 8$
3	$3^3 = 3 \times 3 \times 3 = 27$
4	$4^3 = 4 \times 4 \times 4 = 64$
5	$5^3 = 5 \times 5 \times 5 = 125$
6	$6^3 = 6 \times 6 \times 6 = \ldots\ldots\ldots$
7	$7^3 = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots..$
8	$8^3 = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots..$
9	$9^3 = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots..$
10	$10^3 = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots..$

Think, Discuss and Write

(i) How many perfect cube numbers are present between 1 and 100, 1 and 500, 1 and 1000?

(ii) How many perfect cubes are there between 500 and 1000 ?

Following are the cubes of numbers from 11 to 20

Number	Cube
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000

Do you find anything interesting in the sum of the digits in the cubes of 17 and 18 ?

From the table, we can see that cube of an even number is always an even number. Do you think the same is true for odd numbers also?

We can also observe that, if a number has 1 in the units place, then its cube ends with 1.

Similarly, what can you say about the units digit of the cube of a number having 0, 4, 5, 6 or 9 as the units digit?

Try These:

1. Find the digit in units place of each of the following numbers.

   (i) 75^3 (ii) 123^3 (iii) 157^3 (iv) 198^3 (v) 206^3

6.11 Some interesting patterns:

1. Adding consecutive odd numbers

   Observe the following patterns.

$$1 = 1 = 1^3$$
$$3 + 5 = 8 = 2^3$$
$$7 + 9 + 11 = 27 = 3^3$$
$$13 + 15 + 17 + 19 = \ldots\ldots = \ldots\ldots.$$

Can you guess how many next consecutive odd numbers will be needed to obtain the sum as 5^3?

2. Consider the following pattern

$2^3 - 1^3 = 1 + 2 \times 1 \times 3 = 7$

$3^3 - 2^3 = 1 + 3 \times 2 \times 3 = 19$

$4^3 - 3^3 = 1 + 4 \times 3 \times 3 = 37$

$5^3 - 4^3 = \ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots$

Using the above pattern find the values of the following

(i) $10^3 - 9^3$ (ii) $15^3 - 14^3$ (iii) $26^3 - 25^3$

3. Observe the following pattern and complete it

$1^3 = 1^2$

$1^3 + 2^3 = (1 + 2)^2 = (3)^2$

$1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2 = (\ )^2$

$1^3 + 2^3 + 3^3 + 4^3 = (\underline{\qquad})^2$

$\ldots\ldots\ldots\ldots\ldots\ldots = (1 + 2 + 3 + \ldots + 10)^2$

Hence we can generalize that,

The sum of the cubes of first 'n' natural numbers is equal to the square of their sum.

i.e. $1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = (1 + 2 + 3 + \ldots + n)^2$.

6.12 Cubes and their Prime Factors:

Consider the numbers 64 and 216

Resolving 64 and 216 into prime factors

$64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

In both these cases each factor appears three times. That is the prime factors can be grouped in triples.

Thus, if a number can be expressed as a product of three equal factors then it is said to be a perfect cube or cubic number.

Is 540 a perfect cube?

2	540
2	270
3	135
3	45
3	15
5	5
	1

Resolving 540 into prime factors, we get

$540 = 2 \times 2 \times \underline{3 \times 3 \times 3} \times 5$

Here, 2 and 5 do not appear in groups of three.

Hence 540 is not a perfect cube.

Do These

1. Which of the following are perfect cubes?

(i) 243 (ii) 400 (iii) 500 (iv) 512 (v) 729

Example 13: What is a smallest number by which 2560 is to be multiplied so that the product is a perfect cube?

2	2560
2	1280
2	640
2	320
2	160
2	80
2	40
2	20
2	10
	5

Solution : Resolving 2560 into prime factors, we get

$2560 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 5$

The Prime factor 5 does not appear in a group of three.

So, 2560 is not a perfect cube.

Hence, the smallest number by which it is to be multiplied to make it a perfect cube is $5 \times 5 = 25$

Example 14: What is the smallest number by which 1600 is to be divided. so that the quotient is a perfect cube?

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
	5

Solution : Resolving 1600 into prime factors, we get

$1600 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 5 \times 5$

The prime factor 5 does not appear in a group of three factors. So, 1600 is not a perfect cube.

Hence, the smallest number which is to be divided to make it a perfect cube is $5 \times 5 = 25$

Exercise - 6.4

1. Find the cubes of the following numbers
   (i) 8 (ii) 16 (iii) 21 (iv) 30
2. Test whether the given numbers are perfect cubes or not.
   (i) 243 (ii) 516 (iii) 729 (iv) 8000 (v) 2700
3. Find the smallest number by which 8788 must be multiplied to obtain a perfect cube?
4. What smallest number should 7803 be multiplied with so that the product becomes a perfect cube?
5. Find the smallest number by which 8640 must be divided so that the quotient is a perfect cube?
6. Ravi made a cuboid of plasticine of dimensions 12cm, 8cm and 3cm. How many minimum number of such cuboids will be needed to form a cube?
7. Find the smallest prime number dividing the sum $3^{11} + 5^{13}$.

6.13 Cube roots

We know that, we require 8 unit cubes to form a cube of side 2 units ($2^3 = 8$) similarly, we need 27 unit cubes to form a cube of side 3 units ($3^3 = 27$)

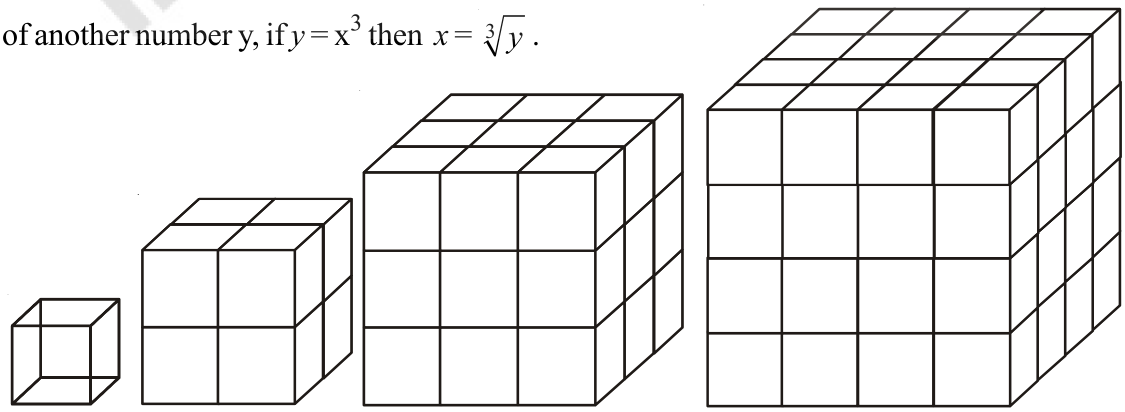
Suppose, a cube is formed with 64 unit cubes. Then what could be the side of the cube?

Let us assume, the length of the side to be 'x'

$\therefore \quad 64 = x^3$

To find the side of a cube, it is necessary to find a number whose cube is 64.
Therefore, finding the number whose cube is known is called finding the cube root. It is the inverse operation of cubing.
As, $4^3 = 64$ then 4 is called cube root of 64

We write $\sqrt[3]{64} = 4$. The symbol $\sqrt[3]{\ }$ denotes cube root. Hence, a number 'x' is the cube root of another number y, if $y = x^3$ then $x = \sqrt[3]{y}$.



(1 unit cube) (2 unit cube) (3 unit cube) (4 unit cube)

Complete the following table:

Cubes			Cube roots		
1^3	=	1	$\sqrt[3]{1}$	=	1
2^3	=	8	$\sqrt[3]{8}$	=	2
3^3	=	27	$\sqrt[3]{27}$	=	3
4^3	=	64	$\sqrt[3]{64}$	=	4
5^3	=	125	$\sqrt[3]{125}$	=	5
6^3	=	…...	$\sqrt[3]{\ }$	=	6
7^3	=	…….	$\sqrt[3]{\ }$	=	7
8^3	=	……	$\sqrt[3]{\ }$	=	8
……	=	………..	……..	=	………..
……	=	………..	…………	=	………

6.14 Finding cube root through Prime Factorization method:

Let us find the cube root of 1728 by prime factorization method.

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

Step1 : Resolve the given number 1728 into prime factors.

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Step2 : Make groups of three equal factors:

$1728 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

Step 3: Choose one factor from each group and multiply by doing so, we get

$\sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

$\therefore \sqrt[3]{1728} = 2 \times 2 \times 3 = 12$

Let us see some more examples

Example 15: Find the cube root of 4096 ?

Solution : Resolving 4096 into Prime Factors, we get

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

$4096 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\sqrt[3]{4096} = 2 \times 2 \times 2 \times 2 = 16$

$\therefore \sqrt[3]{4096} = 16$

6.15 Estimating the cube root of a number

If we know that, the given number is a cube number then to find its cube root the following method can be used.

Let us find the cube root of 9261 through estimation.

Step1: Start making groups of three digits starting from the unit place.

i.e.	9	261
	second group	first group

Step 2: First group i.e. 261 will give us the units digit of the cube root. As 261 ends with 1, its cube root also ends with 1. So, the units place of the cube root will be 1.

Step 3: Now, take second group i.e. 9.

We know that $2^3 < 9 < 3^3$.

As the smallest number is 2, it becomes the tens place of the required cube root

$\therefore \sqrt[3]{9261} = 21$

Exercise - 6.5

1. Find the cube root of the following numbers by prime factorization method.

   (i) 343 (ii) 729 (iii) 1331 (iv) 2744

2. Find the cube root of the following numbers through estimation?

   (i) 1512 (ii) 2197 (iii) 3375 (iv) 5832

3. State true or false?

   (i) Cube of an even number is an odd number

   (ii.) A perfect cube may end with two zeros

   (iii) If a number ends with 5, then its cube ends with 5

   (iv) Cube of a number ending with zero has three zeros at its right

   (v) The cube of a single digit number may be a single digit number.

   (vi) There is no perfect cube which ends with 8

   (vii) The cube of a two digit number may be a three digit number.

4. Find the two digit number which is a square number and also a cubic number.

What we have discussed

- Estimating number of digits in square of a number.
- Square numbers written in different patterns.
- a, b, c are positive integers and if $a^2 + b^2 = c^2$ then (a, b, c) are said to be Pythagorean triplets.
- Finding the square roots by prime factorisation and division method.
- Square root is the inverse operation of squaring.
- Estimating square roots of non perfect square numbers.
- If a number is multiplied three times by itself is called cube number.
- Finding cube root by prime factorisation method.
- Estimating cube roots of a number.
- The square of integer is a integer and a square number, where as square of rational number is a perfect square.

Eternal triangle

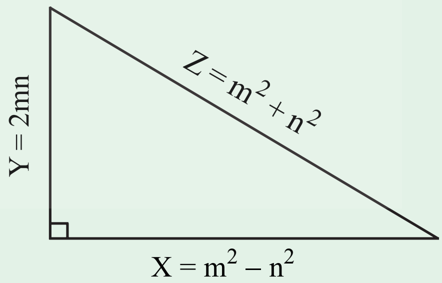
The formulae that will give integral sides of a right - angled triangle have been known since the time of Diophantus and the early Greeks. They are

one leg $X = m^2 - n^2$

second leg $Y = 2mn$

Hypotenuse $Z = m^2 + n^2$

The numbers m and n are integers which may be arbitarily selectus.



Example

m	n	$X = m^2 - n^2$	$Y = 2mn$	$Z = m^2 + n^2$
2	1	3	4	5
3	2	5	12	13
5	2	21	20	29
4	3	7	24	25
4	1	15	8	27