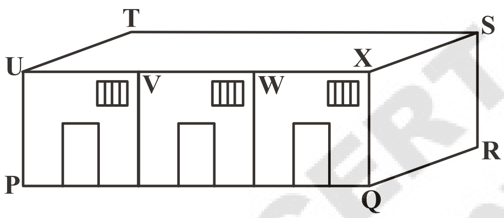
04

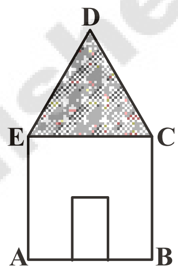
Lines and Angles

4.1 Introduction

Reshma and Gopi have drawn the sketches of their school and home respectively. Can you identify some angles and line segments in these sketches?



(i)



(ii)

In the above figures (PQ, RS, ST, ...) and (AB, BC, CD, ...) are examples of line segments. Where as ∠UPQ, ∠PQR, ... and ∠EAB, ∠ABC, ... are examples of some angles.

Do you know whenever an architect has to draw a plan for buildings, towers, bridges etc., the architect has to draw many lines and parallel lines at different angles.

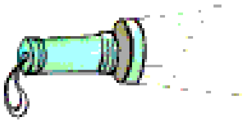
In science say in Optics, we use lines and angles to assume and draw the movement of light and hence the images are formed by reflection, refraction and scattering. Similarly while finding how much work is done by different forces acting on a body, we consider angles between force and displacement to find resultants. To find the height of a place we need both angles and lines. So in our daily life, we come across situations in which the basic ideas of geometry are in much use.

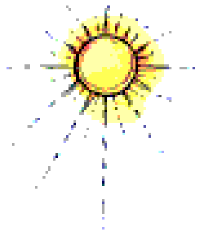
Do This

Observe your surroundings carefully and write any three situations of your daily life where you can observe lines and angles.

Draw the pictures in your note book and collect some pictures.

4.2 Basic Terms in Geometry





Think of a light beam originating from the sun or a torch light. How do you represent such a light beam? It's a ray starting from the sun. Recall that "a ray is a part of a line. It begins at a point and goes on endlessly in a specified direction. While line can be extended in both directions endlessly.

A part of a line with two end points is known as line segment. 

We usually denote a line segment AB by $\overline{AB}$ and its length in denoted by AB. The ray AB is denoted by $\overrightarrow{AB}$ and a line is denoted by $\overleftrightarrow{AB}$. However we normally use $\overleftrightarrow{AB}$, $\overleftrightarrow{PQ}$ for etc. lines and some times small letters l, m, n etc. will also be used to denote lines.

If three or more points lie on the same line, they are called collinear points, otherwise they are called non-collinear points.

Sekhar marked some points on a line and try to count the line segments formed by them. (Note $\overline{PQ}$ and $\overline{QP}$ represents the same line segment)

S.No.	Points on line	Line Segments	Number
1.	P R Q	PQ, PR, RQ	3
2.	P S R Q	PQ, PR, PS, SR, SQ, RQ	6
3.	P S T R Q	....	

Do you find any pattern between the number of points and line segments?

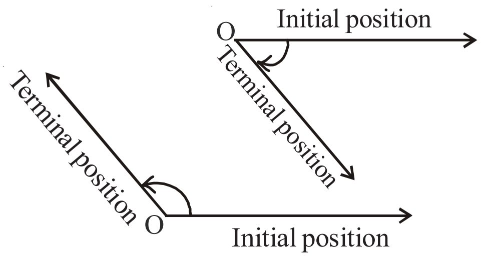
Take some more points on the line and find the pattern:

No. of points on line segment	2	3	4	5	6	7
Total no. of line segments	1	3	6	.....	.....	.....

A circle is divided into 360 equal parts as shown in the figure.

The measure of each part is called one degree.

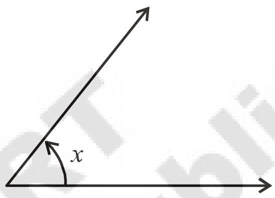
The angle is formed by rotating a ray from an initial position to a terminal position.



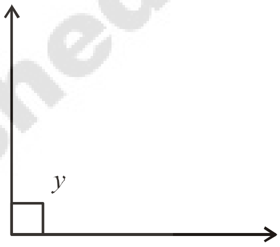
The change of a ray from initial position to terminal position around the fixed point 'O' is called rotation and measure of rotation is called angle.

One complete rotation gives 360^0. We also draw angles with compass.

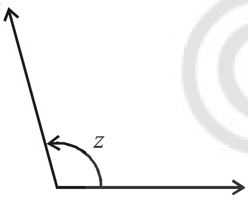
An angle is formed when two rays originate from the same point. The rays making an angle are called **arms** of the angle and the common point is called **vertex** of the angle. You have studied different types of angles, such as acute angle, right angle, obtuse angle, straight angle and reflex angle in your earlier classes.



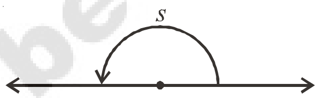
acute angle : $0° < x < 90°$



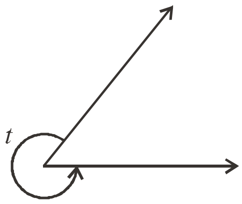
right angle : $y = 90°$



obtuse angle : $90° < z < 180°$



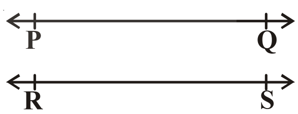
straight angle : $s = 180°$



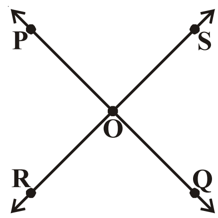
reflex angle : $180° < t < 360°$

4.2.1 Intersecting Lines and Non-intersecting Lines

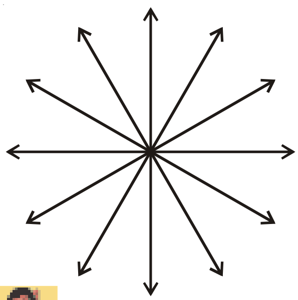
Observe the figure. Do the lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ have any common points? What do we call such lines? They are called parallel lines.



On the other hand if they meet at any point, then they are called intersecting lines.



4.2.2 Concurrent Lines



How many lines can meet at a single point? Do you know the name of such lines? When three or more lines meet at a point, they are called concurrent lines and the point at which they meet is called point of concurrence.

THINK, DISCUSS AND WRITE

What is the difference between intersecting lines and concurrent lines?

EXERCISE - 4.1

1. In the given figure, name:

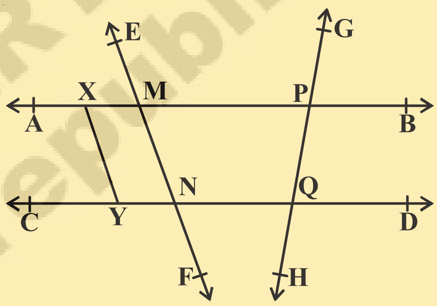
   (i) any six points

   (ii) any five line segments

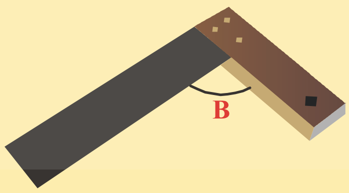
   (iii) any four rays

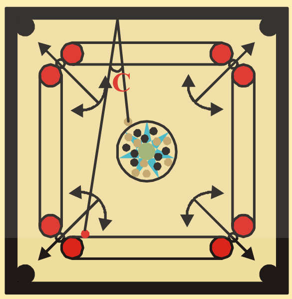
   (iv) any four lines

   (v) any four collinear points



2. Observe the following figures and identify the type of angles in them.





3. State whether the following statements are true or false :

   (i) A ray has no end point.

   (ii) Line $\overleftrightarrow{AB}$ is the same as line $\overleftrightarrow{BA}$.

   (iii) A ray $\overrightarrow{AB}$ is same as the ray $\overrightarrow{BA}$.

   (iv) A line has a define length.

   (v) A plane has length and breadth but no thickness.

(vi) Two distinct points always determine a unique line.

(vii) Two lines may intersect in two points.

(viii) Two intersecting lines cannot both be parallel to the same line.

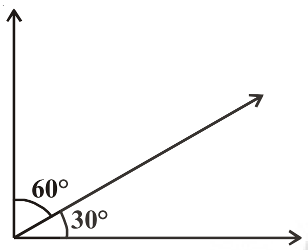
4. What is the angle between two hands of a clock when the time in the clock is

(a) 9'O clock (b) 6'O clock (c) 7:00 PM

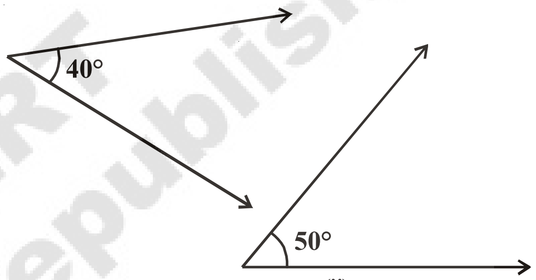
4.3 PAIRS OF ANGLES

Now let us discuss about some pairs of angles.

Observe the following figures and find the sum of angles.



(i)



(ii)

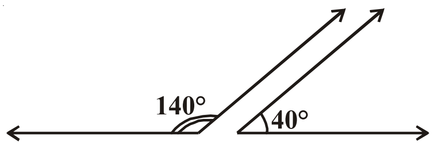
What is the sum of the two angles shown in each figure? It is 90^0. Do you know what do we call such pairs of angles? They are called complementary angles.

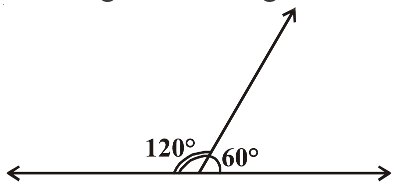
If a given angle is x^0, then what is its complementary angle? The complementary angle of x^0 is $(90^0 - x^0)$.

Example-1. If the measure of an angle is 62°, what is the measure of its complementary angle?

Solution : As the sum is 90°, the complementary angle of 62° is 90° - 62° = 28°

Now observe the following figures and find the sum of angles in each figure.





What is the sum of the two angles shown in each figure? It is 180^0. Do you know what do we call such pair of angles? Yes, they are called supplementary angles. If the given angle is x^0, then what is its supplementary angle ? The supplementary angle of x^0 is $(180° - x°)$.

Example-2. Two complementary angles are in the ratio 4:5. Find the angles.

Solution : Let the required angles be $4x$ and $5x$.

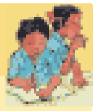
Then $4x + 5x = 90^0$ (Why?)

$9x = 90^0 \Rightarrow x = 10^0$

Hence the required angles are 40^0 and 50^0.

Now observe the pairs of angles such as (120°, 240°) (100°, 260°) (180°, 180°) (50°, 310°) ..... etc. What do you call such pairs? The pair of angles, whose sum is 360° are called conjugate angles. Can you say the conjugate angle of 270°? What is the conjuage angle of $x°$?

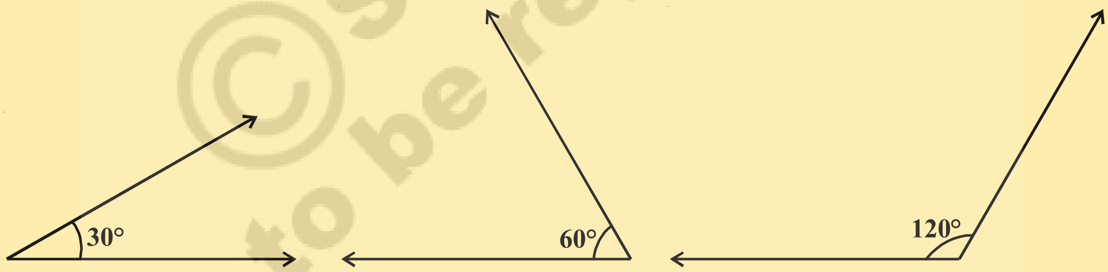
DO THESE



1. Write the complementary, supplementary and conjugate angles for the following angles.

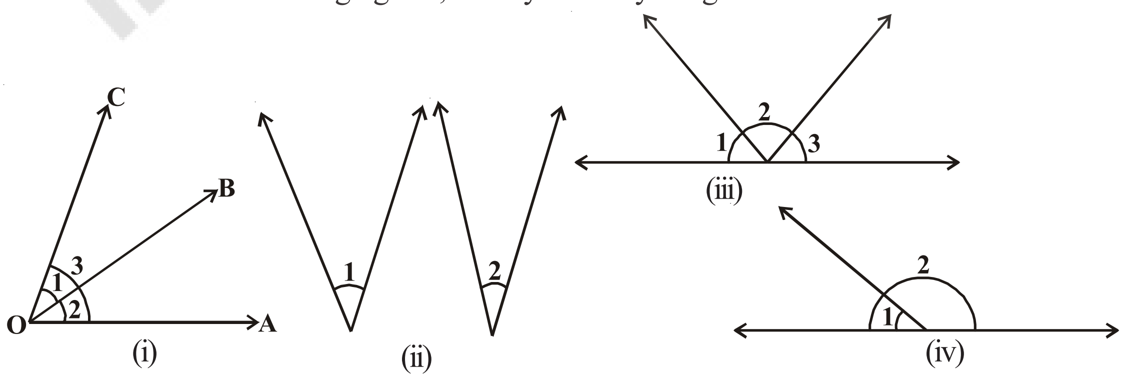
(a)	45^0	(b)	75^0	(c)	215°	(d)	30^0
(e)	60^0	(f)	90°	(g)	180°		

2. Which pairs of following angles become complementary or supplementary angles?



(i) (ii) (iii)

Observe the following figures, do they have any thing in common?



In figure (i) we can observe that vertex 'O' and arm '$\overrightarrow{OB}$' are common to both $\angle 1$ and $\angle 2$. What can you say about the non-common arms and how are they arranged? They are arranged on either side of the common arm. What do you call such pairs of angles?

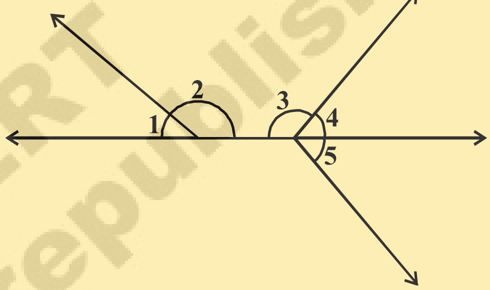
They are called a pair of adjacent angles.

In fig.(ii), two angles $\angle 1$ and $\angle 2$ are given. They have neither a common arm nor a common vertex. So they are not adjacent angles.

TRY THIS

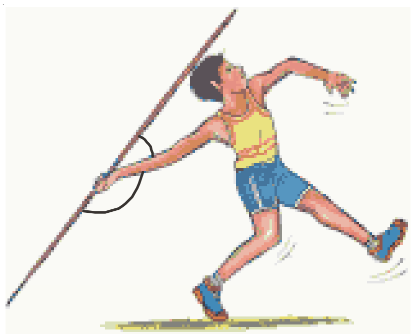
(i) Find pairs of adjacent and non-adjacent angles in the above figures (i, ii, iii & iv).

(ii) List the adjacent angles in the given figure.



From the above, we can conclude that pairs of angles which have a common vertex, a common arm and non common arms lie on either side of common arm are called adjacent angles.

Observe the given figure. The hand of the athlete is making angles with the Javelin. What kind of angles are they? Obviously they are adjacent angles. Further what will be the sum of those two angles? Because they are on a straight line, the sum of the angles is 180^0. What do we call such pair of angles? They are called linear pair. So if the sum of two adjacent angles is 180^0, they are said to be a linear pair.



THINK, DISCUSS AND WRITE

Linear pair of angles are always supplementary. But supplementary angles need not form a linear pair. Why?

ACTIVITY

Measure the angles in the following figure and complete the table.

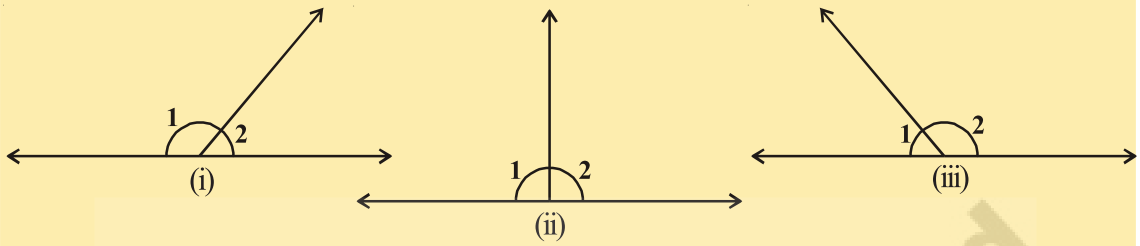
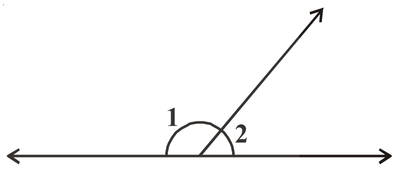


Figure	$\angle 1$	$\angle 2$	$\angle 1 + \angle 2$
(i)			
(ii)			
(iii)			

4.3.1 Linear pair of angles axiom

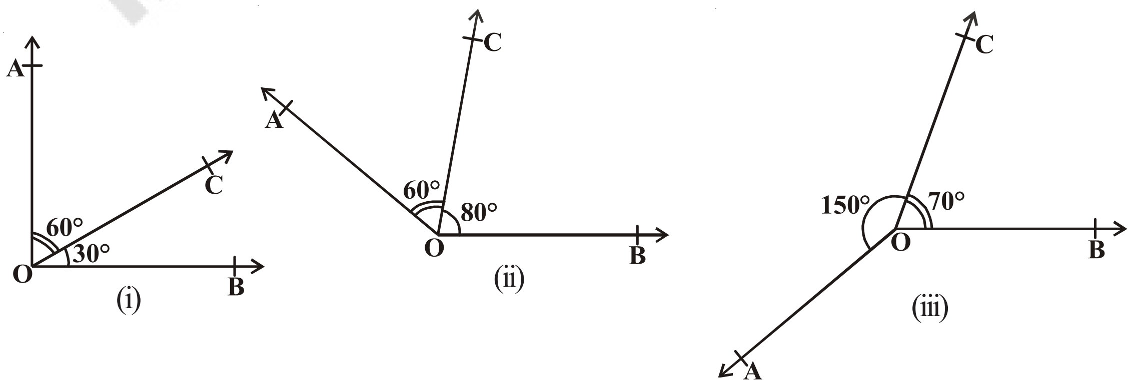
Axiom : If a ray stands on a straight line, then the sum of the two adjacent angles so formed is 180°.

When the sum of two adjacent angles is 180°, they are called a linear pair of angles.

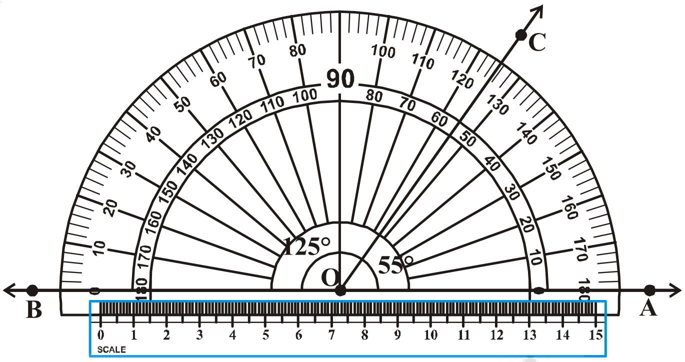


In the given figure, $\angle 1 + \angle 2 = 180^0$

Let us do the following. Draw adjacent angles of different measures as shown in the fig. Keep the ruler along one of the non-common arms in each case. Does the other non-common arm lie along the ruler?

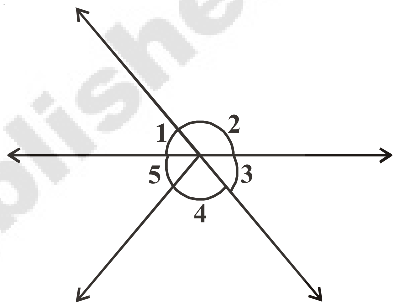


You will find that only in fig. (iv), both the non-common arms lie along the ruler, that is non common arms from a straight line. Also observe that $\angle AOC + \angle COB = 125° + 55° = 180°$. In other figures it is not so.



(iv)

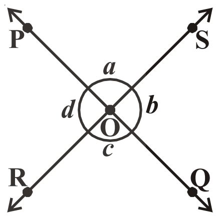
Axiom : If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line. This is the converse of linear pair of angle axiom.



Angles at a point : We know that the sum of all the angles around a point is always 360°.

In the given figure $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360^0$

4.3.2 Angles in intersecting lines



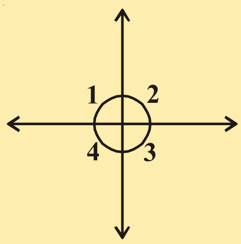
Draw any two intersecting lines and label them. Identify the linear pairs of angles and write down in your note book. How many pairs are formed?

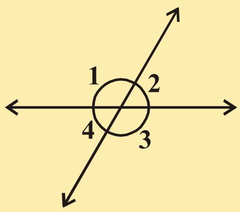
In the figure, $\angle POS$ and $\angle ROQ$ are opposite angles with same vertex and have no common arm. So they are called as **vertically opposite angles.** (Some times called vertical angles).

How many pairs of vertically opposite angles are there? Can you find them? (See figure)

ACTIVITY :

Measure the four angles 1, 2, 3, 4 in each of the above figure and complete the table:





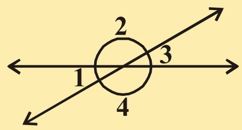
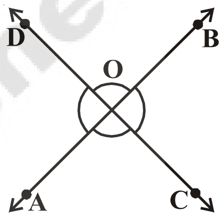


Figure	$\angle 1$	$\angle 2$	$\angle 3$	$\angle 4$
(i)				
(ii)				
(iii)				

What do you observe about the pairs of vertically opposite angles? Are they equal? Now let us prove this result in a logical way.

Theorem-4.1 : If two lines intersect each other, then the pairs of vertically opposite angles thus formed are equal.



Given: AB and CD be two lines intersecting at O

Required to prove (R.T.P.)

(i) $\angle AOC = \angle BOD$

(ii) $\angle AOD = \angle BOC$.

Proof:

Ray $\overrightarrow{OA}$ stands on Line $\overleftrightarrow{CD}$

Therefore, $\angle AOC + \angle AOD = 180°$ [Linear pair angles axiom] .... (1)

Also $\angle AOD + \angle BOD = 180°$ [Why?] .... (2)

$\angle AOC + \angle AOD = \angle AOD + \angle BOD$ [From (1) and (2)]

$\angle AOC = \angle BOD$ [Cancellation of equal angles on both sides]

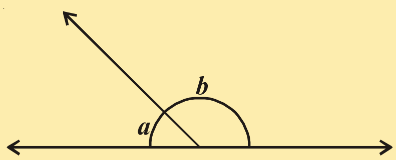
Similarly we can prove

$\angle AOD = \angle BOC$

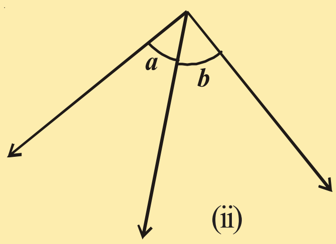
Do it on your own.

DO THIS

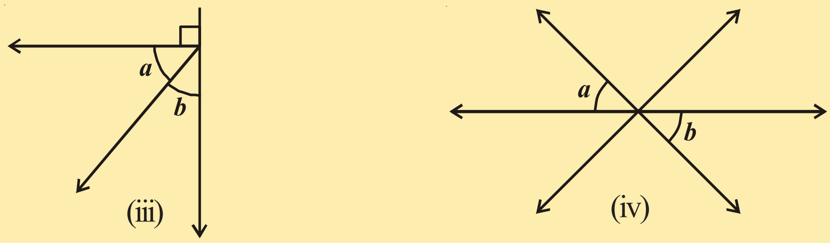
1. Classify the given angles as pairs of complementary, linear pair, vertically opposite and adjacent angles.



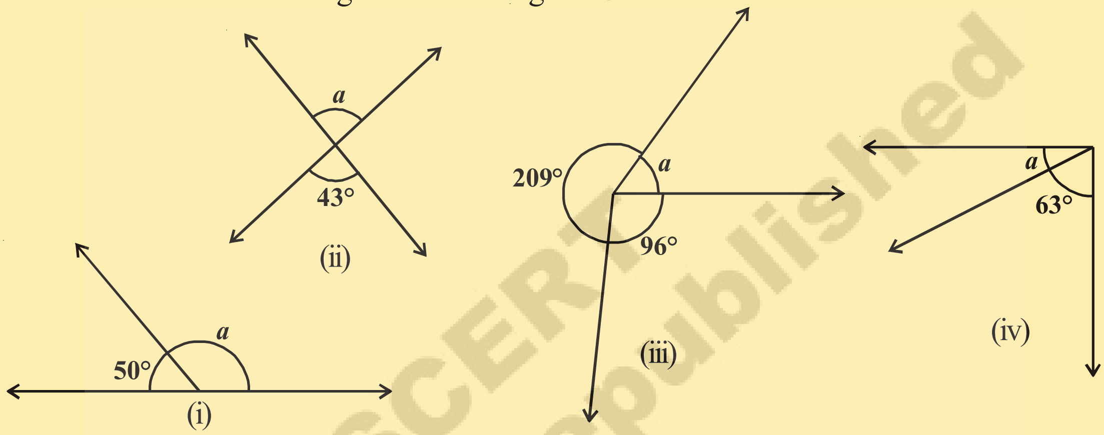
(i)



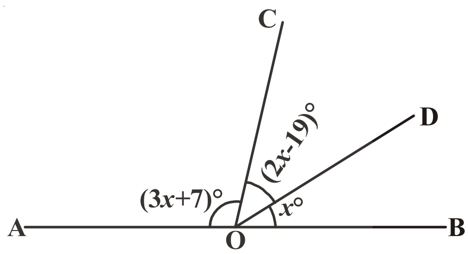
(ii)



2. Find the measure of angle 'a' in each figure. Give reason in each case.



Now, let us do some examples.



Example - 3. In the adjacent figure, $\overleftrightarrow{AB}$ is a straight line. Find the value of x and also find $\angle AOC$, $\angle COD$ and $\angle BOD$.

Solution : Since $\overleftrightarrow{AB}$ is a stright line, the sum of all the angles on $\overleftrightarrow{AB}$ at a point O is 180^0.

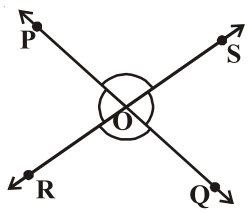
$\therefore (3x + 7)° + (2x - 19)° + x = 180°$ (Linear angles)

$\Rightarrow 6x - 12 = 180 \Rightarrow 6x = 192 \Rightarrow x = 32°$.

So, $\angle AOC = (3x + 7)° = (3 \times 32 + 7)^o = 103^o$,

$\angle COD = (2x - 19)° = (2 \times 32 - 19)^o = 45^o$, $\angle BOD = 32^o$.

Example - 4. In the adjacent figure lines PQ and RS intersect each other at point O. If $\angle POR : \angle ROQ = 5: 7$, find all the angles.



Solution : $\angle POR + \angle ROQ = 180°$ (Linear pair of angles)

But $\angle POR : \angle ROQ = 5 : 7$ (Given)

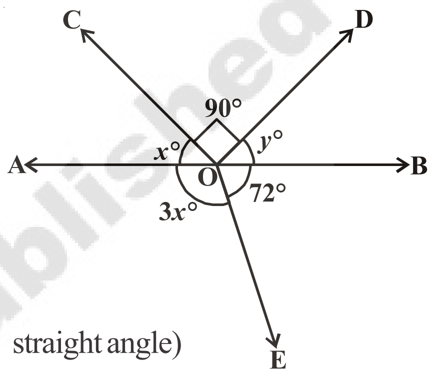
Therefore, $\angle POR = \frac{5}{12} \times 180 = 75°$

Similarly, $\angle ROQ = \frac{7}{12} \times 180 = 105°$

Now, $\angle POS = \angle ROQ = 105°$ (Vertically opposite angles)

and $\angle SOQ = \angle POR = 75°$ (Vertically opposite angles)

Example-5. Calculate $\angle AOC$, $\angle BOD$ and $\angle AOE$ in the adjacent figure given that $\angle COD = 90^o$, $\angle BOE = 72^o$ and AOB is a straight line,



Solution : Since AOB is a straight line, we have :

$\angle AOE + \angle BOE = 180^o$

$= 3x° + 72° = 180°$

$\Rightarrow 3x° = 108° \Rightarrow x = 36°.$

We also know that

$\therefore \angle AOC + \angle COD + \angle BOD = 180^0$ ($\because$ straight angle)

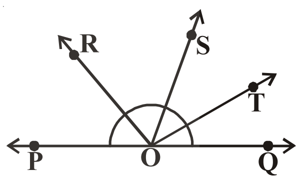
$\Rightarrow x° + 90° + y° = 180°$

$\Rightarrow 36° + 90° + y° = 180°$

$y° = 180° - 126° = 54°$

$\therefore \angle AOC = 36^o$, $\angle BOD = 54^o$ and $\angle AOE = 108^o$.

Example-6. In the adjacent figure ray OS stands on a line PQ. Ray OR and ray OT are angle bisectors of $\angle POS$ and $\angle SOQ$ respectively. Find $\angle ROT$.



Solution : Ray OS stands on the line PQ.

Therefore, $\angle POS + \angle SOQ = 180°$ (Linear pair)

Let $\angle POS = x°$

Therefore, $x° + \angle SOQ = 180°$ (How?)

So, $\angle SOQ = 180° - x°$

Now, ray OR bisects $\angle POS$, therefore,

$$\angle ROS = \frac{1}{2} \times \angle POS$$

$$= \frac{1}{2} \times x = \frac{x}{2}$$

Similarly, $\angle SOT = \frac{1}{2} \times \angle SOQ$

$= \frac{1}{2} \times (180° - x)$

$= 90° - \frac{x°}{2}$

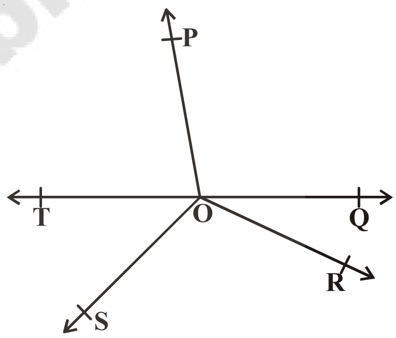
Now, $\angle ROT = \angle ROS + \angle SOT$

$= \frac{x°}{2} + \left(90° - \frac{x°}{2}\right)$

$= 90°$

Example-7. In the adjacent figure $\overrightarrow{OP}$, $\overrightarrow{OQ}$, $\overrightarrow{OR}$ and $\overrightarrow{OS}$ are four rays. Prove that

$\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°$.



Solution : In the given figure, you need to draw opposite ray to any of the rays $\overrightarrow{OP}$, $\overrightarrow{OQ}$, $\overrightarrow{OR}$ or $\overrightarrow{OS}$

Draw ray $\overrightarrow{OT}$ so that $\overleftrightarrow{TOQ}$ is a line. Now, ray OP stands on line $\overleftrightarrow{TQ}$.

Therefore, $\angle TOP + \angle POQ = 180°$ .... (1) (Linear pair axiom)

Similarly, ray $\overrightarrow{OS}$ stands on line $\overleftrightarrow{TQ}$.

Therefore, $\angle TOS + \angle SOQ = 180°$ .... (2) (why?)

But $\angle SOQ = \angle SOR + \angle QOR$

So, (2) becomes

$\angle TOS + \angle SOR + \angle QOR = 180°$ .... (3)

Now, adding (1) and (3), you get

$\angle TOP + \angle POQ + \angle TOS + \angle SOR + \angle QOR = 360°$ .... (4)

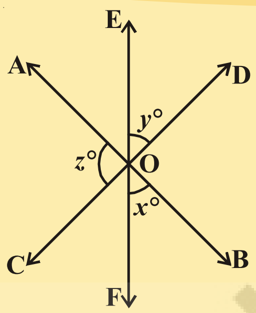
But $\angle TOP + \angle TOS = \angle POS$

Therefore, (4) becomes

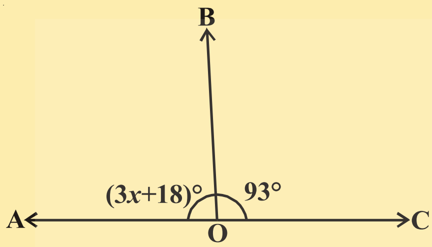
$\angle POQ + \angle QOR + \angle SOR + \angle POS = 360°$

EXERCISE - 4.2

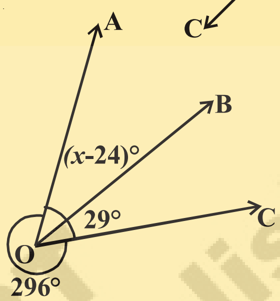
1. In the given figure three lines $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ intersecting at O. Find the values of x, y and z it is being given that x : y : z = 2 : 3 : 5



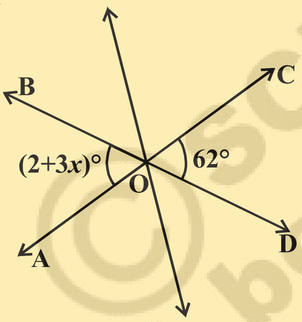
2. Find the value of x in the following figures.



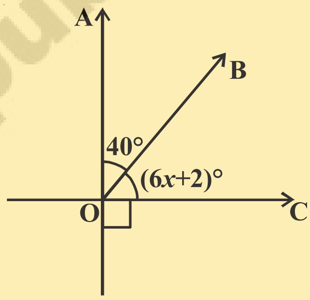
(i)



(ii)

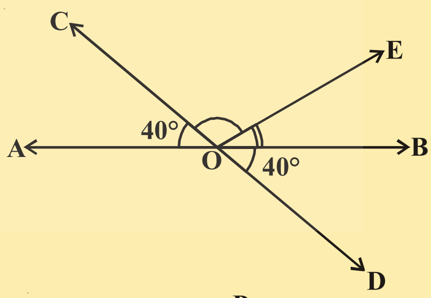


(iii)

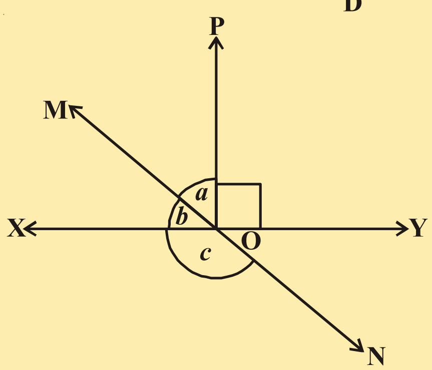


(iv)

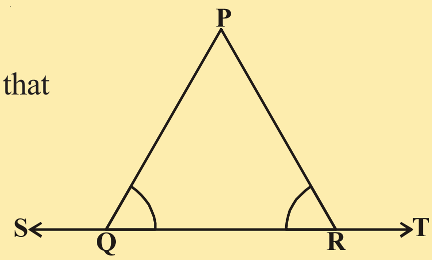
3. In the given figure lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.



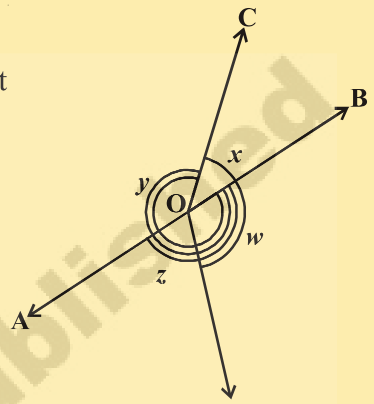
4. In the given figure lines $\overleftrightarrow{XY}$ and $\overleftrightarrow{MN}$ intersect at O. If $\angle POY = 90°$ and a: b = 2 : 3, find c.



5. In the given figure $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.

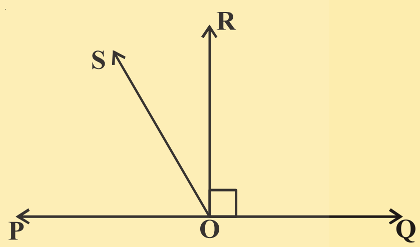


6. In the given figure, if $x + y = w + z$, then prove that AOB is a line.



7. In the given figure $\overleftrightarrow{PQ}$ is a line. Ray $\overrightarrow{OR}$ is perpendicular to line $\overleftrightarrow{PQ}$. $\overrightarrow{OS}$ is another ray lying between rays $\overrightarrow{OP}$ and $\overrightarrow{OR}$.

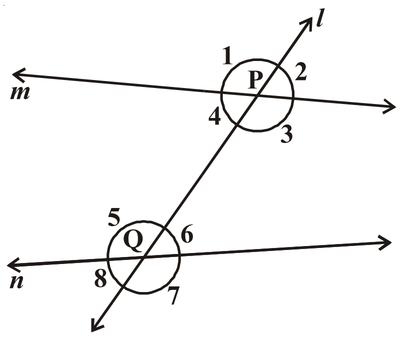
Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$



8. It is given that $\angle XYZ = 64°$ and XY is produced to point P. A ray YQ bisects $\angle ZYP$. Draw a figure from the given information. Find $\angle XYQ$ and reflex $\angle QYP$.

4.4 LINES AND A TRANSVERSAL

Observe the figure. At how many points the line l meets the other lines m and n? Line l meets the lines at two distinct points. What do we call such a line? It is a transversal. It is a line which intersects two distinct lines at two distinct points. Line 'l' intersects lines 'm' and 'n' at points 'P' and 'Q' respectively. So, line l is a transversal for lines m and n.



Observe the number of angles formed when a transversal intersects a pair of lines.

If a transversal meets two lines we get eight angles.

Let us name these angles as $\angle 1, \angle 2 \ldots \angle 8$ as shown in the figure. Can you classify these angles? Some angles are exterior and some are interior. $\angle 1, \angle 2, \angle 7$ and $\angle 8$ are called exterior angles, while $\angle 3, \angle 4, \angle 5$ and $\angle 6$ are called interior angles.

The angles which are non-adjacent and lie on the same side of the transversal of which one is interior and the other is exterior, are called corresponding angles.

From the given figure.

(a) What are corresponding angles?

(i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$

(iii) $\angle 4$ and $\angle 8$ (iv) $\angle 3$ and $\angle 7$, So there are 4 pairs of corresponding angles.

(b) What are alternate interior angles?

(i) $\angle 4$ and $\angle 6$ (ii) $\angle 3$ and $\angle 5$, are two pairs of alternate interior angles.(Why?)

(c) What are alternate exterior angles?

(i) $\angle 1$ and $\angle 7$ (ii) $\angle 2$ and $\angle 8$, are two pairs of alternate exterior angles. (Why?)

(d) What are interior angles on the same side of the transversal?

(i) $\angle 4$ and $\angle 5$ (ii) $\angle 3$ and $\angle 6$ are two pairs of interior angles on the same side of the transversal. (Why?)

Interior angles on the same side of the transversal are also referred to as **consecutive interior angles** or **co-interior angles** or **allied interior angles**.

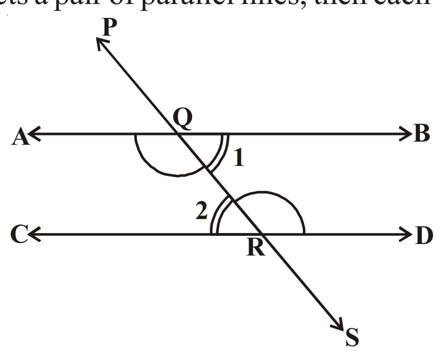
(e) What are exterior angles on the same side of the transversal?

(i) $\angle 1, \angle 8$ (ii) $\angle 2, \angle 7$ are two pairs of exterior angles on the same side of the transversal. (Why?)

Exterior angles on the same side of the transversal are also referred as consecutive exterior angle or co-exterior angles or allied exterior angles?

What can we say about the corresponding angles formed when the two lines l and m are parallel? Check and find. Will they become equal? Yes, they are equal.

Axiom of corresponding angles: If a transversal intersects a pair of parallel lines, then each pair of corresponding angles are equal.



What is the relation between the pairs of alternate interior angles (i) $\angle BQR$ and $\angle QRC$

(ii) $\angle AQR$ and $\angle QRD$ in the figure?

Can we use corresponding angles axiom to find the relation between these alternate interior angles.

P

In the figure, the transversal $\overleftrightarrow{PS}$ intersects two parallel lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ at points Q and R respectively.

Let us prove $\angle BQR = \angle QRC$ and $\angle AQR = \angle QRD$

You know that $\angle PQA = \angle QRC$ ..... (1) (corresponding angles axiom)

And $\angle PQA = \angle BQR$ ..... (2) (Why?)

So, from (1) and (2), you may conclude that $\angle BQR = \angle QRC$.

Similarly, $\angle AQR = \angle QRD$.

This result can be stated as a theorem as follows:

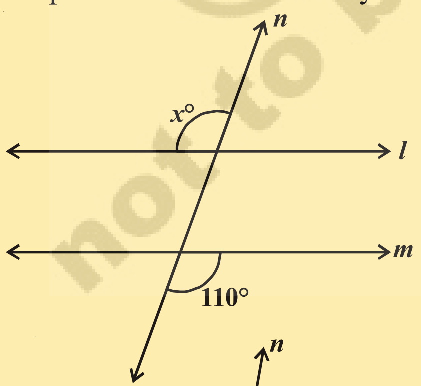
Theorem-4.2 : If a transversal intersects two parallel lines, then each pair of alternate interior angles are equal.

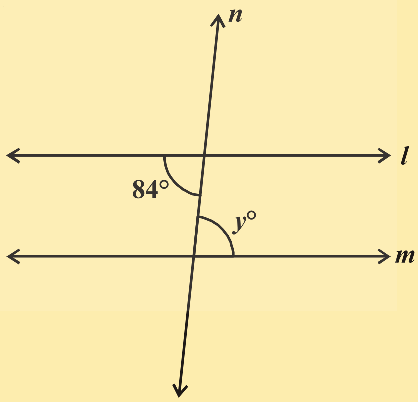
In a similar way, you can obtain the following theorem related to interior angles on the same side of the transversal.

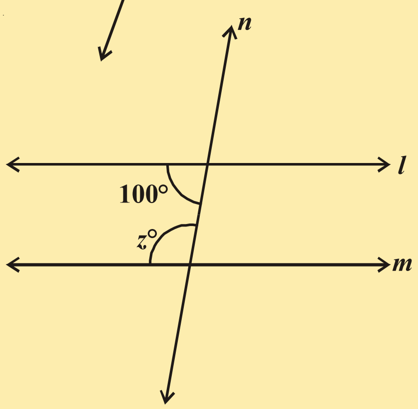
Theorem-4.3 : If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal are supplementary.

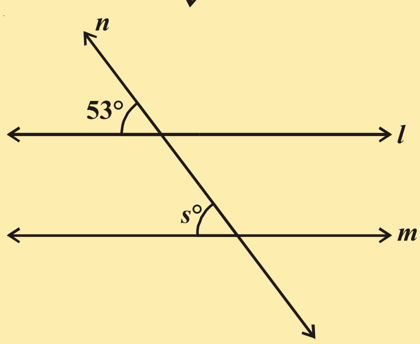
Do These

1. Find the measure of each angle indicated in each figure where l and m are parallel lines intersected by transversal n.

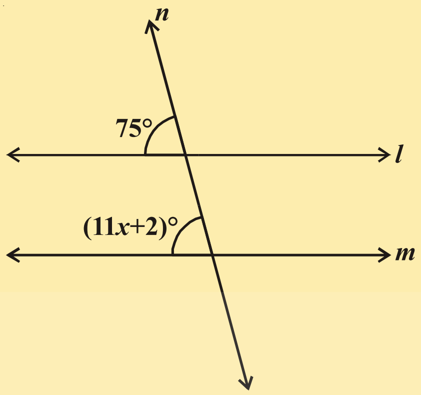


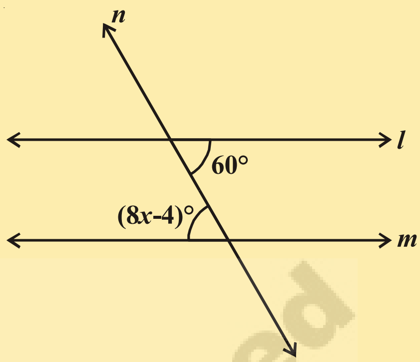


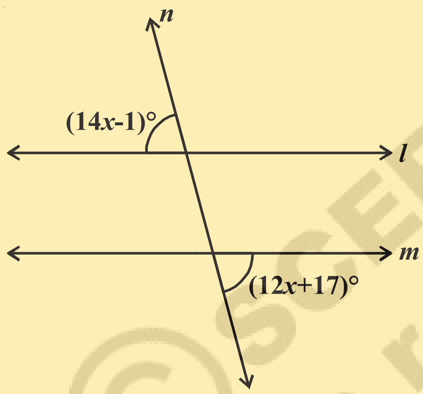


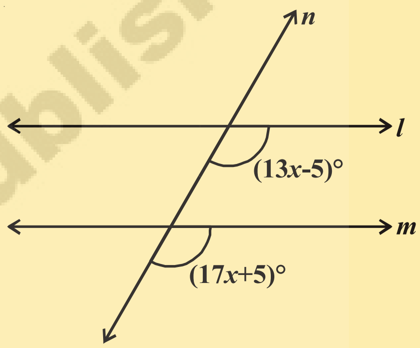


2. Solve for 'x' and give reasons.



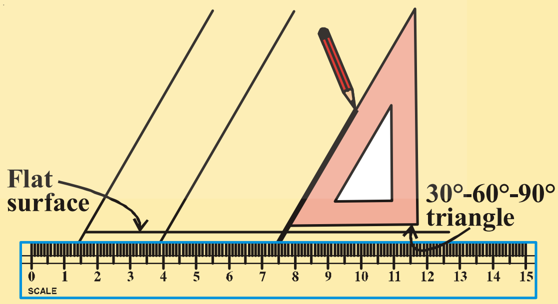






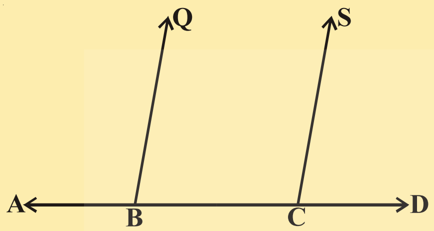
ACTIVITY

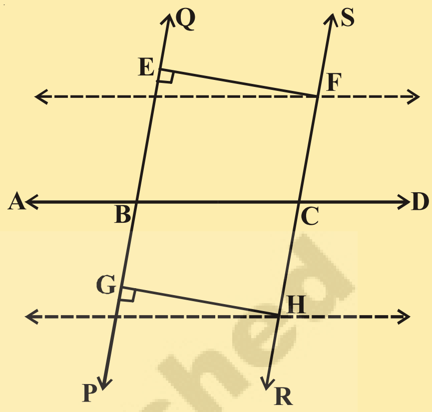
Take a scale and a 'set square'. Arrange the set square on the scale as shown in figure. Along the slant edge of set square draw a line with the pencil. Now slide your set square along its horizontal edge and again draw a line. We observe that the lines are parallel. Why are they parallel? Think and discuss with your friends.



DO THIS

Draw a line $\overleftrightarrow{AD}$ and mark points B and C on it. At B and C, construct $\angle ABQ$ and $\angle BCS$ equal to each other as shown. Produce QB and SC on the other side of AD to form two lines PQ and RS.

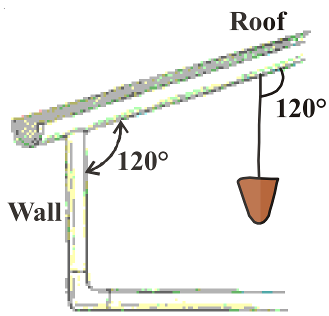




Draw common perpendiculars EF and GH for the two lines PQ and RS. Measure the lengths of EF and GH. What do you observe? What can you conclude from that? Recall that if the perpendicular distance between two lines is the same, then they are parallel lines.

Axiom-1 : If a transversal intersects two lines such that a pair of corresponding angles are equal, then the two lines are parallel to each other.

A plumb bob is a weight hung at the end of a string and the string here is called a plumb line. The weight pulls the string straignt down so that the plumb line is perfectly vertical. Suppose the angle between the wall and the roof is 120^0 and the angle formed by the plumb line and the roof is 120^o. Then the mason concludes that the wall is vertical to the ground. Think, how he has come to this conclusion?



Now, using the converse of the corresponding angles axiom, can we show the two lines are parallel if a pair of alternate interior angles are equal?

In the figure, the transversal PS intersects lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ at points Q and R respectively such that the alternate interior angles $\angle BQR$ and $\angle QRC$ are equal.

i.e. $\angle BQR = \angle QRC$.

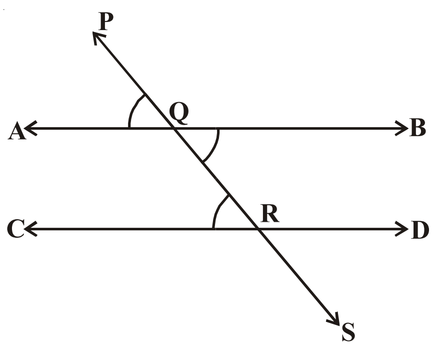
Now we need to prove this AB || CD

$\angle BQR = \angle PQA$ (Why?) ... (1)

But, $\angle BQR = \angle QRC$ (Given) ... (2)

So, from (1) and (2),

$\angle PQA = \angle QRC$



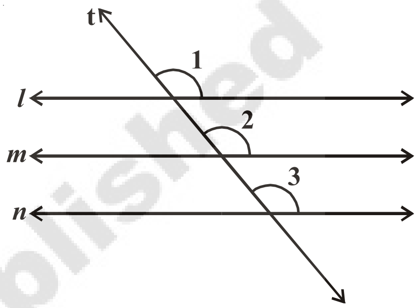
But they are corresponding angles for the pair of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ with transversal $\overleftrightarrow{PS}$.

So, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (Converse of corresponding angles axiom)

This result can be stated as a theorem as given below:

Theorem-4.4 : If a transversal intersects two lines such that a pair of alternate interior angles are equal, then the two lines are parallel.

4.4.1 Lines Parallel to the Same Line



If two lines are parallel to the same line, will they be parallel to each other?

Let us check it. Draw three line l, m and n such that $m \parallel l$ and $n \parallel l$.

Let us draw a transversal 't' on the lines, l, m and n.

Now from the figure $\angle 1 = \angle 2$ and $\angle 1 = \angle 3$ (Corresponding angles axiom)

So, $\angle 2 = \angle 3$ But these two form a pair of corresponding angles for the lines m & n.

Therefore, you can say that $m \parallel n$.

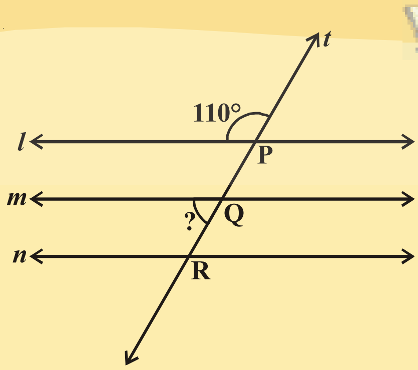
(Converse of corresponding angles axiom)

Theorem-4.5 : Lines which are parallel to the same line are parallel to each other.

TRY THIS

(i) Find the measure of the question marked angle in the given figure.

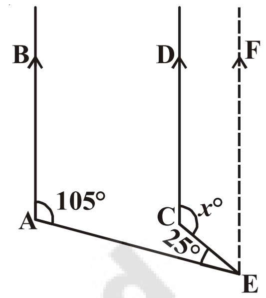
(ii) Find the angles which are equal to $\angle P$.



Now, let us solve some examples related to parallel lines.

Example-8. In the given figure, AB|| CD. Find the value af x.

Solution : From E, draw EF || AB || CD. EF || CD and CE is the transversal.



$\therefore$ $\angle$DCE + $\angle$CEF = 180° [$\because$ Co-interior angles]

$\Rightarrow$ x° + $\angle$CEF = 180° $\Rightarrow$ $\angle$CEF = (180 - x°).

Again, EF || AB and AE is the transversal.

$\angle$BAE + $\angle$AEF = 180° [$\because$ Co-interior angles]

$\Rightarrow$105° + $\angle$AEC + $\angle$CEF =180°

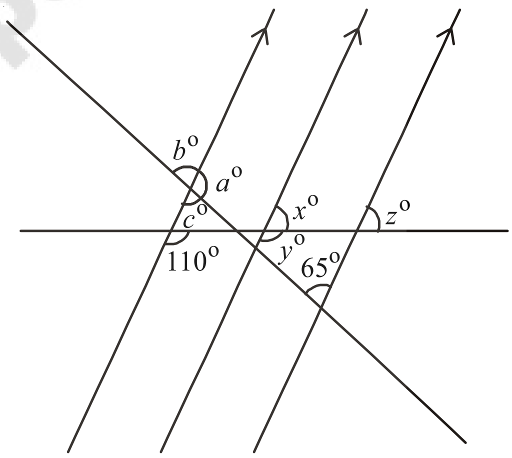
$\Rightarrow$ 105° + 25° + (180° - x°) = 180°

$\Rightarrow$ 310 - x° = 180°

Hence, x = 130°.

Example-9. In the adjacent figure, find the value of x, y, z and a, b, c.

Solution : Clearly, we have



y° = 110° ($\because$ Corresponding angles)

$\Rightarrow$ x° + y° = 180° (Linear pair)

$\Rightarrow$ x° + 110° = 180°

$\Rightarrow$ x° = (180° - 110°) = 70°.

z° = x° = 70° ($\because$ Corresponding angles)

c° = 65° (How?)

a° + c° = 180° [Linear pair]

$\Rightarrow$ a° + 65° = 180°

$\Rightarrow$ a° = (180° - 65°) = 115°.

b° = c° = 65°. [$\because$ Vertically opposite angles]

Hence, a = 115°, b = 65°, c = 65°, x = 70°, y = 110°, z = 70°.

Example 10. In the given figure, lines EF and GH are parallel. Find the value of x if the lines AB and CD are also parallel.

Solution : 4x° = $\angle$APR (Why?)

$\angle$APR = $\angle$PQS (Why?)

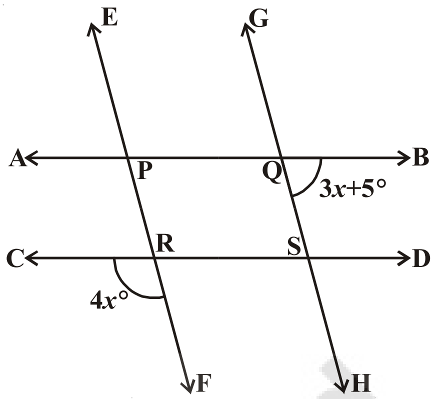
$\angle PQS + \angle SQB = 180°$ (Why?)

$4x° + (3x + 5)° = 180°$

$7x° + 5° = 180°$

$x° = \dfrac{180° - 5°}{7}$

$= 25°$

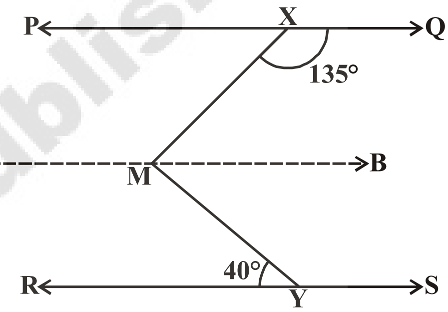


Example-11. In the given figure PQ || RS, $\angle MXQ = 135°$ and $\angle MYR = 40°$, find $\angle XMY$.

Solution : Construct a line AB parallel to PQ, through the point M.

Now, AB || PQ and PQ || RS.

Therefore, AB || RS



Now, $\angle QXM + \angle XMB = 180°$

(AB || PQ, Interior angles on the same side of the transversal XM)

So, $135° + \angle XMB = 180°$

Therefore, $\angle XMB = 45°$ ...(1)

Now, $\angle BMY = \angle MYR$ (Alternate interior angles as AB || RS)

Therefore, $\angle BMY = 40°$ ...(2)

Adding (1) and (2), you get

$\angle XMB + \angle BMY = 45° + 40°$

That is, $\angle XMY = 85°$

Example-12. If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, then prove that the two lines are parallel.

Solution : In the given Figure a transversal $\overleftrightarrow{AD}$ intersects two lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ at two points B and C respectively. Ray $\overrightarrow{BE}$ is the bisector of $\angle ABQ$ and ray $\overrightarrow{CF}$ is the bisector of $\angle BCS$; and BE || CF.

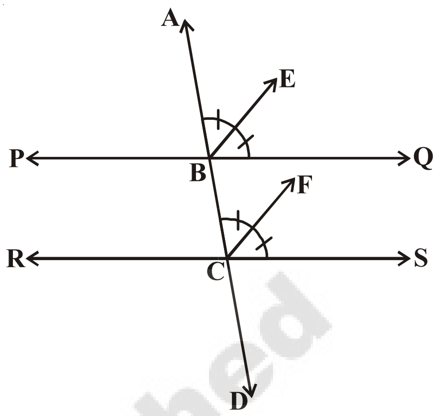
We have to prove that PQ || RS. It is enough to prove any one of the following pair:

i. Corresponding angles are equal.

ii. Pair of interior or exterior angles are equal.

iii. Interior angles same side of the transversal are supplementary.

From the figure, we try to prove the pairs of corresponding angles to be equal.



Since, it is given that ray BE is the bisector of $\angle ABQ$.

$\angle ABE = \frac{1}{2} \angle ABQ$. ... (1)

Similarly, ray CF is the bisector of $\angle BCS$.

Therefore, $\angle BCF = \frac{1}{2} \angle BCS$ ... (2)

But for the parallel lines BE and CF; $\overleftrightarrow{AD}$ is a transversal.

Therefore, $\angle ABE = \angle BCF$

(Corresponding angles axiom) ... (3)

From the equation (1) and (2) in (3), we get

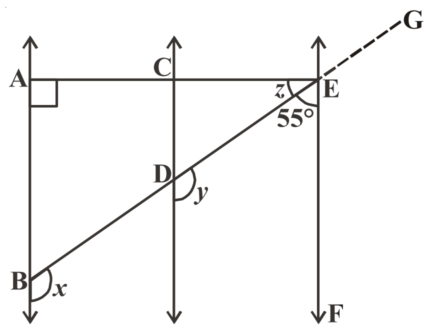
$$\frac{1}{2} \angle ABQ = \frac{1}{2} \angle BCS$$

$$\therefore \quad \angle ABQ = \angle BCS$$

But, these are the corresponding angles made by the transversal $\overleftrightarrow{AD}$ with lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$; and are equal.

Therefore, PQ || RS (Converse of corresponding angles axiom)

Example-13. In the given figure AB || CD and CD || EF. Also EA ⊥ AB. If $\angle BEF = 55°$, find the values of x, y and z.



Solution : Extend BE to G.

Now $\angle GEF = 180° - 55°$ (Why?)

$= 125°$

Also $\angle GEF = x = y = 125°$ (Why?)

Now $z = 90° - 55°$ (Why?)

$= 35°$

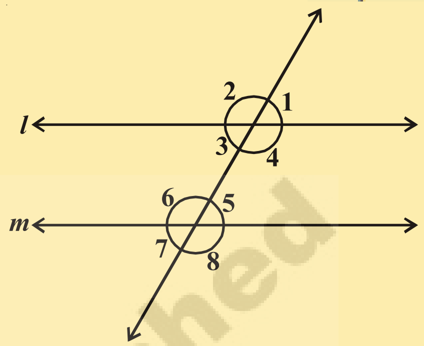
Different ways to prove that two lines are parallel.

1. Showing a pair of corresponding angles are equal.
2. Showing a pair of alternate interior angles are equal.
3. Showing a pair of interior angles on the same side of the transversal are supplementary.
4. In a plane, showing both lines are ⊥ to the same line.
5. Showing both lines are parallel to a third line.

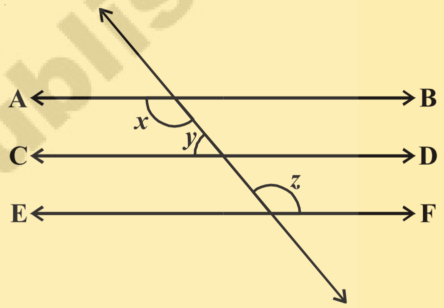
EXERCISE - 4.3

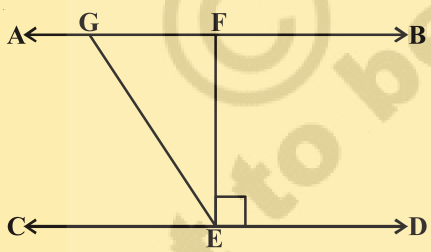
1. It is given that $l \parallel m$ to prove $\angle 1$ is supplement to $\angle 8$. Write reasons for the statement.

	Statement	Reasons
i.	$l \parallel m$	________
ii.	$\angle 1 = \angle 5$	________
iii.	$\angle 5 + \angle 8 = 180°$	________
iv.	$\angle 1 + \angle 8 = 180°$	________
v.	$\angle 1$ is supplement to $\angle 8$	________



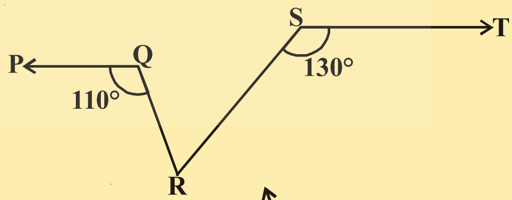
2. In the adjacent figure AB || CD; CD || EF and $y : z = 3 : 7$, find x.



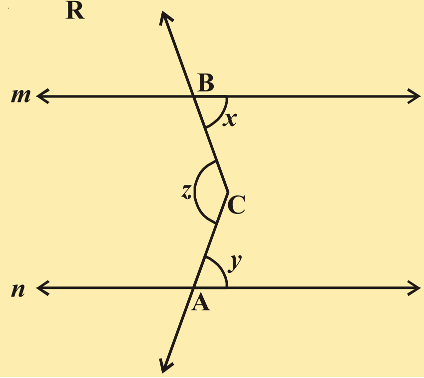


3. In the adjacent figure AB || CD, EF $\perp$ CD and $\angle$GED = 126°, find $\angle$AGE, $\angle$GEF and $\angle$FGE.

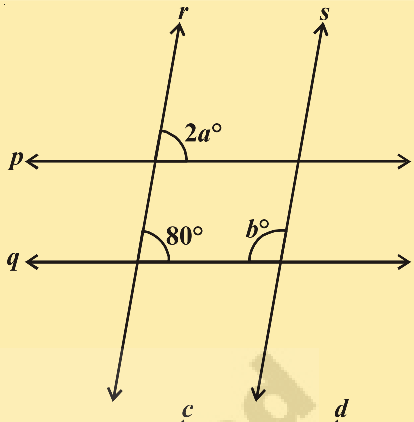
4. In the adjacent figure PQ || ST, $\angle$PQR = 110° and $\angle$RST = 130°, find $\angle$QRS.
[**Hint :** Draw a line parallel to ST through point R.]



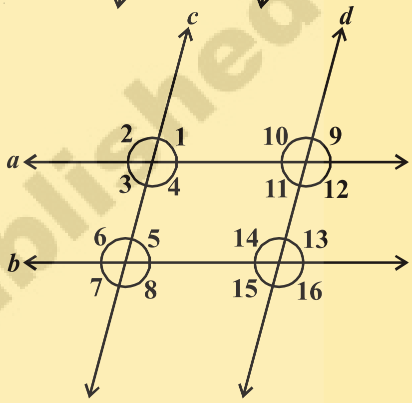
5. In the adjacent figure $m \parallel n$. A, B are any two points on m and n respectively. Let 'C' be an interior, point between the lines m and n. Find $\angle$ACB.

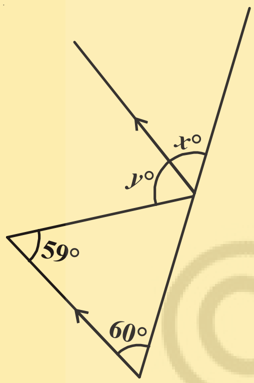


6. Find the value of a and b, given that $p \parallel q$ and $r \parallel s$.



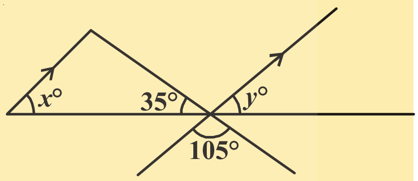
7. If in the figure $a \parallel b$ and $c \parallel d$, then name the angles that are congruent to (i) $\angle 1$ (ii) $\angle 2$.

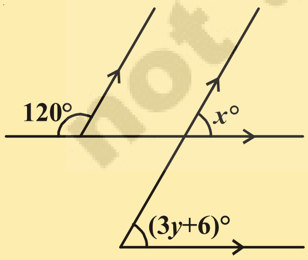




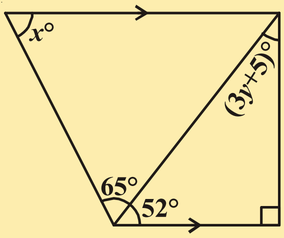
8. In the figure the arrow head segments are parallel. find the value of x and y.

9. In the figure the arrow head segments are parallel then find the value of x and y.





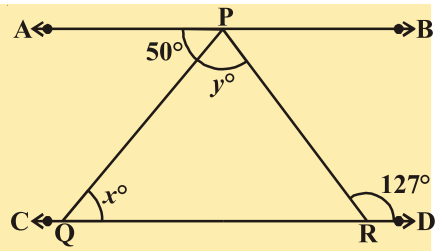
10. Find the value of x and y from the figure.



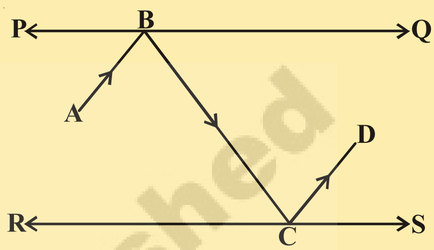
11. From the figure find x and y.

12. Draw figures for the following statement.
"If the two arms of one angle are respectively perpendicular to the two arms of another angle then the two angles are either equal or supplementary".

13. In the given figure, if AB || CD, $\angle APQ = 50°$ and $\angle PRD = 127°$, find x and y.

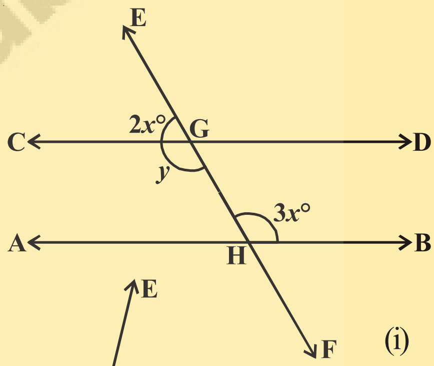


14. In the adjacent figure PQ and RS are two mirrors placed parallel to each other. An incident ray $\overrightarrow{AB}$ strikes the mirror PQ at B, the reflected ray moves along the path $\overrightarrow{BC}$ and strikes the mirror RS at C and again reflected back along $\overrightarrow{CD}$. Prove that AB || CD.

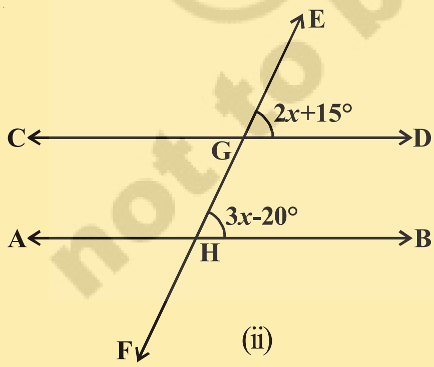


[Hint : Perpendiculars drawn to parallel lines are also parallel.]

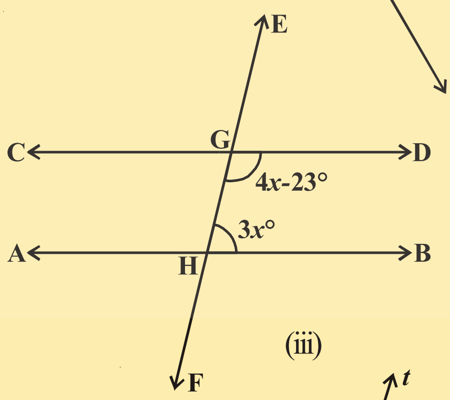
15. In the figures given below AB || CD. EF is the transversal intersecting AB and CD at G and H respectively. Find the values of x and y. Give reasons



(i)

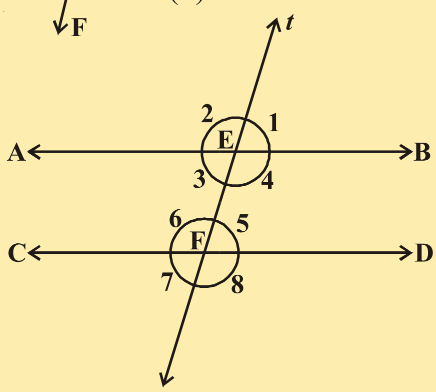


(ii)

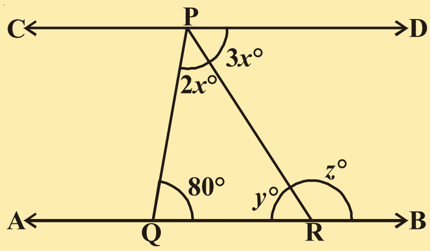


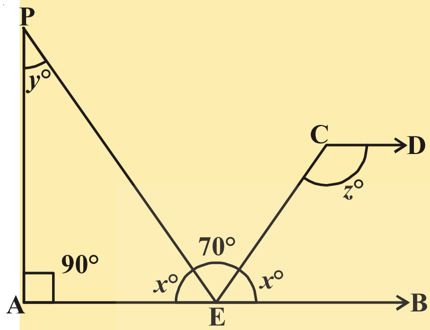
(iii)

16. In the adjacent figure, AB || CD, 't' is a transversal intersecting E and F respectively. If $\angle 2 : \angle 1 = 5 : 4$, find the measure of each marked angles.



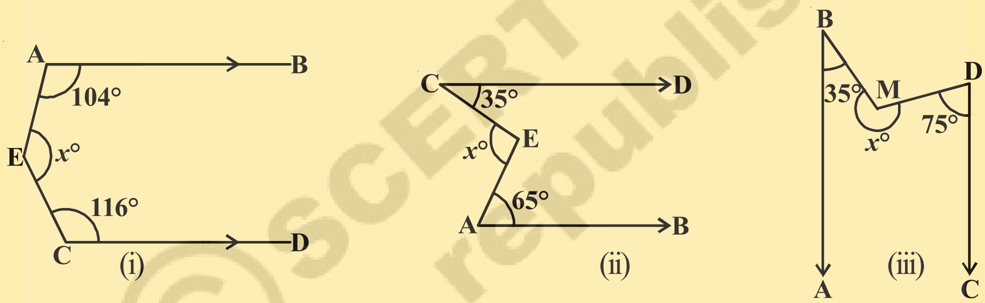
17. In the adjacent figure AB || CD. Find the value of x, y and z.





18. In the adjacent figure AB || CD. Find the values of x, y and z.

19. In each of the following figures AB || CD. Find the value of x in each case.



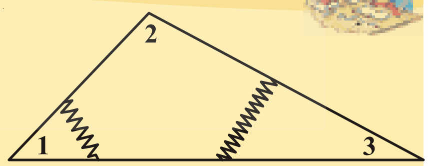
(i) (ii) (iii)

4.5 Angle Sum Property of a Triangle

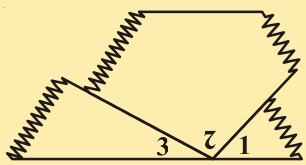
Let us now prove that the sum of the interior angles of a triangle is 180°.

Activity

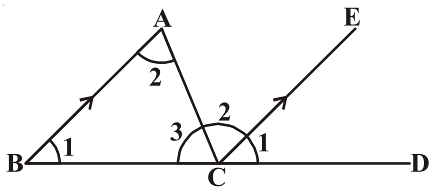
- Draw and cut out a large triangle as shown in the figure.
- Number the angles and tear them off.
- Place the three angles adjacent to each other to form one angle. as shown at the right.



1. Identify angle formed by the three adjacent angles? What is its measure?
2. Write about the sum of the measures of the angles of a triangle.



Now let us prove this statement using the axioms and theorems related to parallel lines.



Theorem-4.6 : The sum of the angles of a triangle is 180°.

Given : ABC is a triangle.

R.T.P. : $\angle A + \angle B + \angle C = 180^0$

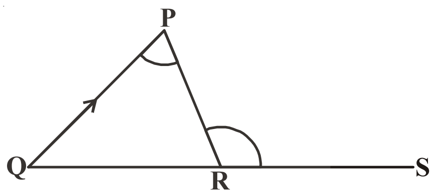
Construction : Produce BC to a point D

Through 'C' draw a line CE parallel to BA

Proof :

BA‖CE	[By construction]
$\angle ABC = \angle ECD$ .....(1)	[By corresponding angles axiom.]
$\angle BAC = \angle ACE$ .....(2)	[Alternate interior angles for the parallel lines AB and CE]
$\angle ACB = \angle ACB$ .....(3)	[Same angle]
$\angle ABC + \angle BAC + \angle ACB =$ $\angle ECD + \angle ACE + \angle ACB$	[Adding the above three equations]
But $\angle ECD + \angle ACE + \angle ACB = 180^0$	[angles on a straight line]
$\therefore \angle ABC + \angle BAC + \angle ACB = 180^0$	
$\angle A + \angle B + \angle C = 180^0$	

You know that when a side of a triangle is produced there forms an exterior angle of the triangle



When side QR is produced to point S, $\angle PRS$ is called an exterior angle of ΔPQR.

Is $\angle PRQ + \angle PRS = 180°$? (Why?) .....(1)

Also, see that

$\angle PRQ + \angle PQR + \angle QPR = 180°$ (Why?) .....(2)

From (1) and (2), we can see that $\angle PRQ + \angle PRS = \angle PRQ + \angle PQR + \angle QPR$

$\therefore \angle PRS = \angle PQR + \angle QPR$

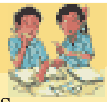
This result can be stated in the form of a theorem as given below

Theorem-4.7 : If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

It is obvious from the above theorem that an exterior angle of a triangle is always greater than either of its interior opposite angles.

Now, let us solve some examples based on the above

THINK, DISCUSS AND WRITE



If the sides of a triangle are produced in order, what will be the sum of exterior angles formed?

Example-14. The angle of a triangle are $(2x)°$, $(3x+5)°$ and $(4x - 14)°$.

Find the value of x and the measure of each angle of the triangle.

Solution : We know that the sum of the angles of a triangle is 180°.

$$\therefore\ 2x° + 3x° + 5° + 4x° - 14° = 180° \Rightarrow 9x° - 9° = 180°$$

$$\Rightarrow 9x° = 180° + 9° = 189°$$

$$\Rightarrow x = \frac{189°}{9°} = 21.$$

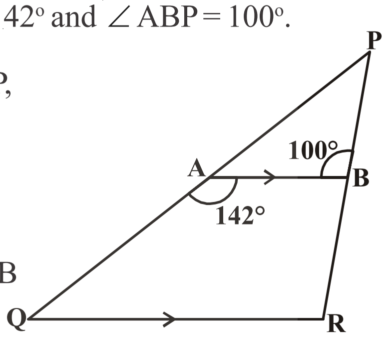
$$\therefore\ 2x° = (2\times21)° = 42°,\ (3x+5)° = [(3\times21+5)]° = 68°.$$

$$(4x - 14)° = [(4\times21) - 14]° = 70°$$

Hence, the angles of the triangle are 42°, 68° and 70°.

Example-15. In the adjacent figure, AB || QR, $\angle BAQ = 142°$ and $\angle ABP = 100°$.

Find (i) $\angle APB$ (ii) $\angle AQR$ and (iii) $\angle QRP$,



Solution : (i) Let $\angle APB = x°$,

Side PA of ΔPAB is produced to Q.

$\therefore$ Exterior angle $\angle BAQ = \angle ABP + \angle APB$

$\Rightarrow 142° = 100° + x°$

$\Rightarrow x° = (142° - 100°) = 42°.$

$\therefore\ \angle APB = 42°$,

(ii) Now, AB || QR and PQ is a transversal.

$\therefore\ \angle BAQ + \angle AQR = 180°$ [Sum of co-interior angles is 180°]

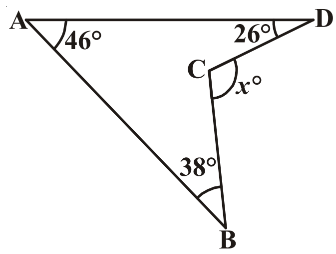
$\Rightarrow\ 142° + \angle AQR = 180°$,

$\therefore\ \angle AQR = (180° - 142°) = 38°$.

(iii) Since AB || QR and PR is a transversal.

$\angle QRP = \angle ABP = 100°$ [Corresponding angles]

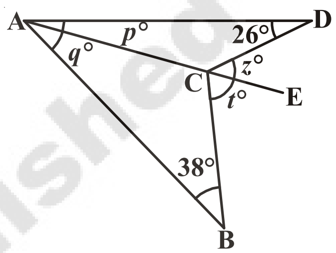
Example-16. Using information given in the adjacent figure, find the value of x.



Solution : In the given figure, ABCD is a quadrilateral. Let us try to make it as two triangles.

Join AC and produce it to E.

Let $\angle DAE = p^\circ$, $\angle BAE = q^\circ$, $\angle DCE = z^\circ$ and $\angle ECB = t^\circ$. Since the exterior angle of a triangle is equal to the sum of the interior opposite angles, we have :



$z^\circ = p^\circ + 26^\circ$

$t^\circ = q^\circ + 38^\circ$

$\therefore\ z^\circ + t^\circ = p^\circ + q^\circ + (26 + 38)^\circ = p^\circ + q^\circ + 64^\circ$

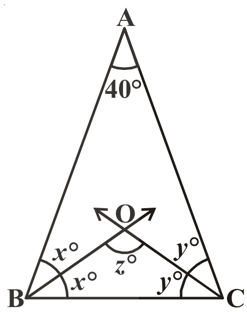
But, $p^\circ + q^\circ = 46$. $\quad (\because \angle DAB = 46^\circ)$

So, $z^\circ + t^\circ = 46 + 64 = 110^\circ$.

Hence $x^\circ = z^\circ + t^\circ = 110^\circ$.

Example-17. In the given figure $\angle A = 40°$. If $\overrightarrow{BO}$ and $\overrightarrow{CO}$ are the bisectors of $\angle B$ and $\angle C$ respectively. Find the measure of $\angle BOC$.

Solution : We know that BO is the bisector of $\angle B$ and CO is the bisector of $\angle C$.



Let $\angle CBO = \angle ABO = x°$ and $\angle BCO = \angle ACO = y°$.

Then, $\angle B = (2x)°$, $\angle C = (2y)°$ and $\angle A = 40°$.

But, $\angle A + \angle B + \angle C = 180°$. (How?)

$2x° + 2y° + 40° = 180°$

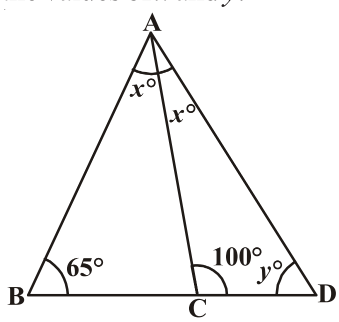
$\Rightarrow 2(x + y)° = 140°$

$= x° + y° = \dfrac{140°}{2} = 70°$.

Hence, $\angle BOC = 180° - 70° = 110°$.

Example-18. Using information given in the adjacent figure, find the values of x and y.

Solution : Side BC of ΔABC has been produced to D.



Exterior $\angle ACD = \angle ABC + \angle BAC$

$\therefore\ 100° = 65° + x°$

$\Rightarrow x° = (100° - 65°) = 35°$.

$\therefore\ \angle CAD = \angle BAC = 35°$

In ΔACD, we have :

$$\angle CAD + \angle ACD + \angle CDA = 180^\circ \text{ (Angle sum property of triangle)}$$

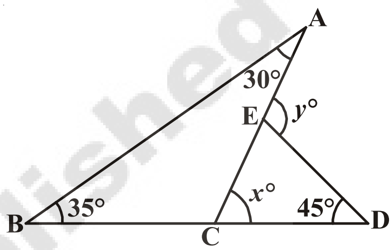
$$\Rightarrow 35^\circ + 100^\circ + y^\circ = 180^\circ$$

$$\Rightarrow 135^\circ + y^\circ = 180^\circ$$

$$\Rightarrow y^\circ = (180^\circ - 135^\circ) = 45^\circ$$

Hence, $x = 35^\circ$, $y = 45^\circ$.

Example-19. Using information given in the adjacent figure, find the value of x and y.



Solution : Side BC of ΔABC has been produced to D.

$\therefore$ Exterior angle $\angle ACD = \angle BAC + \angle ABC$

$\Rightarrow \quad x^\circ = 30^\circ + 35^\circ = 65^\circ$.

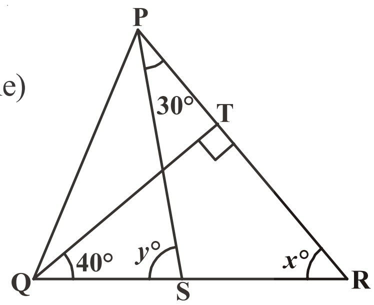
Again, side CE of ΔDCE has produced to A.

$\therefore$ Exterior angle $\angle DEA = \angle EDC + \angle ECD$

$\Rightarrow \quad y = 45 + x^\circ = 45^\circ + 65^\circ = 110^\circ$.

Hence, $x = 65^\circ$ and $y = 110^\circ$.

Example-20. In the adjacent fig. if $QT \perp PR$, $\angle TQR = 40^\circ$ and $\angle SPR = 30^\circ$, find x and y.



Solution : In ΔTQR,

$90^\circ + 40^\circ + x = 180^\circ$ (Angle sum property of a triangle)

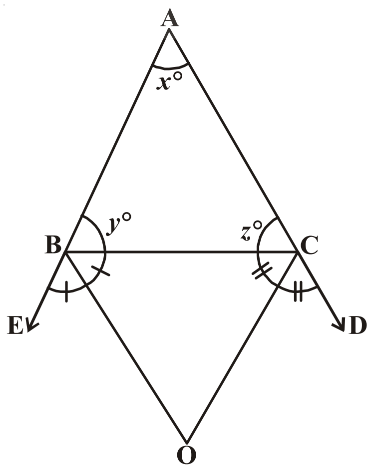
Therefore, $\quad x^\circ = 50^\circ$

Now, $\quad y^\circ = \angle SPR + x^\circ$ (Exterior angle of traingle)

Therefore, $\quad y^\circ = 30^\circ + 50^\circ$

$\quad = 80^\circ$

Example-21. In the adjacent figure the sides AB and AC of ΔABC are produced to points E and D respectively. If bisectors BO and CO of $\angle CBE$ and $\angle BCD$ respectively meet at point O, then prove that $\angle BOC = 90^\circ - \frac{1}{2}\angle BAC$.



Solution : Ray BO is the bisector of $\angle CBE$.

Therefore, $\angle CBO = \frac{1}{2}\angle CBE$

$$= \frac{1}{2}(180^\circ - y^\circ)$$

$$= 90^\circ - \frac{y^\circ}{2} \qquad ...(1)$$

Similarly, ray CO is the bisector of $\angle$BCD.

Therefore, $\angle BCO = \frac{1}{2}\angle BCD$

$$= \frac{1}{2}(180° - z°)$$

$$= 90° - \frac{z°}{2} \qquad ...(2)$$

In ΔBOC, $\angle BOC + \angle BCO + \angle CBO = 180°$ ...(3)

Substituting (1) and (2) in (3), you get

$$\angle BOC + 90° - \frac{z°}{2} + 90° - \frac{y°}{2} = 180°$$

So, $\angle BOC = \frac{z°}{2} + \frac{y°}{2}$

or, $\angle BOC = \frac{1}{2}(y° + z°)$ ... (4)

But, $x° + y° + z° = 180°$ (Angle sum property of a triangle)

Therefore, $y° + z° = 180° - x°$

Therefore, (4) becomes

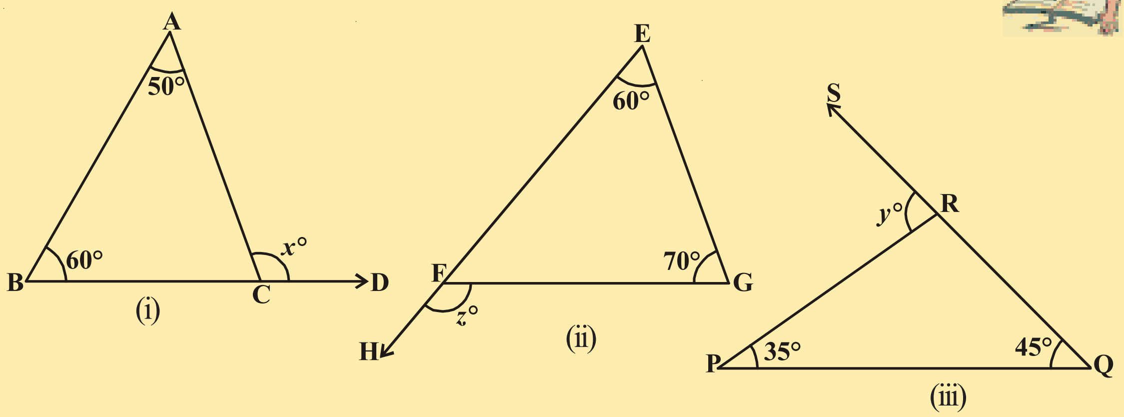
$$\angle BOC = \frac{1}{2}(180° - x)°$$

$$= 90° - \frac{x°}{2}$$

$$= 90° - \frac{1}{2}\angle BAC$$

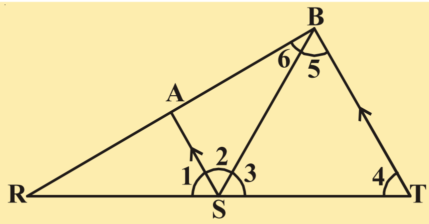
EXERCISE 4.4

1. In the given triangles, find out $\angle x$, $\angle y$ and $\angle z$.

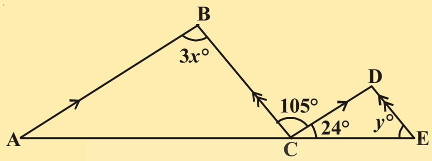


2. In the given figure AS || BT; $\angle 4 = \angle 5$

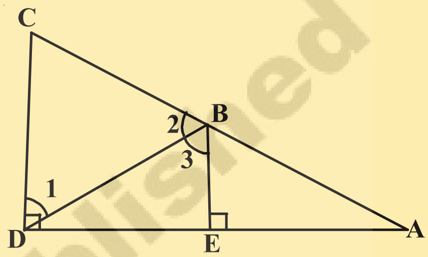
$\overrightarrow{SB}$ bisects $\angle AST$. Find the measure of $\angle 1$



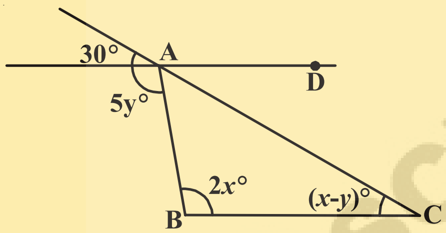
3. In the given figure AB || CD; BC || DE then find the values of x and y.



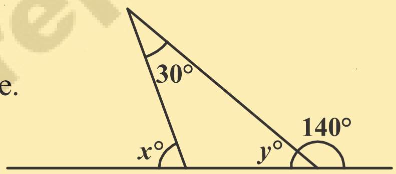
4. In the adjacent figure BE $\perp$ DA and CD$\perp$DA then prove that $m\angle 1 \cong m\angle 3$.



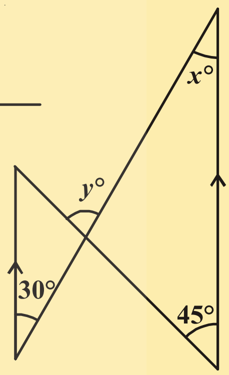
5. Find the values of x, y for which the lines AD and BC become parallel.



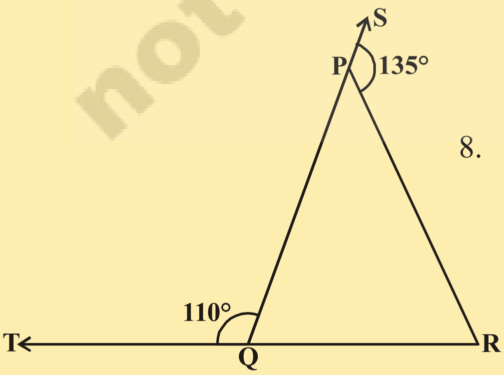
6. Find the values of x and y in the figure.



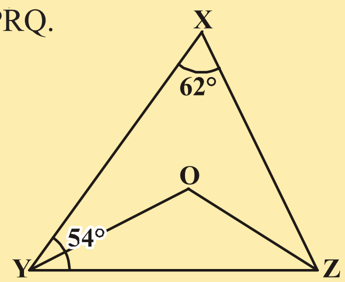
7. In the given figure segments shown by arrow heads are parallel. Find the values of x and y.



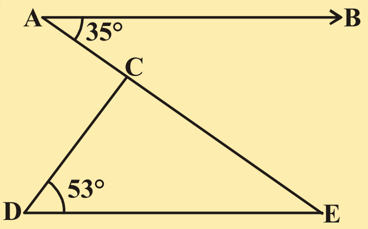
8. In the given figure sides QP and RQ of $\angle PQR$ are produced to points S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, find $\angle PRQ$.

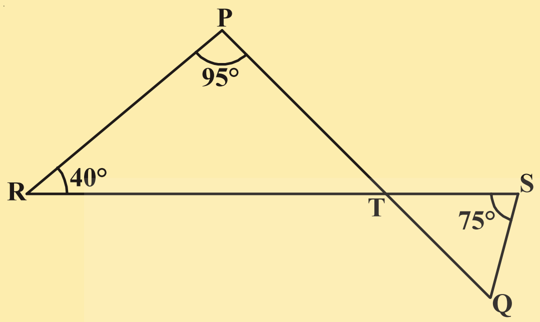


9. In the given figure, $\angle X = 62°$, $\angle XYZ = 54°$. In ΔXYZ If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively find $\angle OZY$ and $\angle YOZ$.



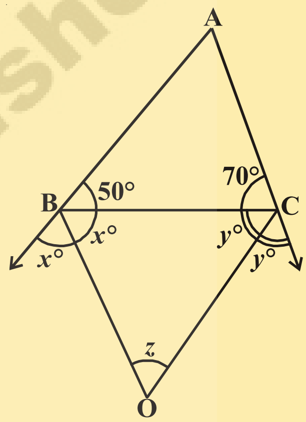
10. In the given figure if AB || DE, $\angle BAC = 35°$ and $\angle CDE = 53°$, find $\angle DCE$.

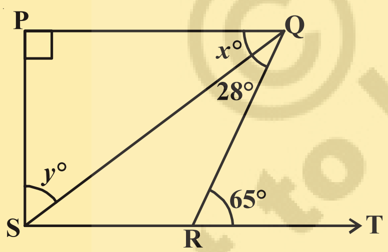




11. In the given figure if line segments PQ and RS intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$, find $\angle SQT$.

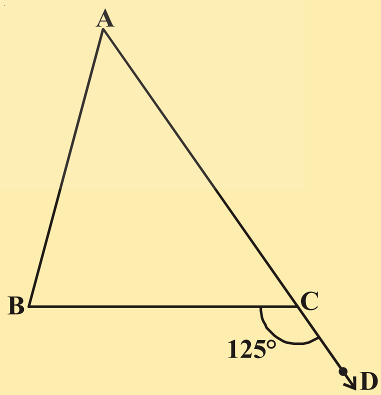
12. In the adjacent figure, ABC is a triangle in which $\angle B = 50°$ and $\angle C = 70°$. Sides AB and AC are produced. If 'z' is the measure of the angle between the bisectors of the exterior angles so formed, then find 'z'.

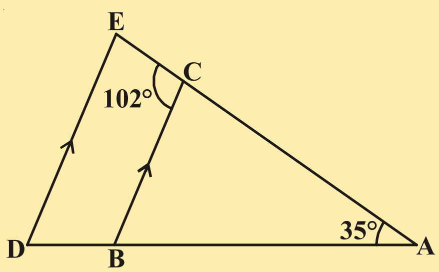




13. In the given figure if PQ $\perp$ PS, PQ || SR, $\angle SQR = 28°$ and $\angle QRT = 65°$, then find the values of x and y.

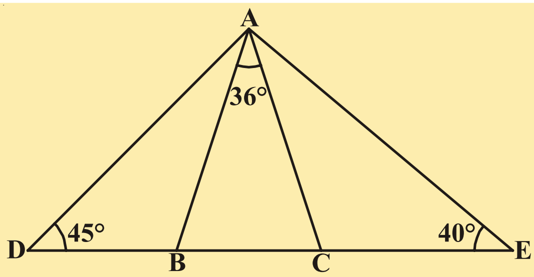
14. In the given figure ΔABC side AC has been produced to D. $\angle BCD = 125°$ and $\angle A : \angle B = 2 : 3$, find the measure of $\angle A$ and $\angle B$.

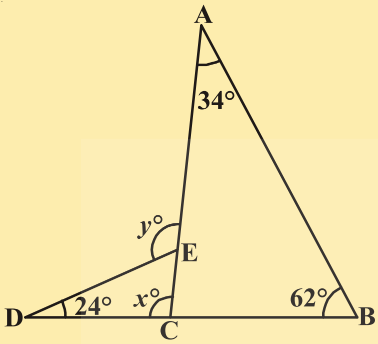




15. In the adjacent figure, it is given that, BC || DE, $\angle BAC = 35°$ and $\angle BCE = 102°$. Find the measure of (i) $\angle BCA$ (ii) $\angle ADE$ and (iii) $\angle CED$.

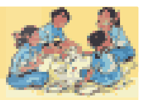
16. In the adjacent figure, it is given that AB = AC, $\angle BAC = 36^\circ$, $\angle ADB = 45^\circ$ and $\angle AEC = 40^\circ$. Find (i) $\angle ABC$ (ii) $\angle ACB$ (iii) $\angle DAB$ (iv) $\angle EAC$.





17. Using information given in the figure, calculate the value of x and y.

WHAT WE HAVE DISCUSSED



- **Linear pair axiom:** If a ray stands on a straight line, then the sum of the two adjacent angles so formed is 180°.
- **Converse of linear pair axiom:**

  If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.
- **Theorem**: If two lines intersect each other, then the vertically opposite angles are equal.
- **Axiom of corresponding angles:** If a transversal intersects two parallel lines, then each pair of corresponding angles are equal.
- **Theorem**: If a transversal intersects two parallel lines, then each pair of alternate interior angles are equal.
- **Theorem**: If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal are supplementary.
- **Converse of axiom of corresponding angles**:

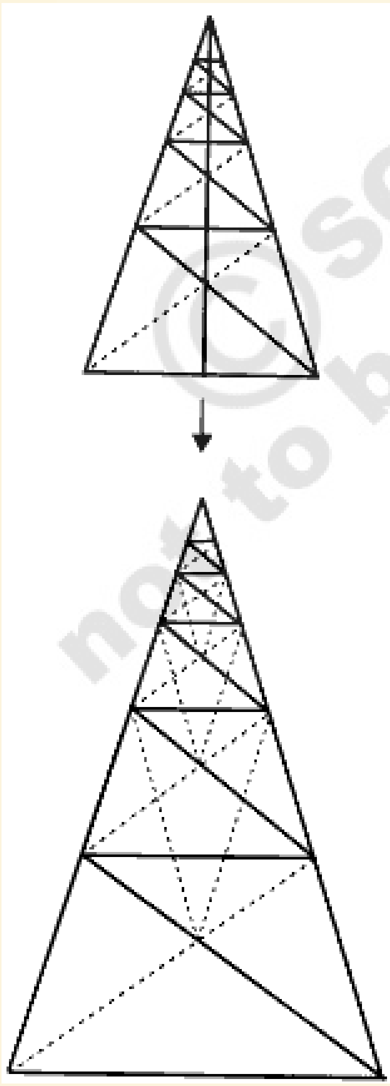
  If a transversal intersects two lines such that a pair of corresponding angles are equal, then the two lines are parallel to each other.
- **Theorem**: If a transversal intersects two lines such that a pair of alternate

  interior angles are equal, then the two lines are parallel.

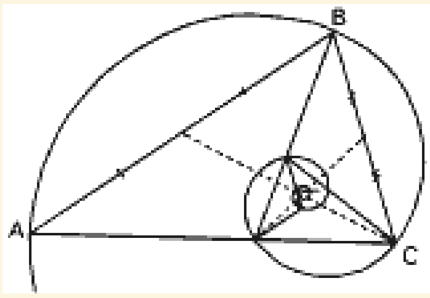
- **Theorem**: If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal are supplementary, then the two lines are parallel.
- **Theorem**: Lines which are parallel to a given line are parallel to each other.
- **Theorem:** The sum of the angles of a triangle is 180º.
- **Theorem**: If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

Do You Know?

The Self-generating Golden Triangle



The golden triangle is an isosceles triangle with base angles 72^{o} and the vertex angle 36^{o}. When both of these base angles are bisected the two new triangles produced arc also golden triangles. This process can be continued indefinitely up the legs of the original golden triangle, and an infinite number of golden triangles will appear as if they are unfolding.



As this diagram shows, the golden triangle also produces the equi-angular spiral and the golden ratio, $\phi = |AB| / |BC| = 1.618$ ...

From these infinite climbing golden triangles one can also construct inside them an infinite number of climbing pentagrams. Note the five points of the penta-gram are also golden triangles.