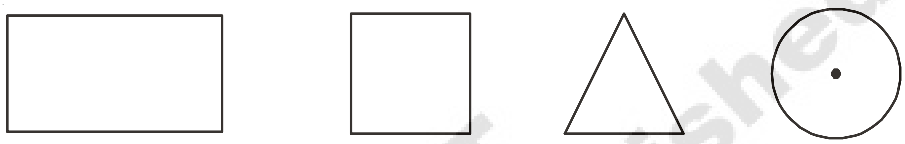
10 Surface Areas and Volumes

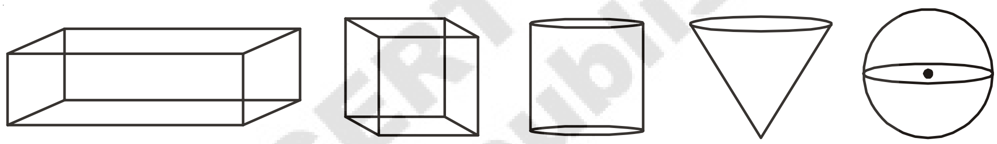
10.1 Introduction

Observe the following figures

(a)



(b)

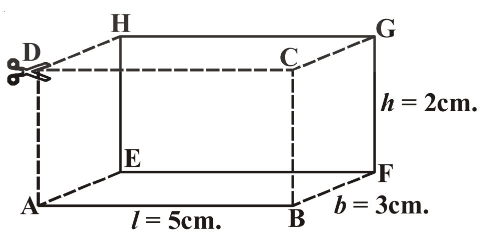


Have you noticed any differences between the figures of group (a) and (b)?

From the above, figures of group (a) can be drawn easily on our note books. These figures have length and breadth only and are named as two dimensional figures or 2-D objects. In group (b) the figures, which have length, breadth and height are called as three dimensional figures or 3-D objects. These are called solid figures. Usually we see solid figures in our surroundings. You have learned about plane figures and their areas. We shall now learn to find the surface areas and volumes of 3-dimensional objects such as cylinders, cones and spheres.

10.2 Surface area of Cuboid

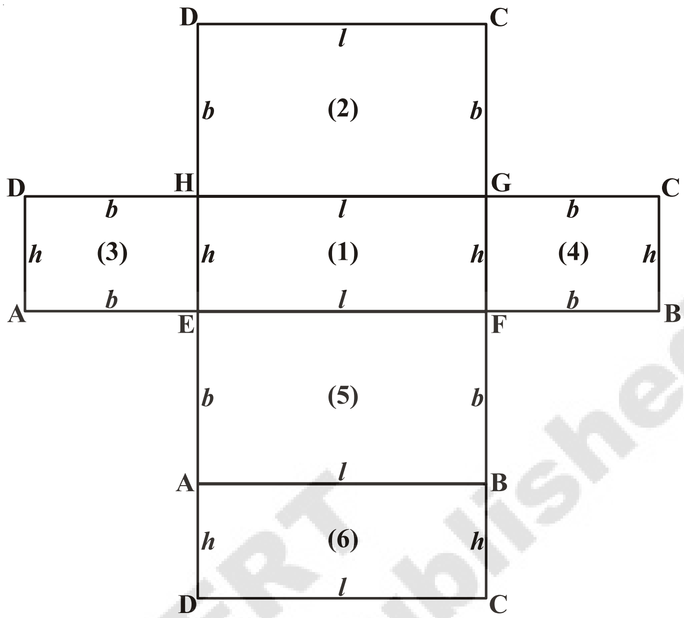
Observe the cuboid and find how many faces it has? How many corners and how many edges it has? See the faces are surfaces? Which pair of faces are equal in size? Do you get any idea to find the surface area of the cuboid?



Now let us find the surface area of a cuboid.

In the above figure length (l) = 5 cm; breadth (b) = 3 cm; height (h) = 2 cm

If we cut and open the given cuboid along CD, ADHE and BCGF. The figure we obtained is shown below:



This shows that the surface area of a cuboid is made up of six rectangles of three identical pairs of rectangles. To get the total surface area of cuboid, we have to add the areas of all six rectangular faces. The sum of these areas gives the total surface area of a cuboid.

Area of the rectangle EFGH = $l \times h = lh$ .....(1)

Area of the rectangle HGCD = $l \times b = lb$ .....(2)

Area of the rectangle AEHD = $b \times h = bh$ .....(3)

Area of the rectangle FBCG = $b \times h = bh$ .....(4)

Area of the rectangle ABFE = $l \times b = lb$ .....(5)

Area of the rectangle DCBA = $l \times h = lh$ .....(6)

On adding the above areas, we get the surface area of cuboid.

Surface Area of a cuboid = Areas of (1) + (2) + (3) + (4) + (5) + (6)

$= lh + lb + bh + bh + lb + lh$

$= 2\,lb + 2lh + 2bh$

$= 2(lb + bh + lh)$

(1), (3), (4), (6) are lateral surfaces of the cuboid

Lateral Surface Area of a cuboid = Area of (1) + (3) + (4) + (6)

$= lh + bh + bh + lh$

$= 2lh + 2bh$

$= 2h\,(l + b)$

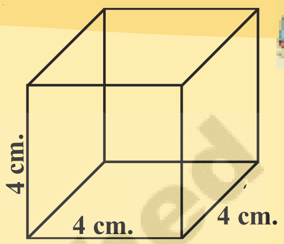
Now let us find the surface areas of cuboid for the above figure. Thus total surface area is 62 cm.^{2} and lateral surface area is 32 cm.^{2}.

Try This

Take a cube of edge '*l*' cm. and cut it as we did in the previous activity and find total surface area and lateral surface area of cube.

Do This

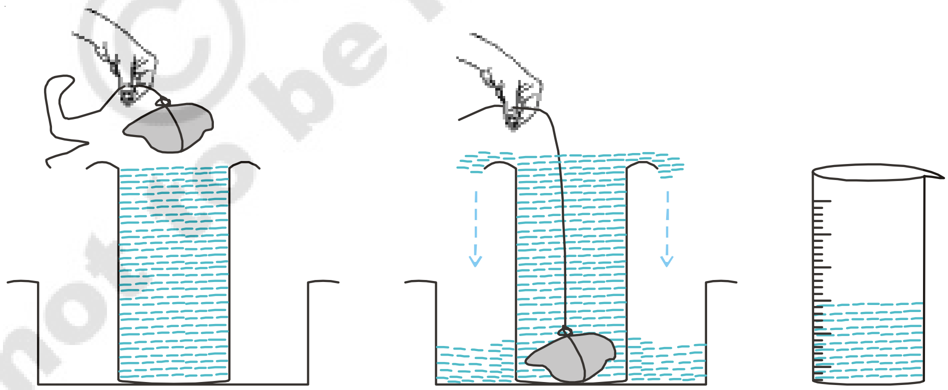
1. Find the total Surface area and lateral surface area of the Cube with side 4 cm. By using the formulae deduced in above try this.
2. Each edge of a cube is increased by 50%. Find the percentage increase in the surface area.



10.2.1 Volume

To recall the concept of volume, Let us do the following activity.

Take a glass jar, place it in a container. Fill the water in glass jar up to the its brim. Slowly drop a solid object (a stone) in it. Some of the water from the jar will overflow into the container. Take the overflowed water into measuring jar. It gives an idea of space occupied by a solid object called volume.

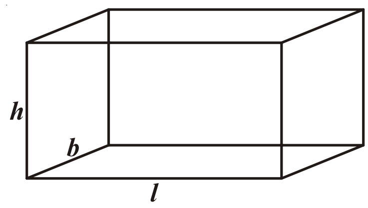


Every object occupies some space, the space occupied by an object is called its volume. Volume is measured in cubic units.

10.2.2 Capacity of the container

If the object is hollow, then interior is empty and it can be filled with air or any other liquid, that will take the shape of its container. Volume of the substance that can fill the interior is called the capacity of the container.

Volume of a Cuboid : Cut some rectangles from a cardboard of same dimensions and arrange them one over other. What do you say about the shape so formed?



The shape is a cuboid.

Now let us find volume of a cuboid.

Its length is equal to the length of the rectangle, and breadth is equal to the breadth of the rectangle.

The height up to which the rectangles are stacked is the height of the cuboid is 'h'

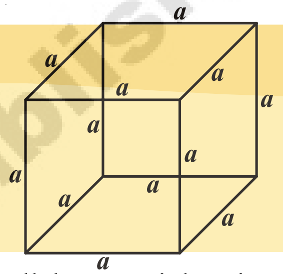
Space occupied by the cuboid = Area of plane region occupied by rectangle × height

Volume of the cuboid = $l\,b \times h = l\,bh$

$\therefore$ Volume of the cuboid = $l\,bh$

Where l, b, h are length, breadth and height of the cuboid.

TRY THESE



(a) Find the volume of a cube whose edge is 'a' units.

(b) Find the edge of a cube whose volume is 1000 cm^3.

Consider that cuboid and cube are the solids. Do we call them as right prisms? You have observed that cuboid and cube are also called right prisms as their lateral faces are rectangle and perpendicular to base.

We know that the volume of a cuboid is the product of the area of its base and height.

Remember that volume of the cuboid = Area of base × height

$= lb \times h$

$= lbh$

In cube $= l = b = h = s$ (All the dimensions are same)

volume of the cube $= s^2 \times s$

$= s^3$

It is natural to guess that the formula for the volume of a cuboid should hold good for all right prisms.

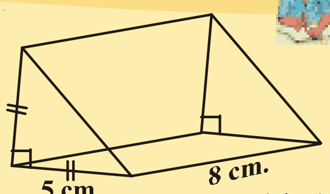
Hence volume of right prism = Area of the base × height

In particular, if the base of a right prism is an equilateral triangle its volume is $\frac{\sqrt{3}}{4}a^2 \times h$ cu.units.

Where, 'a' is the length of each side of the base and 'h' is the height of the prim.

DO THESE

1. Find the volume of cuboid if $l = 12$ cm., $b = 10$ cm. and $h = 8$ cm.
2. Find the volume of cube, if its edge is 10 cm.
3. Find the volume of isosceles right angled triangular prism in (fig. 1).



(Fig 1)

Like prism, pyramid is also a three dimentional solid figure. This figure has fascinated human beings from the ancient times. You might have read about pyramids of Egypt, which are, one of the seven wonders of the world. They are remarkable accurate examples of pyramids on square bases. How are they built? It is a mystery. No one really knows that how these massive structures were built.

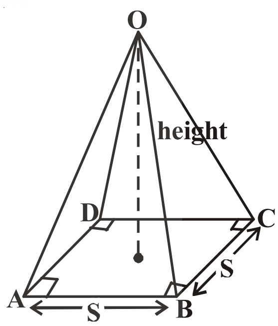
Can you draw the shape of a pyramid?

What is the difference you have observed between the prism and pyramid?

What do we call a pyramid of square base?

Here OABCD is a square pyramid of side 'S' units and height 'h' units.

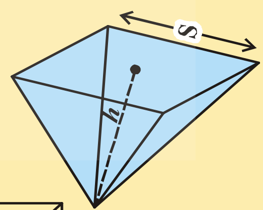
Can you guess the volume of a square pyramid in terms of volume of cube if their bases and height are equal?

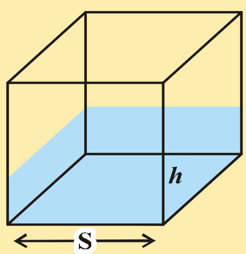


ACTIVITY

Take the square pyramid and cube containers of same base and with equal heights.

Fill the pyramid with a liquid and pour into the cube (prism) completely. How many times it takes to fill the cube? From this, what inference can you make?





Thus volume of pyramid

$$= \frac{1}{3} \text{ of the volume of right prism.}$$

$$= \frac{1}{3} \times \text{Area of the base} \times \text{height.}$$

Note : A Right prism has bases perpendicular to the lateral edges and all lateral faces are rectangles.

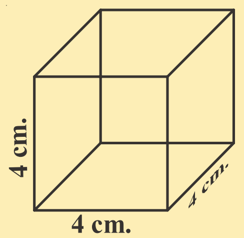
DO THESE

1. Find the volume of a pyramid whose square base is 10 cm. and height 8 cm.
2. The volume of cube is 1200 cubic cm. Find the volume of square pyramid of the same height.

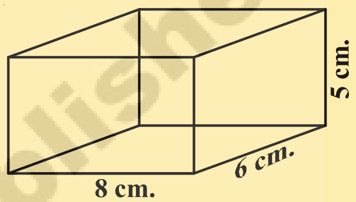
EXERCISE - 10.1

1. Find the later surface area and total surface area of the following right prisms.

(i)



(ii)



2. The total surface area of a cube is 1350 sq.m. Find its volume.

3. Find the area of four walls of a room (Assume that there are no doors or windows) if its length 12 m., breadth 10 m. and height 7.5 m.

4. The volume of a cuboid is 1200 cm^3. The length is 15 cm. and breadth is 10 cm. Find its height.

5. How does the total surface area of a box change if

(i) Each dimension is doubled? (ii) Each dimension is tripled?

Express in words. Can you find the area if each dimension is raised to n times?

6. The base of a prism is triangular in shape with sides 3 cm., 4 cm. and 5 cm. Find the volume of the prism if its height is 10 cm.

7. A regular square pyramid is 3 m. height and the perimeter of its base is 16 m. Find the volume of the pyramid.

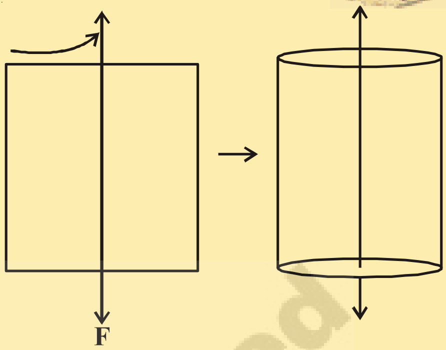
8. An Olympic swimming pool is in the shape of a cuboid of dimensions 50 m. long and 25 m. wide. If it is 3 m. deep throughout, how many liters of water does it hold?

ACTIVITY

Cut out a rectangular sheet of paper. Paste a thick string along the line as shown in the figure. Hold the string with your hands on either sides of the rectangle and rotate the rectangle sheet about the string as fast as you can.

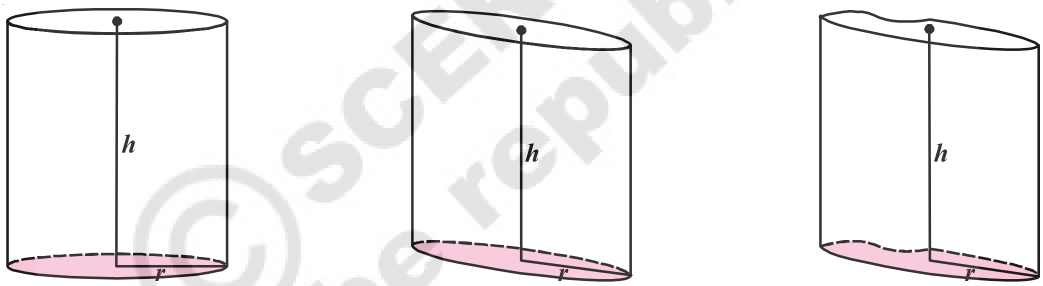
Do you recognize the shape that the rotating rectangle is forming ?

Does it remind you the shape of a cylinder ?



10.3 RIGHT CIRCULAR CYLINDER

Observe the following cylinders:



(i) (ii) (iii)

(i) What similiarties you have observed in figure (i), (ii) and (iii)?

(ii) What differences you have observed between fig. (i), (ii) and (iii)?

(iii) In which figure, the line segment is perpendicular to the base?

Every cylinder is made up of one curved surface and with two congruent circular faces on both ends. If the line segment joining the centre of circular faces, is perpendicular to its base, such a cylinder is called right circular cylinder.

Find out which is right circular cylinder in the above figures? Which are not? Give reasons.

Let us do an activity to generate a cylinder

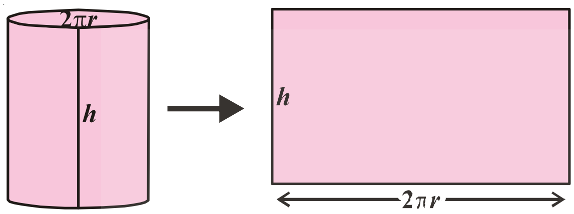
10.3.1 Curved Surface area of a cylinder

Take a right circular cylinder made up of cardboard. Cut the curved face vertically and unfold it. While unfolding cylinder, observe its transformation of its height and the circular base. After unfolding the cylinder what shape do you find?

You will find it is in rectangular shape. The area of rectangle is equal to the area of curved surface area of cylinder. Its height is equal to the breadth of the rectangle, and the circumference of the base is equal to the length of the rectangle.

Height of cylinder = breadth of rectangle ($h = b$)

Circumferance of base of cylinder with radius 'r' = length of the rectangle ($2\pi r = l$)



Curved surface area of the cylinder = Area of the rectangle

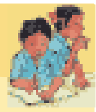
$= \text{length} \times \text{breadth}$

$= 2\ \pi r \times h$

$= 2\pi rh$

Therefore, Curved surface area of a cylinder = $2\pi rh$

DO THIS

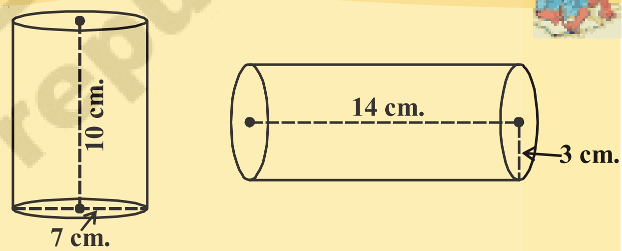


Find CSA of each of following cylinders

(i) $r = x$ cm., $h = y$ cm.

(ii) $d = 7$ cm., $h = 10$ cm.

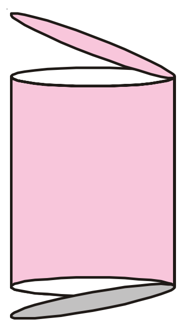
(iii) $r = 3$ cm., $h = 14$ cm.



10.3.2 Total Surface area of a Cylinder

Observe the adjacent figure.

Do you find that it is a right circular cylinder? What surfaces you have to add to get its total surface area? They are the curved surface area and area of two circular faces.



Now the total surface area of a cylinder

= Curved surface area + Area of top + Area of base

$= 2\pi rh + \pi r^2 + \pi r^2$

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r)$

$= 2\pi r\,(r + h)$

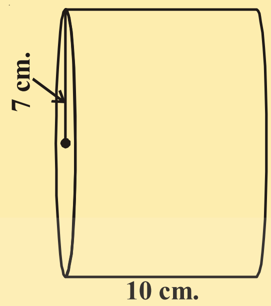
$\therefore$ The total surface area of a cylinder $= 2\pi r\,(r + h)$

Where 'r' is the radius of the cylinder and 'h' is its height.

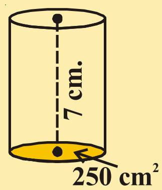
DO THESE

Find the Total surface area of each of the following cylinders.

(i)



(ii)



10.3.3 Volume of a cylinder

Take circles with equal radii and arrange one over other.

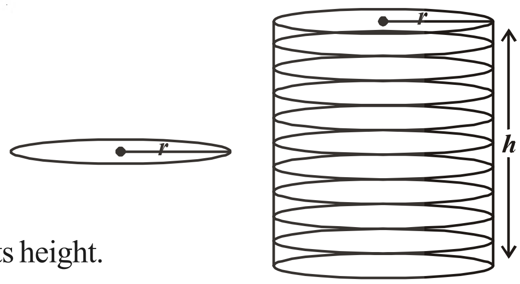
Do this activity and find whether it form a cylinder or not.

In the figure 'r' is the radius of the circle, and the 'h' is the height up to which the circles are stacked.

Volume of a cylinder $= \pi r^2 \times$ height

$= \pi r^2 \times h$

$= \pi r^2 h$



So volume of a cylinder = $\pi r^2 h$

Where 'r' is the radius of cylinder and 'h' is its height.

Example-1. A Rectangular paper of width 14 cm is rolled along its width and a cylinder of radius 20 cm is formed. Find the volume of the cylinder (Fig 1) ? $\left(\text{Take } \pi = \frac{22}{7}\right)$

Solution : A cylinder is formed by rolling a rectangle about its width. Hence the width of the paper becomes height of cylinder and radius of the cylinder is 20 cm.

Height of the cylinder = h = 14 cm.

radius (r) = 20 cm.

Volume of the cylinder V = $\pi r^2 h$

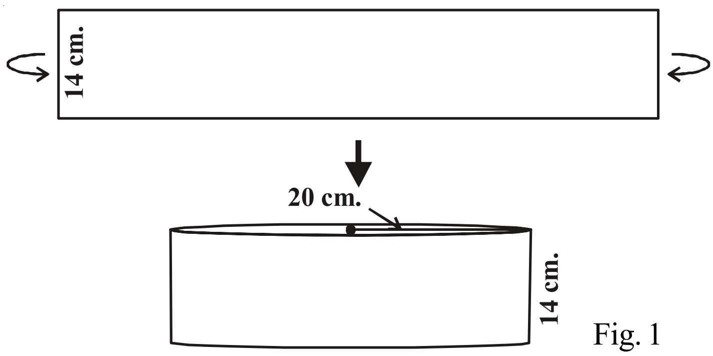


Fig. 1

$$= \frac{22}{7} \times 20 \times 20 \times 14$$

$$= 17600 \text{ cm}^3.$$

Hence the volume of the cylinder is 17600 cm^3.

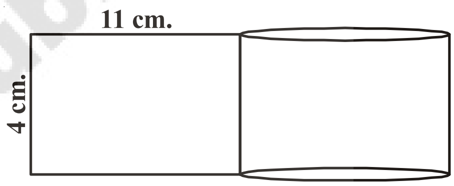
Example-2. A Rectangular piece of paper 11 cm × 4 cm is folded without overlapping to make a cylinder of height 4 cm. Find the volume of the cylinder.

Solution : Length of the paper becomes the circumference of the base of the cylinder and width becomes height.

Let radius of the cylinder = r and height = h

Circumference of the base of the cylinder = $2\pi r$ = 11 cm.

$$2 \times \frac{22}{7} \times r = 11$$



$$\therefore \qquad r = \frac{7}{4} \text{ cm.}$$

$$h = 4 \text{ cm}$$

Volume of the cylinder (V) = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 4 \text{ cm}^3$$

$$= 38.5 \text{ cm}^3.$$

Example-3. A rectangular sheet of paper 44 cm × 18 cm is rolled along the length to form a cylinder. Assuming that the cylinder is solid (Completely filled), find its total surface area and the radius.

Solution : Height of the cylinder = 18 cm

Circumference of base of cylinder = 44 cm

$$2\pi r = 44 \text{ cm}$$

$$r = \frac{44}{2 \times \pi} = \frac{44 \times 7}{2 \times 22} = 7\text{cm.}$$

Total surface area $= 2\pi r\,(r + h)$

$$= 2\times\frac{22}{7}\times 7(7+18)\text{cm}^2$$

$= 1100\ \text{cm}^2$.

Example-4. Circular discs 5 mm thickness, are placed one above the other to form a cylinder of curved surface area 462 cm^2. Find the number of discs, if the radius is 3.5 cm.

Solution : Thickness of disc $= 5\ \text{mm} = \frac{5}{10}\text{cm} = 0.5\,\text{cm}$

Radius of disc = 3.5 cm.

Curved surface area of cylinder = 462 cm^2.

$\therefore\ 2\pi rh = 462$ ..... (i)

Let the no of discs be x

$\therefore$ Height of cylinder = h = Thickness of disc × no of discs

$= 0.5\,x$

$$\therefore\ 2\,\pi rh = 2\times\frac{22}{7}\times 3.5\times 0.5x \quad \text{..... (ii)}$$

From (i) are (ii) we get

$$2\times\frac{22}{7}\times 3.5\times 0.5x = 462$$

$$\therefore\ x = \frac{462\times 7}{2\times 22\times 3.5\times 0.5} = 42\ \text{discs}$$

Example-5. A hollow cylinder having external radius 8 cm and height 10 cm has a total surface area of 338 π cm^2. Find the thickness of the hollow metallic cylinder.

Solution : External radius = R = 8 cm

Internal radius = r

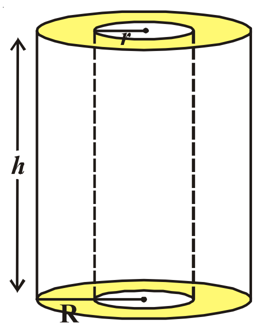
Height = 10 cm

TSA = $338\pi\ \text{cm}^2$.

But TSA = Area of external cylinder (CSA)

+ Area of internal cylinder (CSA)

+ Twice Area of base (ring)



$= 2\pi Rh + 2\pi rh + 2\pi (R^2 - r^2)$

$= 2\pi (Rh + rh + R^2 - r^2)$

$\therefore\ 2\pi (Rh + rh + R^2 - r^2) = 338\pi$

$Rh + rh + R^2 - r^2 = 169$

$\Rightarrow\ (10 \times 8) + (r \times 10) + 8^2 - r^2 = 169$

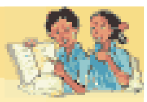
$\Rightarrow\ r^2 - 10r + 25 = 0$

$\Rightarrow\ (r - 5)^2 = 0$

$\therefore\ r = 5$

$\therefore$ Thickness of metal $= R - r = (8 - 5)$ cm $= 3$ cm.

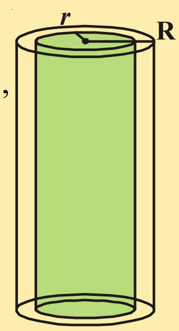
TRY THESE



1. If the radius of a cylinder is doubled keeping its lateral surface area the same, then what is its height ?
2. A hot water system (Geyser) consists of a cylindrical pipe of length 14 m and diameter 5 cm. Find the total radiating surface of hot water system.

EXERCISE - 10.2

1. A closed cylindrical tank of height 1.4 m. and radius of the base is 56 cm. is made up of a thick metal sheet. How much metal sheet is required (Express in square meters)
2. The volume of a cylinder is 308 cm.^{3}. Its height is 8 cm. Find its later surface area and total surface area.
3. A metal cuboid of dimension 22 cm. × 15 cm. × 7.5 cm. was melted and cast into a cylinder of height 14 cm. What is its radius?
4. An overhead water tanker is in the shape of a cylinder has capacity of 616 litres. The diameter of the tank is 5.6 m. Find the height of the tank.
5. A metal pipe is 77 cm. long. The inner diameter of a cross section is 4 cm., the outer diameter being 4.4 cm. (see figure) Find its
   (i) inner curved surface area
   (ii) outer curved surface area
   (iii) Total surface area.



6. A cylindrical piller has a diameter of 56 cm and is of 35 m high. There are 16 pillars around the building. Find the cost of painting the curved surface area of all the pillars at the rate of ₹5.50 per 1 m^2.

7. The diameter of a roller is 84 cm and its length is 120 cm. It takes 500 complete revolutions to roll once over the play ground to level. Find the area of the play ground in m^2.

8. The inner diameter of a circular well is 3.5 m. It is 10 m deep. Find
   (i) its inner curved surface area
   (ii) The cost of plastering this curved surface at the rate of Rs. 40 per m^2.

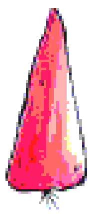
9. Find
   (i) The total surface area of a closed cylindrical petrol storage tank whose diameter 4.2 m. and height 4.5 m.
   (ii) How much steel sheet was actually used, if $\frac{1}{12}$ of the steel was wasted in making the tank.

10. A one side open cylinderical drum has inner radius 28 cm. and height 2.1 m. How much water you can store in the drum. Express in litres. (1 litre = 1000 cc.)

11. The curved surface area of the cylinder is 1760 cm.^{2} and its volume is 12320 cm^3. Find its height.

10.4 Right Circular Cone

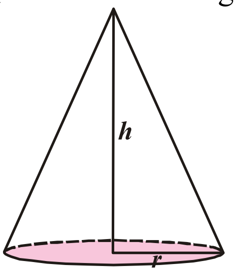




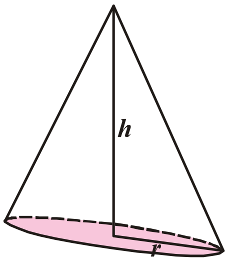

Observe the above figures and which solid shape they resemble?

These are in the shape of a cone.

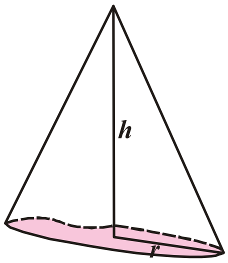
Observe the following cones:



(i)



(ii)



(iii)

(i) What common properties do you find among these cones?

(ii) What difference do you notice among them?

In fig.(i), lateral surface is curved and base is circle. The line segment joining the vertex of the cone and the centre of the circular base (vertical height) is perpendicular to the radius of the base. This type of cone is called Right Circular Cone.

In fig.(ii) although it has circular base, but its vertical height is not perpendicular to the radius of the cone.

Such type of cones are not right circular cones.

In the fig. (iii) although the vertical height is perpendicular to the base, but the base is not in circular shape.

Therefore, this cone is not a right circular cone.

10.4.1 Slant Height of the Cone

In the adjacent figure (cone), $\overline{AO}$ is perpendicular to $\overline{OB}$

ΔAOB is a right angled triangle.

$\overline{AO}$ is the height of the cone (h) and $\overline{OB}$ is equal to the radius of the cone (r)

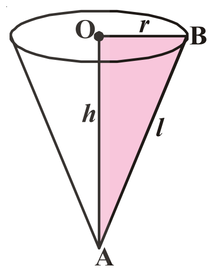
From ΔAOB

$AB^2 = AO^2 + OB^2$

$AB^2 = h^2 + r^2$ (AB is called slant height $= l$)

$l^2 = h^2 + r^2$

$l = \sqrt{h^2 + r^2}$



ACTIVITY

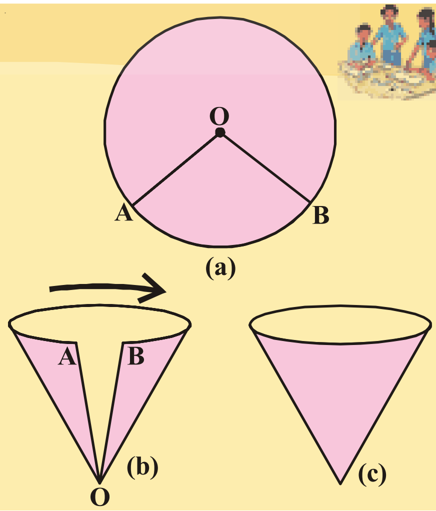
Making a cone from a sector

Follow the instructions and do as shown in the figure.

(i) Draw a circle on a thick paper Fig(a)

(ii) Cut a sector AOB from it Fig(b).

(iii) Fold the ends A, B nearer to each other slowly and join AB. Remember A, B should not overlap on each other. After joining A, B attach them with cello tape Fig(c).

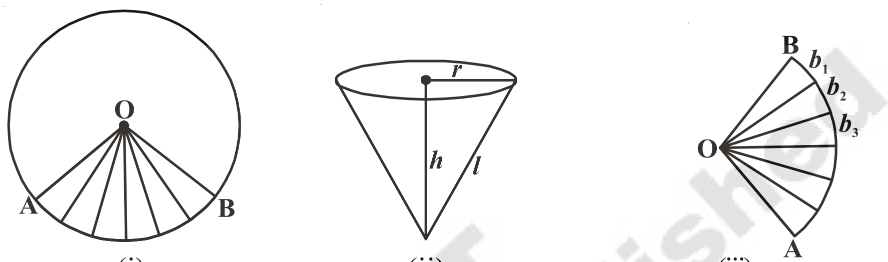


(iv) What kind of shape you have obtained?

Is it a right cone?

While making a cone observe what happened to the edges ‘OA’ and ‘OB’ and length of arc AB of the sector?

10.4.2 Curved Surface area of a cone



(i) (ii) (iii)

Let us find the surface area of a right circular cone that we made out of the paper as discussed in the activity.

While folding the sector into cone you have noticed that OA, OB of sector coincides and becomes the slant height of the cone, whereas the length of $\overset{\frown}{AB}$ becomes the circumference of the base of the cone.

Now unfold the cone and cut the sector AOB as shown in the figure as many as you can, then you can see each cut portion is almost a small triangle with base b_1, b_2, b_3 ..... etc. and height ‘l’ i.e. equal to the slant height of the cone.

If we find the area of these triangles and adding these, it gives area of the sector. We know that sector forms a cone, so the area of a sector is equal to the surface area of the cone formed with it.

Area of the cone = Sum of the areas of triangles.

$$= \frac{1}{2}b_1 l + \frac{1}{2}b_2 l + \frac{1}{2}b_3 l + \frac{1}{2}b_4 l + ...$$

$$= \frac{1}{2}l(b_1 + b_2 + b_3 + b_4 + ...)$$

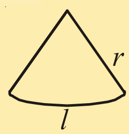
$$= \frac{1}{2}l \text{ (length of the curved part from A to B or circumference of the base of the cone)}$$

$$= \frac{1}{2}l(2\pi r) \quad (\because\ b_1 + b_2 + b_3 + ... = 2\pi r, \text{ where ‘r’ is the radius of the cone})$$

as $\overset{\frown}{AB}$ forms a circle.

TRY THIS

A sector with radius ‘r’ and length of its arc ‘l’ is cut from a circular sheet of paper. Fold it as a cone. How can you derive the formula of its curved surface area $A = \pi rl$



Thus, lateral surface area or curved surface area of the cone = πrl

Where 'l' is the slant height of the cone and 'r' is its radius

10.4.3 Total surface area of the cone

If the base of the cone is to be covered, we need a circle whose radius is equal to the radius of the cone.

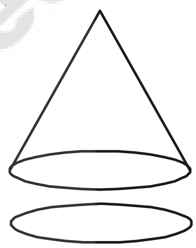
How to obtain the total surface area of cone? How many surfaces you have to add to get total surface area?

The area of the circle $= \pi r^2$

Total surface area of a cone = lateral surface area + area of its base

$= \pi rl + \pi r^2$

$= \pi r (l + r)$



Total surface area of the cone = $\pi r (l + r)$

Where 'r' is the radius of the cone and 'l' is its slant height.

DO THIS

1. Cut a right angled triangle, stick a string along its perpendicular side, as shown in fig. (i) hold the both the sides of a string with your hands and rotate it with constant speed.

   What do you observe ?

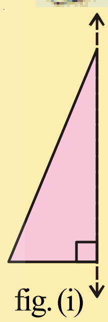
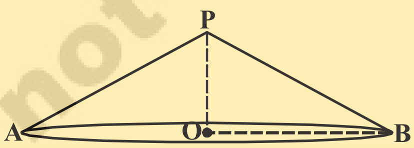
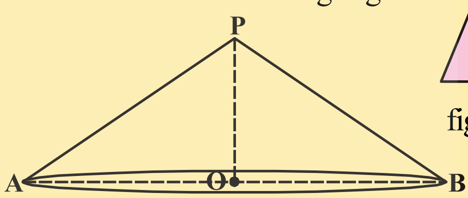


fig. (i)

2. Find the curved surface area and total surface area of the each following Right Circular Cones.

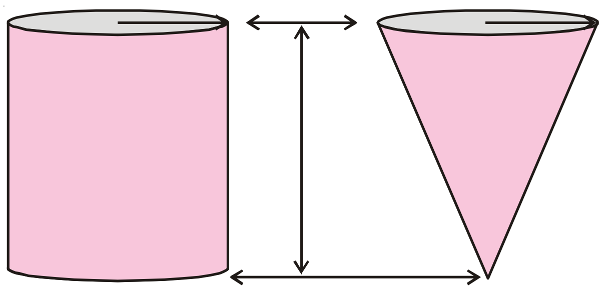


OP = 2cm.; OB = 3.5cm.

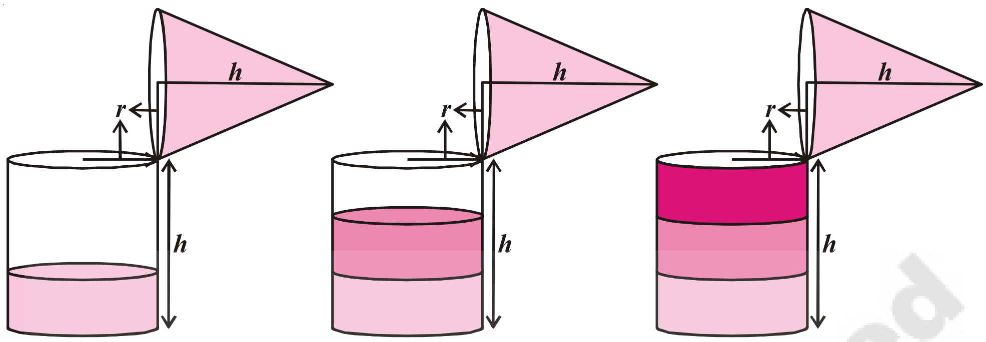


OP = 3.5cm.; AB = 10cm.

10.4.4 Volume of a right circular cone



Make a hollow cylinder and a hollow cone with the equal radius and equal height and do the following experiment, that will help us to find the volume of a cone.



i. Fill water in the cone up to the brim and pour into the hollow cylinder, it will fill up only some part of the cylinder.

ii. Again fill up the cone up to the brim and pour into the cylinder, we see the cylinder is still not full.

iii. When the cone is filled up for the third time and emptied into the cylinder, observe whether the cylinder is filled completely or not.

With the above experiment do you find any relation between the volume of the cone and the volume of the cylinder?

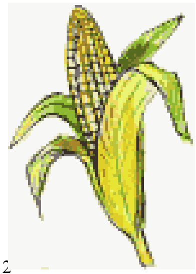
We can say that three times the volume of a cone makes up the volume of cylinder, which both have the same base and same height.

So the volume of a cone is one third of the volume of the cylinder.

$$\therefore \text{ Volume of a cone} = \frac{1}{3}\pi r^2 h$$

where 'r' is the radius of the base of cone and 'h' is its height.

Example-6. A corn cob see fig(1), shaped like a cone, has the radius of its broadest end as 1.4 cm and length (height) as 12 cm. If each 1cm^2 of the surface of the cob carries an average of four grains, find how many grains approximately you would find on the entire cob.



Solution : Here $l = \sqrt{r^2 + h^2} = \sqrt{(1.4)^2 + (12)^2}\, cm.$

$$= \sqrt{145.96} = 12.08 \text{ cm. (approx.)}$$

Therefore the curved surface area of the corn cob = $\pi r l$

$$= \frac{22}{7} \times 1.4 \times 12.08\ cm^2$$

$= 53.15 \text{ cm}^2$

$= 53.2 \text{ cm}^2$ (approx)

Number of grains of corn on 1 cm^2 of the surface of the corn cob = 4.

Therefore, number of grains on the entire curved surface of the cob.

$= 53.2 \times 4 = 212.8 = 213$ (approx)

So, there would be approximately 213 grain of corn on the cob.

Example-7. Find the slant height and vertical height of a Cone with radius 5.6 cm and curved surface area 158.4 cm^2.

Solution : Radius = 5.6 cm, vertical height = h, slant height = l

CSA of cone $= \pi r l = 158.4 \text{ cm}^2$

$$\Rightarrow \frac{22}{7} \times 5.6 \times l = 158.4$$

$$\Rightarrow l = \frac{158.4 \times 7}{22 \times 5.6} = \frac{18}{2} = 9\text{cm}$$

we know $l^2 = r^2 + h^2$

$h^2 = l^2 - r^2 = 9^2 - (5.6)^2$

$= 81 - 31.36$

$= 49.64$

$h = \sqrt{49.64}$

$h = 7.05$ cm (approx)

Example-8. A tent is in the form of a cylinder surmounted by a cone having its diameter of the base equal to 24 m. The height of cylinder is 11 m and the vertex of the cone is 5m above the cylinder. Find the cost of making the tent, if the rate of canvas is ₹10 per m^2.

Solution : Diametre of base of cylinder = diametre of cone = 24m

$\therefore$ Radius of base = 12 m

Height of cylinder = 11 m = h_1

Height of Cone = 5m = h_2

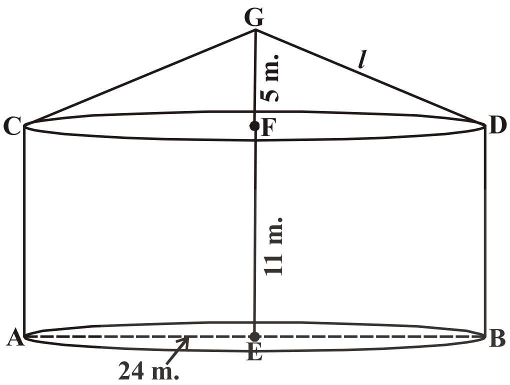
Let slant height of cone be l

l = GD = $\sqrt{r^2+h^2} = \sqrt{12^2+5^2} = 13m$

Area of canvas required = CSA of cylinder + CSA of cone

$= 2\pi rh_1 + \pi rl$

$= \pi r\,(2h_1 + l)$



$= \dfrac{22}{7} \times 12(2\times11+13)m^2$

$= \dfrac{22\times12}{7} \times 35m^2$

$= 22 \times 60\ m^2$

$= 1320\ m^2$

Rate of canvas = ₹10 per m^2

∴ Cost of canvas = Rate × area of canvas

= ₹10 × 1320

= ₹13,200.

Example-9. A conical tent was erected by army at a base camp with height 3m. and base diameter 8m. Find;

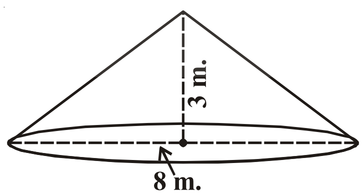
(i) The cost of canvas required for making the tent, if the canvas cost ₹ 70 per 1 sq.m.

(ii) If every person requires 3.5 m.^{3} air, how many can be seated in that tent.

Solution : Diameter of the tent = 8 m.

$r = \dfrac{d}{2} = \dfrac{8}{2} = 4$ m.

height = 3 m.



Slant height (l) = $\sqrt{h^2+r^2}$

$= \sqrt{3^2+4^2}$

$= \sqrt{25} = 5$ m.

∴ Curved surface area of tent= πrl

$= \dfrac{22}{7} \times 4 \times 5 = \dfrac{440}{7}\ m^2$

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3$

$= \frac{352}{7}$ m^3

(i) Cost of canvas required for the tent

= CSA × Unit cost

$= \frac{440}{7} \times 70$

= ₹4400

(ii) No. of persons can be seated in the tent

$= \frac{\text{Volume of conical tent}}{\text{air required for each}}$

$= \frac{352}{7} \div 3.5$

$= \frac{352}{7} \times \frac{1}{3.5} = 14.36$

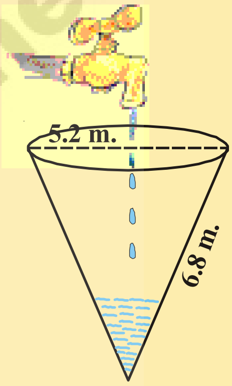
= 14 men (approx.)

EXERCISE - 10.3

1. The base area of a cone is 38.5 cm^2. Its volume is 77 cm^3. Find its height.
2. The volume of a cone is 462 m^3. Its base radius is 7 m. Find its height.
3. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm Find.

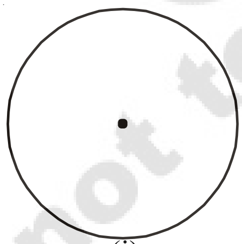
   (i) radius of the base (ii) Total surface area of the cone.
4. The cost of painting the total surface area of a cone at 25 paise per cm^2 is ₹176. Find the volume of the cone, if its slant height is 25 cm.
5. From a circle of radius 15 cm., a sector with angle 216° is cut out and its bounding radii are bent so as to form a cone. Find its volume.
6. The height of a tent is 9 m. Its base diameter is 24 m. What is its slant height? Find the cost of canvas cloth required if it costs ₹14 per sq.m.

7. The curved surface area of a cone is $1159\frac{5}{7}$ cm^2. Area of its base is $254\frac{4}{7}$ cm^2. Find its volume.
8. A tent is cylindrical to a height of 4.8 m. and conical above it. The radius of the base is 4.5m. and total height of the tent is 10.8 m. Find the canvas required for the tent in square meters.
9. What length of tarpaulin 3 m wide will be required to make a conical tent of height 8m and base radius 6m ? Assume that extra length of material that will be required for stitching margins and wastage in cutting is approximately 20 cm (use $\pi = 3.14$)
10. A Joker's cap is in the form of a right circular cone of base radius 7 cm and height 27 cm. Find the area of the sheet required to make 10 such caps.
11. Water is pouring into a conical vessel as shown in the adjoining figure at the rate of 1.8 m^3 per minute. How long will it take to fill the vessel?



12. Two similiar cones have volumes 12π cu. units and 96π cu. units. If the curved surface area of the smaller cone is 15π sq. units, what is the curved surface area of the larger one?

10.5 SPHERE


(i)


(ii)


(iii)

All the above figures are well known to you. Can you identify the difference among them.

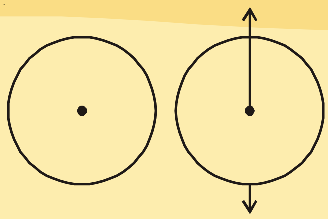
Figure (i) is a circle. You can easily draw it on a plane paper. Because it is a plane figure. A circle is plane closed figure whose every point lies at a constant distance (radius) from a fixed point (centre)

The remaining above figures are solids. These solids are circular in shape and are called spheres.

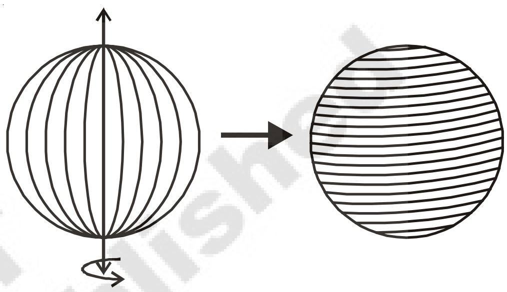
A sphere is a three dimensional figure, which is made up of all points in the space, which is at a constant distance from a fixed point. This fixed point is called centre of the sphere. The distance from the centre to any point on the surface of the sphere is its radius.

ACTIVITY

Draw a circle on a thick paper and cut it neatly. Stick a string along its diameter. Hold the both the ends of the string with hands and rotate with constant speed and observe the figure so formed.



10.5.1 Surface area of a sphere



Let us find the surface area of the figure with the following activity.

Take a tennis ball as shown in the figure and wind a string around the ball, use pins to keep the string in place. Mark the starting and ending points of the string. Slowly remove the string from the surface of the sphere.

Find the radius of the sphere and draw four circles of radius equal to the radius of the ball as shown in the pictures. Start filling the circles one after one with the string you had wound around the ball.

What do you observe?

The string, which had completely covered the surface area of the sphere (ball), has been used to completely fill the area of four circles, all have same radius as of the sphere.

With this we can understand that the surface area of a sphere of radius (r) is equal to the four times of the area of a circle of radius (r).

$\therefore$ Surface area of a sphere $= 4 \times$ the area of circle

$= 4\pi r^2$

Surface area of a sphere = $4\pi r^2$

Where 'r' is the radius of the sphere

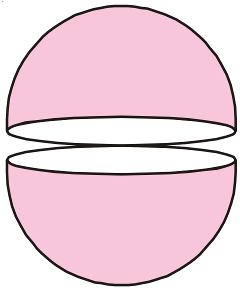
TRY THIS

Can you find the surface area of sphere in any other way?

10.5.2 Hemisphere

Take a solid sphere and cut it through the middle with a plane that passes through its centre.

Then it gets divided into two equal parts as shown in the figure



Each equal part is called a hemisphere.

A sphere has only one curved face. If it is divided into two equal parts, then its curved face is also divided into two equal curved faces.

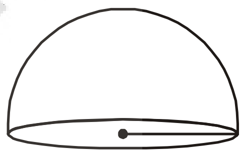
What do you think about the surface area of a hemisphere ?

Obviously,

Curved surface area of a hemisphere is equal to half the surface area of the sphere

So, surface area of a hemisphere = $\frac{1}{2}$ surface area of sphere

$$= \frac{1}{2} \times 4\pi r^2$$



$$= 2\,\pi r^2$$

$\therefore$ **surface area of a hemisphere = $2\,\pi r^2$**

The base of hemisphere is a circular region.

Its area is equal to = πr^2

Let us add both the curved surface area and area of the base, we get total surface area of hemisphere.

Total surface area of hemisphere = Its curved surface area + area of its base

$$= 2\,\pi r^2 + \pi r^2$$

$$= 3\,\pi r^2.$$

Total surface area of hemisphere = $3\pi r^2$.

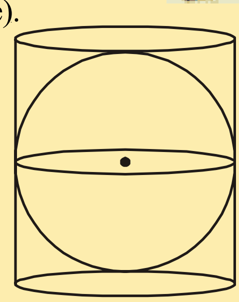
DO THESE

1. A right circular cylinder just encloses a sphere of radius r (see figure).

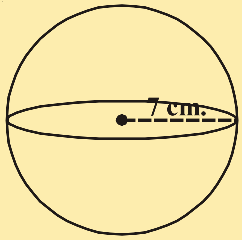
   Find : (i) surface area of the sphere

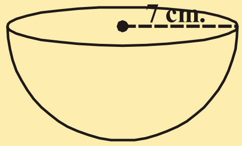
   (ii) curved surface area of the cylinder

   (iii) ratio of the areas obtained in (i) and (ii)



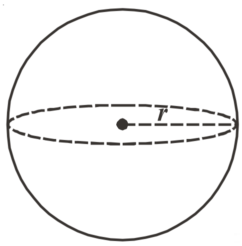
2. Find the surface area of each the following figure.

(i) 

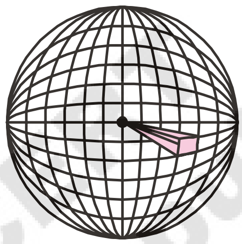
(ii) 

10.5.3 Volume of Sphere

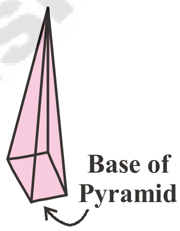
To find the volume of a sphere, imagine that a sphere is composed of a great number of congruent pyramids with all their vertices join at the centre of the sphere, as shown in the figure.



(i)



(ii)



(iii)

Let us follow the steps:

1. Let 'r' be the radius of the solid sphere as in fig. (i).
2. Assume that a sphere with radius 'r' is made of 'n' number of pyramids of equal sizes as shown in the fig. (ii).
3. Consider a part (pyramid) among them. Each pyramid has a base and let the area of the base of pyramids are A_1, A_2, A_3.....

   The height of the pyramid is equal to the radius of sphere, then the

   Volume of one pyramid $= \frac{1}{3} \times$ Area of the base $\times$ height

   $= \frac{1}{3} A_1 r$

you can take any polygon as base of a pyramid

4. As there are 'n' number of pyramids, then

   Volume of 'n' pyramids $= \frac{1}{3} A_1 r + \frac{1}{3} A_2 r + \frac{1}{3} A_3 r + \ldots$.. n times

   $= \frac{1}{3} r [A_1 + A_2 + A_3 + \ldots$.. n times]

$$= \frac{1}{3} \times A\,r$$

$A = A_1 + A_2 + A_3 + \ldots\ldots$ n times
$=$ Surface areas of 'n' pyramids

5. As the sum of volumes of all these pyramids is equal to the volume of sphere and the sum of the areas of all the bases of the pyramids is very close to the surface area of the sphere, (i.e. $4\pi r^2$).

So, volume of sphere $= \frac{1}{3}(4\pi r^2)\,r$

$= \frac{4}{3}\pi r^3$ cub. units

Volume of a sphere $= \frac{4}{3}\pi r^3$

Where 'r' is the radius of the sphere

How can you find volume of hemisphere? It is half the volume of sphere.

$\therefore$ Volume of hemisphere $= \frac{1}{2}$ of volume of a sphere

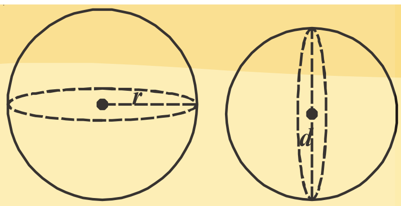
$$= \frac{1}{2} \times \frac{4}{3}\pi r^3$$

$$= \frac{2}{3}\pi r^3$$

[**Hint :** You can try to derive these formulae using water melon or any other like that]

Do This

1. Find the volume of the sphere given in the adjacent figures.
2. Find the volume of sphere of radius 6.3 cm.



$r = 3$cm. $d = 5.4$cm.

Example-10. If the surface area of a sphere is 154 cm^2, find its radius.

Solution : Surface area of sphere $= 4\pi r^2$

$$4\pi r^2 = 154 \Rightarrow 4 \times \frac{22}{7} \times r^2 = 154$$

$$\Rightarrow r^2 = \frac{154 \times 7}{4 \times 22} = \frac{7^2}{2^2}$$

$$\Rightarrow r = \frac{7}{2} = 3.5\,\text{cm}$$

Example-11. A hemispherical bowl is made up of stone whose thickness is 5 cm. If the inner radius is 35 cm, find the total surface area of the bowl.

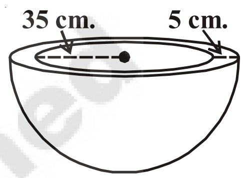
Solution : Let R be outer radius and 'r' be inner radius Thickness of ring = 5 cm

$\therefore$ R = (r + 5) cm = (35 + 5) cm = 40 cm

Total Surface Area = CSA of outer hemisphere + CSA of inner hemisphere + area of the ring.

$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= \pi(2R^2 + 2r^2 + R^2 - r^2)$



$$= \frac{22}{7}(3R^2 + r^2) = \frac{22}{7}(3 \times 40^2 + 35^2)\,cm^2$$

$$= \frac{6025 \times 22}{7} cm^2$$

$= 18935.71\ cm^2$ (approx).

Example-12. The hemispherical dome of a building needs to be painted (see fig 1). If the circumference of the base of dome is 17.6 m, find the cost of painting it, given the cost of painting is Rs.5 per 100 cm^2.

Solution : Since only the rounded surface of the dome is to be painted we need to find the curved surface area of the hemisphere to know the extent of painting that needs to be done. Now, circumference of base of the dome = 17.6 m Therefore 17.6 = $2\pi r$

So, The radius of the dome $= 17.6 \times \frac{7}{2 \times 22}$ m

$= 2.8$ m

The curved surface area of the dome $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 2.8 \times 2.8 m^2$$

$= 49.28\ m^2$.

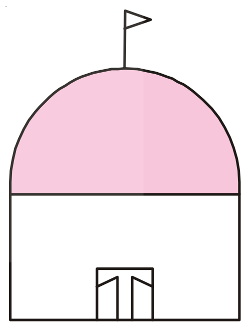


fig 1

Now, cost of painting 100 cm^2 is Rs.5

So, cost of painting 1m^2 = Rs. 500

Therefore, cost of painting the whole dome

= Rs.500 × 49.28

= Rs. 24640.

Example-13. The hollow sphere, in which the circus motor cyclist performs his stunts, has a diameter of 7m. Find the area available to the motor cyclist for riding.

Solution : Diameter of the sphere = 7 m. Therefore, radius is 3.5 m. So, the riding space available for the motorcyclist is the surface area of the 'sphere' which is given by

$$4\pi r^2 = 4 \times \frac{22}{7} \times 3.5 \times 3.5 m^2$$

$$= 154 \text{ m}^2.$$

Example-14. A shotput is a metallic sphere of radius 4.9 cm. If the density of the metal is 7.8 g. per cm^3, find the mass of the shotput.

Solution : Since the shot-put is a solid sphere made of metal and its mass is equal to the product of its volume and density, we need to find the volume of the sphere.

$$\text{Now, volume of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 4.9 \times 4.9 \times 4.9 \text{ cm}^3$$

$$= 493 \text{ cm}^3 \text{ (nearly)}$$

Further, mass of $1cm^3$ of metal is 7.8 g

$$\text{Therefore, mass of the shot-put} = 7.8 \times 493 \text{ g}$$

$$= 3845.44\text{g} = 3.85 \text{ kg (nearly)}$$

Example-15. A hemispherical bowl has a radius of 3.5 cm. What would be the volume of water it would contain ?

Solution : The volume of water the bowl can contains = Volume of hemisphere

$$= \frac{2}{3}\pi r^3$$

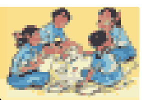
$$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 \text{ cm}^3$$

$$= 89.8 \text{ cm}^3. \text{ (approx).}$$

EXERCISE - 10.4

1. The radius of a sphere is 3.5 cm. Find its surface area and volume.
2. The surface area of a sphere is $1018\frac{2}{7}$ sq.cm. What is its volume?
3. The length of equator of the globe is 44 cm. Find its surface area.
4. The diameter of a spherical ball is 21 cm. How much leather is required to prepare 5 such balls.
5. The ratio of radii of two spheres is 2 : 3. Find the ratio of their surface areas and volumes.
6. Find the total surface area of a hemisphere of radius 10 cm. (use $\pi = 3.14$)
7. The diameter of a spherical balloon increases from 14 cm. to 28 cm. as air is being pumped into it. Find the ratio of surface areas of the balloons in the two cases.
8. A hemispherical bowl is made of brass, 0.25 cm. thickness. The inner radius of the bowl is 5 cm. Find the ratio of outer surface area to inner surface area.
9. The diameter of a lead ball is 2.1 cm. The density of the lead used is 11.34 g/c^3. What is the weight of the ball?
10. A metallic cylinder of diameter 5 cm. and height $3\frac{1}{3}$ cm. is melted and cast into a sphere. What is its diameter.
11. How many litres of milk can a hemispherical bowl of diameter 10.5 cm. hold?
12. A hemispherical bowl has diameter 9 cm. The liquid is poured into cylindrical bottles of diameter 3 cm. and height 3 cm. If a full bowl of liquid is filled in the bottles, find how many bottles are required.

WHAT WE HAVE DISCUSSED



1. Cuboid and cube are regular prisms having six faces and of which four are lateral faces and the base and top.
2. If length of cuboid is l, breadth is 'b' and height is 'h' then,

   Total surface area of a cuboid $= 2\,(lb + bh + lh)$

   Lateral surface area of a cuboid $= 2\,h\,(l + b)$

   Volume of a cuboid $= lbh$

3. If the length of the edge of a cube is 'l' units, then

   Total surface area of a cube $= 6l^2$

   Lateral surface area of a cube $= 4l^2$

   Volume of a cube $= l^3$

4. The volume of a pyramid is $\frac{1}{3}$rd volume of a right prism if both have the same base and same height.

5. A cylinder is a solid having two circular ends with a curved surface area. If the line segment joining the centres of base and top is perpendicular to the base, it is called right circular cylinder.

6. If the radius of right circular cylinder is 'r' and height is 'h' then;

   - Curved surface area of a cylinder $= 2\pi rh$
   - Total surface area of a cylinder $= 2\pi r\,(r + h)$
   - Volume of a cylinder $= \pi r^2h$

7. Cone is a geometrical shaped object with circle as base, having a vertext at the top. If the line segment joining the vertex to the centre of the base is perpendicular to the base, it is called right circular cone.

8. The length joining the vertex to any point on the circular base of the cone is called slant height (l)

   $l^2 = h^2 + r^2$

9. If 'r' is the radius, 'h' is the height, 'l' is the slant height of a cone, then

   - Curved surface area of a cone $= \pi rl$
   - Total surface area of a cone $= \pi r\,(r + l)$

10. The volume of a cone is $\frac{1}{3}$rd the volume of a cylinder of the same base and same height

    i.e. volume of a cone $= \frac{1}{3}\,\pi r^2h$.

11. A sphere is an geometrical object formed where the set of points are equidistant from the fixed point in the space. The fixed point is called centre of the sphere and the fixed distance is called radius of the sphere.

12. If the radius of sphere is 'r' then,

- Surface area of a sphere = $4\pi r^2$
- Volume of a sphere = $\frac{4}{3}\pi r^3$

13. A plane through the centre of a sphere divides it into two equal parts, each of which is called a hemisphere.

- Curved surface area of a hemisphere = $2\pi r^2$
- Total surface area of a hemisphere = $3\pi r^2$
- Volume of a hemisphere = $\frac{2}{3}\pi r^3$

Do You Know?

Making an 8 × 8 Magic Square

Simply place the numbers from 1 to 64 sequentially in the square grids, as illustrated on the left. Sketch in the dashed diagonals as indicated. To obtain the magic square on the bottom, replace any number which lands on a dashed line with its compliment (two numbers of a magic square are compliments if they total the same value as the sum of the magic's square smallest and largest numbers).

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

64	2	3	61	60	6	7	57
9	55	54	12	13	51	50	16
17	47	46	20	21	43	42	24
40	26	27	37	36	30	31	33
32	34	35	29	28	38	39	25
41	23	22	44	45	19	18	48
49	15	14	52	53	11	10	56
8	58	59	5	4	62	63	1

* A magic square is an array of numbers arrange in a square shape in which any row, column total the same amount. You can try more such magic squares.