14 Probability

Probability theory is nothing but common sense reduced to calculation.

- Pierre-Simon Laplace

14.1 Introduction

Siddu and Vivek are classmates. One day during their lunch they are talking to each other.

Observe their conversation

Siddu : Hello Vivek , What are you going to do in the evening today?

Vivek : Most likely, I will watch India v/s Australia cricket match.

Siddu : Whom do you think will win the toss ?

Vivek : Both teams have equal chance to win the toss.
Do you watch the cricket match at home?

Siddu : There is no chance for me to watch the cricket at my home. Because my T.V. is under repair.

Vivek : Oh! then come to my home, we will watch the match together.

Siddu : I will come after doing my home work.

Vivek : Tomorrow is 2nd october. We have a holiday on the occasion of Gandhiji's birthday. So why don't you do your home work tomorrow?

Siddu : No, first I will finish the homework then I will come to your home.

Vivek : Ok.

Consider the following statements from the above conversation:

Most likely, I will watch India v/s Australia cricket match

There is ***no chance*** for me to watch the cricket match.

Both teams have ***equal chance*** to win the toss.

Here Vivek and Siddu are making judgements about the chances of the particular occurrence.

In many situations we make such statements and use our past experience and logic to take decisions. For example

It is a bright and pleasant sunny day. I need not carry my umbrella and will take a chance to go.

However, the decisions may not always favour us. Consider the situation. "Mary took her umbrella to school regularly during the rainy season. She carried the umbrella to school for many days but it did not rain during her walk to the school. However, by chance, one day she forgot to take the umbrella and it rains heavily on that day".

Usually the summer begins from the month of March, but oneday in that month there is a heavy rainfall in the evening. Luckly Mary escaped becoming wet, because she carried umbrella on that day as she does daily.

Thus we take a decision by guessing the future happening that is whether an event occurs or not. In the above two cases Mary guessed the occurances and non-occurance of the event of raining on that day. Our decision comes true and sometimes it may not. (Why?)

We try to measure numerically the chance of occurance or non-occurance of some events just as we measure many other things in our daily life. This kind of measurement helps us to take decision in a more systematic manner. Therefore we study probability to figure out the chance of something happening.

Before measuring numerically the chance of happening that we have discussed in the above situations, we grade it using the following terms given in the table. Let us observe the following table.

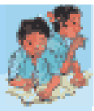
Term	Chance	Examples from conversation
certain	something that must occur	Gandhiji's birthday is on 2nd October.
more likely	something that would occur with great chance	Vivek watching the cricket match
equally likely	somethings that have the same chance of occurring	Both teams winning the toss.
less likely	Some thing that would occur with less chance	Vivek doing homework on the day of cricket match.
impossible	Something that cannot happen.	Sidhu watching the circket match at his home.

Equally likely

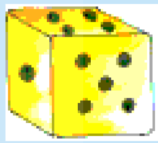
(Getting rain in the month March)

(Getting the cold wave in the last week of December)

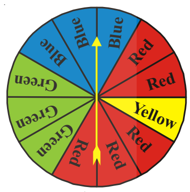
Do This



1. Observe the table given in the previous page and give some other example for each term.
2. Classify the following statements into the categories ***less likely, equally likely, more likely.***
   a) Rolling a die* and getting a number 5 on the top face.
   b) Getting a cold wave in your village in the month of November.
   c) India winning the next soccer(foot ball)world cup
   d) Getting a tail or head when a coin is tossed.
   e) You buy a lottery ticket and win the jackpot.



14.2 Probability



spinner

14.2.1 Random experiment and outcomes

To understand and measure the chance, we perform the experiments like tossing a coin, rolling a die and spining the spinner etc.

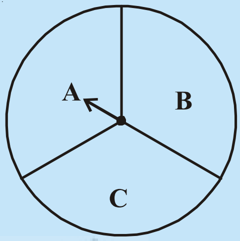
When we toss a coin we have only two possible results, head or tail. Suppose you are the captain of a cricket team and your friend is the captain of the other cricket team. You toss the coin and ask your friend to choose head or tail. Can you control the result of the toss? Can you get a head or tail that you want? In an ordinary coin that is not possible. The chance of getting either is same and you cannot say what you would get. Such tossing is like an experiment known as 'random experiment'. In such experiments though we know the possible outcomes before conducting the experiment, we cannot predict the exact outcome that occurs at a particular time, in advance. The outcomes of random experiments may be equally likely or may not be. In the coin tossing experiment head or tail are two possible outcomes.

* A die (plural dice) is a well balanced cube with its six faces marked with numbers from 1 to 6, one number on each face. Sometimes dots appear in place of numbers.

TRY THESE

1. If you try to start a scooter , What are the possible outcomes?
2. When you roll a die, What are the six possible outcomes?
3. When you spin the wheel shown, What are the possible outcomes?

   (Out comes here means the possible sector where the pointer stops)



4. You have a jar with five identical balls of different colours (White, Red, Blue, Grey and Yellow)and you have to pickup (draw) a ball without looking at it. List the possible outcomes you get.

THINK, DISCUSS AND WRITE

In rolling a die.

- Does the first player have a greater chance of getting a six on the top face?
- Would the player who played after him have a lesser chance of getting a six on the top face?
- Suppose the second player got a six on the top face. Does it mean that the third player would not have a chance of getting a six on the top face?

14.2.2 Equally likely outcomes

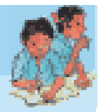
When we toss a coin or roll a die , we assume that the coin and the die are fair and unbiased i.e. for each toss or roll the chance of all possibilities is equal. We conduct the experiment many times and collect the observations. Using the collected data, we find the measure of chance of occurrence of a particular happening.

A coin is tossed several times and number of times we get head or tail is noted. Let us look at the result sheet where we keep on increasing the tosses.

Number of tosses	Tally marks (Heads)	Number of heads	Tally mark (Tails)	Number of tails
50	𝍸 𝍸 𝍸 𝍸 ||	22	𝍸 𝍸 𝍸 𝍸 𝍸 |||	28
60	𝍸 𝍸 𝍸 𝍸 𝍸 |	26	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	34
70	......	30	......	40
80	......	36	......	44
90	......	42	......	48
100	......	48	......	52

We can observe from the above table as you increase the number of tosses more and more, the number of heads and the number of tails come close to each other.

DO THIS



Toss a coin for number of times as shown in the table. And record your findings in the table.

No. of Tosses	Number of heads	No. of tails
10		
20		
30		
40		
50		

What happens if you increase the number of tosses more and more.

This could also be done with a die, roll it for large number of times and observe.

No. of times Die rolled	Number of times each outcome occured (i.e. each number appearing on the top face)					
	1	2	3	4	5	6
25	4	3	9	3	3	3
50	9	5	12	9	8	7
75	14	10	16	12	10	13
100	17	19	19	16	13	16
125	25	20	24	18	16	22
150	28	24	28	23	21	26
175	31	30	33	27	26	28
200	34	34	36	30	32	34
225	37	38	40	34	38	38
250	40	40	43	40	43	44
275	44	41	47	47	47	49
300	48	47	49	52	52	52

From the above table, it is evident that rolling a die for a larger number of times, the each of six outcomes, becomes almost equal to each other.

From the above two experiments, we may say that the different outcomes of the experiment are equally likely. This means each of the outcome has equal chance of occurring.

14.2.3 Trail and Events

In the above experiments each toss of a coin or each roll of die is a **Trial** or **Random experiment**.

Consider a trial of rolling a die,

How many possible outcomes are there to get a number more than 5 on the top face?

It is only one (i.e., 6)

How many possible outcomes are there to get an even number on the top face?

They are 3 outcomes (2,4, and 6).

Thus each specific outcome or the collection of specific outcomes make an **Event**.

In the above trail getting a number more than 5 and getting an even number on the top face are two events. Note that event need not necessarily a single outcome. But, every outcome of a random experiment is an event.

Here we understand the basic idea of the event, more could be learnt on event in higher classes.

14.2.4 Linking the chance to Probability

Consider the experiment of tossing a coin once. What are the outcomes? There are only two outcomes Head or Tail and both outcomes are equally likely.

What is the chance of getting a head?

It is one out of two possible outcomes i.e. $\frac{1}{2}$. In other words it is expressed as the probability of getting a head when a coin is tossed is $\frac{1}{2}$, which is represented by

$$P(H) = \frac{1}{2} = 0.5 \text{ or } 50\%$$

What is the probability of getting a tail?

Now take the example of rolling a die. What are the possible outcomes in one roll? There are six equally likely outcomes 1,2,3,4,5,or 6.

What is the probability of getting an odd number on the top face?

1, 3 or 5 are the three favourable outcomes out of six total possible outcomes. It is $\frac{3}{6}$ or $\frac{1}{2}$

We can write the formula for Probability of an event 'A'

$$P(A) = \frac{\text{Number of favourable outcomes for event 'A'}}{\text{Number of total possible outcomes}}$$

Now let us see some examples :

Example 1: If two identical coins are tossed for one time simultaneously. Find (a) the possible outcomes, (b) the number of total outcomes, (c) the probability of getting two heads, (d) probability of getting atleast one head, (e) probability of getting no heads and (f) probability of getting only one head.

Solution : (a) The possible outcomes are

Coin 1	Coin 2
Head	Head
Head	Tail
Tail	Head
Tail	Tail

b) Number of total possible outcomes is 4

c) Probability of getting two heads

$$= \frac{\text{Number of favourable outcomes of getting two heads}}{\text{Number of total possible outcomes}} = \frac{1}{4}$$

d) Probability of getting atleast one head $= \frac{3}{4}$

[At least one head means getting a head one or more number of times]

e) Probability of getting no heads $= \frac{1}{4}$.

e) Probability of getting only one head $= \frac{2}{4} = \frac{1}{2}$.

DO THIS

1. If three coins are tossed simultaneously
   a) Write all possible outcomes
   b) Number of possible outcomes
   c) Find the probability of getting at least one head
      (getting one or more than one head)
   d) Find the Probability of getting at most two heads
      (getting Two or less than two heads)
   e) Find the Probability of getting no tails

Example 2 : (a) Write the probability of getting each number top face when a die was rolled in the following table. (b) Find the sum of the probabilities of all outcomes.

Solution : (a) Out of six possibilities only once the number 4 occurs hence probability is 1/6. Similarly we fill the remaining

Outcome	1	2	3	4	5	6
Probability (P)				1/6		

(b) The sum of all probabilities

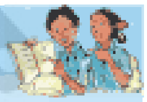
$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6)$$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$

We can generalize that

Sum of the probabilities of all outcomes of a random experiment is always 1

TRY THIS



Find the probability of each event when a die is roll once

Event	Favourable outcome(s)	Number of favourable outcome(s)	Total possible outcomes	Number of total possible outcomes	Probability = $\frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$
Getting a number 5 on the top face	5	1	1, 2, 3, 4, 5 and 6	6	1/6
Getting a number more than 3 on the top face					
Getting a prime numberon the top face					
A number less than 5 on the top face					
A number that is factor of 6 on the top face					
A number more than 7 on the top face					
A number Multiple of 3 on the top face					
Getting a number 6 or less than 6 on the top face					

You can observe that

The probability of an event always lies between 0 and 1 (0 and 1 inclusive)

$$0 \leq \text{probability of an event} \leq 1$$

a) The probability of an event which is certain = 1

b) The probability of an event which is impossible = 0

14.2.5 Conduct your own experiments

1. We would work here in groups of 3-4 students each. Each group would take a coin of the same denomination and of the same type. In each group one student of the group would toss the coin 20 times and record the data. The data of all the groups would be placed in the table below (Examples are shown in the table).

Group No.	No. of tosses	Cumulative tosses of groups	Number of heads	Cumulative No. of heads	$\frac{\text{Cumulative heads}}{\text{total times tossed}}$	$\frac{\text{Cumulative tails}}{\text{total times tossed}}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	20	20	7	7	$\frac{7}{20}$	$\frac{20-7}{20} - \frac{13}{20}$
2	20	40	14	21	$\frac{21}{40}$	$\frac{40-21}{40} = \frac{19}{40}$
3	20	60				
4	20	80				
5	20	100				
6	.....	....				
7	....	....				

What happens to the value of the fractions in (6) and (7) when the total number of tosses of the coin increases? Could you see that the values are moving close to the probability of getting a head and tail respectively.

2. In this activity also we would work in groups of 3-4. Students each one student from each group would roll a die for 30 times. Other students would record the data in the following table. All the groups should have the same kind of die so all throws will be treated as the throws of the same die.

No. of times Die rolled	Number of times these outcomes turn up					
	1	2	3	4	5	6
30						

Using the data from all the groups complete the following table :

Group(s)	Number of times 1 turned up	Total number of times a die is rolled	$\frac{\text{Number of times 1 turned up}}{\text{Total number of times a die is rolled}}$
(1)	(2)	(3)	(4)
1^{st}			
$1^{st}+2^{nd}$			
$1^{st}+2^{nd}+3^{rd}$			
$1^{st}+2^{nd}+3^{rd}+4^{th}$			
$1^{st}+2^{nd}+3^{rd}+4^{th}+5^{th}$			

What do you observe as the number of rolls gets larger; the fractions in cloumn (4) move closer to $\frac{1}{6}$. We did the above experiment for the outcome 1. Check the same for number of times outcome 2 turned up and outcome 5 turned up.

What can you conclude about the fraction values you get in column (4) and compare these with the probabilities of getting 1, 2, and 5 on rolling a die.

3. What would happen if we toss two coins simultaneously? We could have either both coins showing head, both showing tail or one showing head and one showing tail. Would the possibility of occurence of these three be the same? Think about this while you do this group activity.

Divide class into small groups of 4 each. Each group take two coins. Note all the coins used in the class should be of the same denomination and of the same type. Each group would throw the two coins simultaneously 20 times and record the observations in a table.

No. of times two coins tossed	No. of times no head comes up	Number of times one head comes up	Number of times two heads come up
20			

All the groups should now make a cummulative table:

Group(s)	Number of times two coins are tossed	Number of times no head comes up	Number of times one head comes up	Number of times two heads come up
1^{st}				
$1^{st} + 2^{nd}$				
$1^{st} + 2^{nd} + 3^{rd}$				
$1^{st} + 2^{nd} + 3^{rd} + 4^{th}$				
....				

Now we find the ratio of the number of times no head comes up to the total number of times two coins are tossed. Do the same for the one head and two heads up events also.

Fill the following table:

Group(s)	$\frac{\text{No. of times no head}}{\text{Total tosses}}$	$\frac{\text{No. of times one head}}{\text{Total tosses}}$	$\frac{\text{No. of times two heads}}{\text{Total tosses}}$
(1)	(2)	(3)	(4)
Group 1^{st}			
Group $1 + 2^{nd}$			
Group $1 + 2 + 3^{rd}$			
Group $1 + 2 + 3 + 4^{th}$			
....			

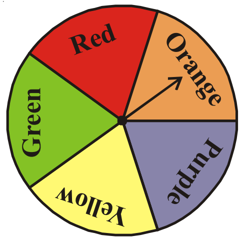
As the number of tosses increases, the values of the columns (2), (3) and (4) get closer to 0.25, 0.5 and 0.25 respectively.

Example-3: A spinner was spun 1000 times and the frequency of outcomes was recorded as in given table:

Out come	Red	Orange	Purple	Yellow	Green
Frequency	185	195	210	206	204

Find (a) List the possible outcomes that you can see in the spinner (b) Compute the probability of each outcome. (c) Find the ratio of each outcome to the total number of times that the spinner spun (use the table)

Solution :



spinner

(a) The possible outcomes are 5. They are red, orange, purple, yellow, green. Here all the five colours occupy same area in the spinner. They are all equally likely.

(b) Compute the probability of each event.

$$P(\text{Red}) = \frac{\text{Favourable outcomes of red}}{\text{Total number of possible outcomes}}$$

$$= \frac{1}{5} = 0.2.$$

Similarly

P(Orange), P(Purple), P(Yellow) and P(Green) is also $\frac{1}{5}$ or 0.2.

(c) From the experiment the frequency was recorded in the table

$$\text{Ratio for red} = \frac{\text{No. of outcomes of red in the above experiment}}{\text{Number of times the spinner was spun}}$$

$$= \frac{185}{1000} = 0.185$$

Similarly, we can find the corresponding ratios for orange, purple, yellow and green are 0.195, 0.210, 0.206 and 0. 204 respectively.

Can you see that each of the ratio is approximately equal to the probability which we have obtained in (b) [i.e. before conducting the experiment]

Example-4. The following table gives the ages of people in the audience at a movie theatre. Each person was given a serial number and a person was selected randomly for the bumper prize by choosing a serial number. Now find the probability of each event.

Age	Male	Female
Under 2	3	5
3 - 10 years	24	35
11 - 16 years	42	53
17 - 40 years	121	97
41- 60 years	51	43
Over 60	18	13

Toatal number of audience : 505

Find the probability of each event given below.

Solution :

a) The probability of audience of age less than or equal to 10 years

The audience of age less than or equal to 10 years = 24 + 35 + 5 + 3 = 67

Total number of people = 505

$$P(\text{audience of age} \leq 10 \text{ years}) = \frac{67}{505}$$

b) The probability of female audience of age 16 years or younger

The female audience with age less than or equal 16 years = 53 + 35 + 5 = 93

P(female audience of age $\leq$ 16 years) = 93/505

c) The probability of male audience of age 17 years or above

$= 121 + 51 + 18 = 190$

$$P(\text{male audience of age} \geq 17 \text{ years}) = \frac{190}{505} = \frac{38}{101}$$

d) The probability of audience of age above 40 years

$= 51+43+18+ 13 = 125$

$$P(\text{audience of age} > 40 \text{ years}) = \frac{125}{505} = \frac{25}{101}$$

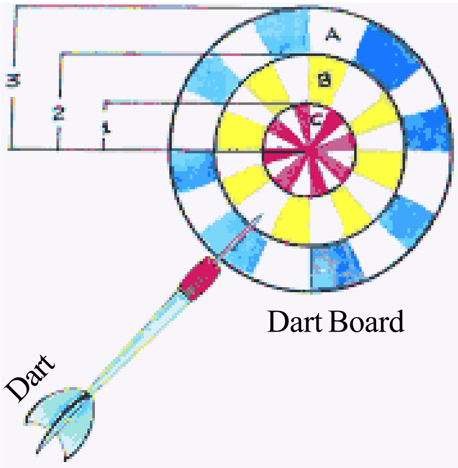
e) The probability of the person watching the movie is not a male

$= 5 + 35 + 53 + 97 + 43 + 13 = 246$

$$P(\text{A person watching movie is not a male}) = \frac{246}{505}$$

Example-5 : Assume that a dart will hit the dart board and each point on the dart board is equally likely to be hit in all the three concentric circles where radii of concetric circles are 3 cm, 2 cm and 1 cm as shown in the figure below.

Find the probability of a dart hitting the board in the region A. (The outer ring)



Solution : Here the event is hitting in region A.

The Total area of the circular region with radius 3 cm

$= \pi(3)^2$

Area of circular region A (i.e. ring A) = $\pi(3)^2 - \pi(2)^2$

Probability of dart hitting dart board in region A is

$$P(A) = \frac{\text{Area of circular region A}}{\text{Total Area}}$$

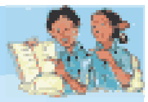
$$= \frac{\pi(3)^2 - \pi(2)^2}{\pi(3)^2}$$

$$= \frac{9\pi - 4\pi}{9\pi}$$

$$\frac{5}{9} = 0.556 = 55.6\%$$

> Remember
>
> Area of circular is πr^2
>
> Area of ring = $\pi R^2 - \pi r^2$

TRY THESE



From the figure given in example 5.

1. Find the probability of the dart hitting the board in the circular region B (i.e. ring B).
2. Without calculating, write the percentage of probability of the dart hitting the board in circular region C (i.e. ring C).

14.3 USES OF PROBABILITY IN REAL LIFE

- Meteorological department predicts the weather by observing trends from the data collected over many years in the past.
- Insurance companies calculate the probability of happening of an accident or casuality to determine insurance premiums.
- "An exit poll" is taken after the election . This involves asking voted people to which party they have voted. This gives an idea of winning chances of each candidate and predictions are made accordingly.

EXERCISE - 14.1

1. A die has six faces numbered from 1 to 6. It is rolled and number on the top face is noted. When this is treated as a random trial.

   a) What are the possible outcomes ?

   b) Are they equally likely? Why?

   c) Find the probability of a composite number turning up on the top face.

2. A coin is tossed 100 times and the following outcomes are recorded

   Head:45 times   Tails:55 times from the experiment

   a) Compute the probability of each outcomes.

   b) Find the sum of probabilities of all outcomes.

3. A spinner has four colours as shown in the figure. When we spin it once, find

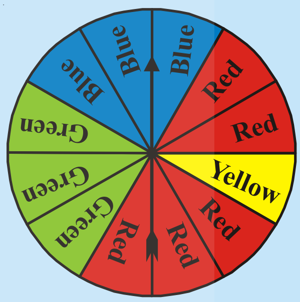
   a) At which colour, is the pointer more likely to stop?

   b) At which colour, is the pointer less likely to stop?

   c) At which colours, is the pointer equally likely to stop?

   d) What is the chance the pointer will stop on white?

   e) Is there any colour at which the pointer certainly stops?



4. A bag contains five green marbles, three blue marbles, two red marbles, and two yellow marbles. One marble is drawn out randomly.

   a) Are the four different colour outcomes equally likely? Explain.

   b) Find the probability of drawing each colour marble

   i.e. , P(green), P(blue), P(red) and P(yellow)

   c) Find the sum of their probabilities.

5. A letter is chosen from English alphabet. Find the probability of the letters being

   a) A vowel   b) a letter comes after P

   c) A vowel or a consonant   d) Not a vowel

6. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):

   4.97, 5.05, 5.08, 5.03, 5.00, 5.06, 5.08, 4.98, 5.04, 5.07, 5.00

   Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

7. An insurance company selected 2000 drivers at random (i.e., without any preference of one driver over another) in a particular city to find a relationship between age and accidents. The data obtained is given in the following table:

Age of Drivers (in years)	Accidents in one year				More than 3 accidents
	0	1	2	3	
18-29	440	160	110	61	35
30- 50	505	125	60	22	18
Over 50	360	45	35	15	9

   Find the probabilities of the following events for a driver chosen at random from the city:

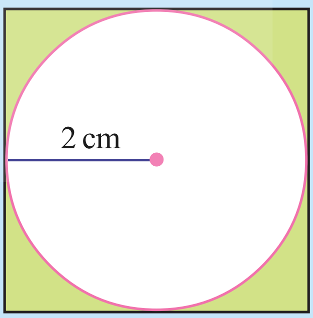
   (i) The driver being in the age group 18-29 years and having exactly 3 accidents in one year.

   (ii) The driver being in the age group of 30-50 years and having one or more accidents in a year.

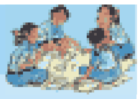
   (iii) Having no accidents in the year.

8. What is the probability that a randomly thrown dart that hits the square board in shaded region

   (Take $\pi = \frac{22}{7}$ and express in percentage)



WHAT WE HAVE DISCUSSED



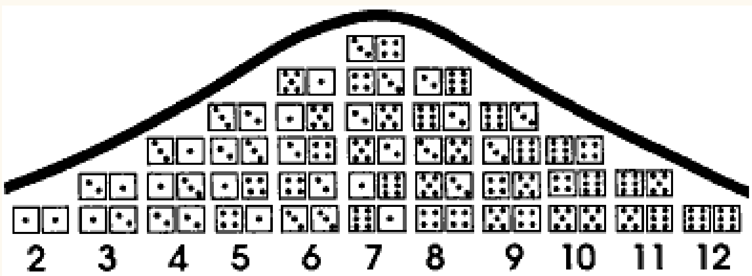
- There is use of words like most likely, no chance, equally likely in daily life, are showing the manner of chance and judgement.
- There are certain experiments whose outcomes have equal chance of occuring. Outcomes of such experiments are known as **equally likely** outcomes.
- An event is a collection of a specific outcome or some of the specific outcomes of the experiment.
- In some random experiments all outcomes have equal chance of occuring.
- As number of trials increase, the probability of all equally likely outcomes come very close to each other.
- The probability of an event A

$$P(A) = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

- The probability of an event which is certain = 1.
- The probability of an event which is impossible = 0
- The probability of an event always lies between 0 and 1 (0 and 1 inclusive).

Do you Know?

The diagram below shows the 36 possible outcomes when a pair of dice are thrown. It is interesting to notice how the frequency of the outcomes of different possible numbers (2 through 12) illustrate the Gaussian curve.



This curve illustrate the Gaussian curve, name after 19th century famous mathematician Carl Friedrich Gauss.

15 Proofs in Mathematics

15.1 Introduction

We come across many statements in our daily life. We gauge the worth of each statement. Some statements we consider to be appropriate and true and some we dismiss. There are some we are not sure of. How do we make these judgements? In case there is a statement of conflict about loans or debts. You want to claim that bank owes your money then you need to present documents as evidence of the monetary transaction. Without that people would not believe you. If we think carefully we can see that in our daily life we need to prove if a statement is true or false. In our conversations in daily life we sometimes do not consider it relevant to prove or check statements and accept them without serious examination. That however is not at all accepted in mathematics. Consider the following:

1. The sun rises in the east.
2. $3 + 2 = 5$
3. New York is the capital of USA.
4. $4 > 8$
5. How many siblings do you have?
6. Goa has better football team than Bengal.
7. Rectangle has 4 lines of symmetry.
8. $x + 2 = 7$
9. Please come in.
10. The probability of getting two consecutive 6's on throws of a 6 sided dice is ?
11. How are you?
12. The sun is not stationary but moving at high speed all the time.
13. $x < y$
14. Where do you live?

Out of these some sentences we know are false. For example, 4>8. Similarly we know that at present New York is not the capital of USA. Some we can say are correct from our present knowledge. These include "sun rises in the east." The probability............"

The Sun is not stationary......................

Besides those there are some other sentences that are true for some known cases but not true for other cases, for example $x + 2 = 7$ is true only when $x = 5$ and $x < y$ is only true for those values of x and y where x is less than y.