06 Linear Equations in Two Variables

6.1 Introduction

We have come across many problems like

(i) If five pens cost ₹ 60, then find the cost of one pen.

(ii) A number when added to 7 gives 51. Find that number.

Here, in situation (i) the cost of the pen is unknown, while in situation (ii) the number is unknown. How do we solve questions of this type? We take letters x, y or z for the unknown quantities and write an equation for these situations.

For situation (i) we can write

$5 \times$ cost of one pen $= 60$

If the cost of one pen is ₹ y

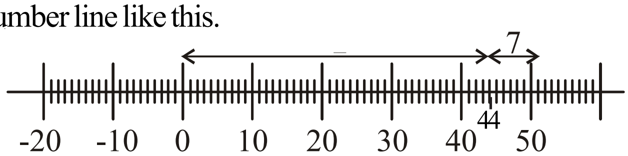
Then, $5y = 60$

Now solve it for y.

Likewise we can make an equation for situation (ii) and find the unknown number. Such type of equations are linear equations.

Equations like $x + 3 = 0$, $x + \sqrt{3} = 0$ and $\sqrt{2}x + 5 = 0$ are examples of linear equations in one variable. You also know that such equations have unique (implying one and only one) solution. You may also remember how to represent the solution on number line.

Atif represented the solution of situation (ii) on the number line like this.



6.2 LINEAR EQUATIONS IN TWO VARIABLES

Now consider the following situation :

One day Kavya went to a bookshop with her father to buy. 4 notebooks and 2 pens. Her father paid ₹ 100 for all these.

Kavya did not know the cost of the note book and the pen separately. Now can you express this information in the form of an equation ?

Here, you can see that the cost of the single note book and also of the pen is unknown, i.e. there are two unknown quantities. Let us use x and y to denote them. So, the cost of a single note book is ₹ x and the cost of a single pen is ₹ y.

We represent the above as an equation in the form of $4x + 2y = 100$,

Have you observed the exponents of x and y in the equation ?

Thus the above equation is in linear form with variables 'x' and 'y'.

If a linear equation has two variables then it is called a linear equation in two variables.

Therefore $4x + 2y = 100$ is an example of linear equation in two variables.

It is usually to denote the variables by 'x' and 'y'. But other letters may also be used.

$p + 3q = 50$, $\sqrt{3}u + \sqrt{2}v = \sqrt{11}$, $\frac{s}{2} - \frac{t}{3} = 5$ and $3 = \sqrt{5}x - 7y$ are examples of linear equation in two variables.

Note that you can put above equations in the form of $p + 3q - 50 = 0$, $\sqrt{3}u + \sqrt{2}v - \sqrt{11} = 0$, $\frac{s}{2} - \frac{t}{3} = 5$ and $\sqrt{5}x - 7y - 3 = 0$ respectively.

Therefore the general form of linear equation in two variables x, y is $ax + by + c = 0$. Where a, b, c are real numbers, and a, b are not simultaneously zero.

Example 1. Sachin and Sehwag scored 137 runs together. Express the information in the form of an equation.

Solution : Let runs scored by Sachin be 'x' and runs scored by Sehwag be 'y' .

Then the above information in the form of an equation is

$$x + y = 137$$

Example 2. Hema's age is 4 times the age of Mary. Write a linear equation in two variables to represent this information.

Solution : Let Hema's age be 'x' years and of Mary be 'y' years,

If Mary's age is y then Hema's age is '$4y$'.

According to the given information we have $x = 4y$

$$\Rightarrow x - 4y = 0 \text{ (how?)}$$

Example 3. A number is 27 more than the number obtained by reversing its digits. If its unit's and ten's digits are x and y respectively, write the linear equation representing the above statement.

Solution : Units digit is represented by x and tens digit by y, then the number is 10y + x

If we reverse the digits then the new number would be 10x + y (Recall the place value of digits in a two digit number).

Therefore according to the given condition the equation is

(two digit number) – (number formed by reversing the digits) = 27.

i.e., $10y + x - (10x + y) = 27$

$\Rightarrow 10y + x - 10x - y - 27 = 0$

$\Rightarrow 9y - 9x - 27 = 0$

$\Rightarrow y - x - 3 = 0$

$\Rightarrow x - y + 3 = 0$ is the required equation.

Example 4. Express each of the following equations in the form of ax + by + c = 0 and write the values of a, b and c.

i) $3x + 4y = 5$ ii) $x - 5 = \sqrt{3}y$

iii) $3x = y$ iv) $\frac{x}{2} + \frac{y}{2} = \frac{1}{6}$

v) $3x - 7 = 0$

Solution : (i) $3x + 4y = 5$ can be written as

$3x + 4y - 5 = 0.$

Here $a = 3$, $b = 4$ and $c = -5$.

(ii) $x - 5 = \sqrt{3}y$ can be written as

$1.x - \sqrt{3}y - 5 = 0.$

Here $a = 1$, $b = -\sqrt{3}$ and $c = -5$.

(iii) The equation $3x = y$ can be written as

$3x - y + 0 = 0.$

Here $a = 3$, $b = -1$ and $c = 0$.

(iv) The equation $\frac{x}{2} + \frac{y}{2} = \frac{1}{6}$ can be written as

$\frac{x}{2} + \frac{y}{2} - \frac{1}{6} = 0$;

$a = \frac{1}{2}$, $b = \frac{1}{2}$ and $c = \frac{-1}{6}$

(v) $3x - 7 = 0$ can be written as

$3x + 0.\ y - 7 = 0.$

$a = 3$, $b = 0$; $c = -7$

Example-5. Write each of the following in the form of $ax + by + c = 0$ and find the values of a, b and c

i) $x = -5$

ii) $y = 2$

iii) $2x = 3$

iv) $5y = -3$

Solution :

S.No.	Given equation	Expressed as $ax + by + c = 0$	Value of a, b, c: a	b	c
1	$x = -5$	$1.x + 0.y + 5 = 0$	1	0	5
2	$y = 2$	$0.x + 1.y - 2 = 0$	0	1	-2
3	$2x = 3$	---	---	---	---
4	$5y = -3$	----	----	---	---

TRY THIS

1. Express the following linear equations in the form of $ax + by + c = 0$ and indicate the values of a, b, c in each case?

   i) $3x + 2y = 9$ ii) $-2x + 3y = 6$ iii) $9x - 5y = 10$

   iv) $\frac{x}{2} - \frac{y}{3} - 5 = 0$ v) $2x = y$

EXERCISE - 6.1

1. Express the following linear equation in the form of $ax+by+c=0$ and indicate the values of a, b and c in each case.

   i) $8x + 5y - 3 = 0$ ii) $28x - 35y = -7$ iii) $93x = 12 - 15y$

   iv) $2x = -5y$ v) $\frac{x}{3} + \frac{y}{4} = 7$ vi) $y = \frac{-3}{2}x$

   vii) $3x + 5y = 12$

2. Write each of the following in the form of $ax + by + c = 0$ and find the values of a, b and c

   i) $2x = 5$ ii) $y - 2 = 0$ iii) $\frac{y}{7} = 3$ iv) $x = \frac{-14}{13}$

3. Express the following statements as a linear equation in two variables.
   (i) The sum of two numbers is 34.
   (ii) The cost of a ball pen is ₹ 5 less than half the cost of a fountain pen.
   (iii) Bhargavi got 10 more marks than double of the marks of Sindhu.
   (iv) The cost of a pencil is ₹ 2 and one ball point pen costs ₹ 15. Sheela pays ₹ 100 for the pencils and pens she purchased.
   (v) Yamini and Fatima of class IX together contributed ₹ 200/- towards the Prime Minister's Relief Fund.
   (vi) The sum of a two digit number and the number obtained by reversing the order of its digits is 121. If the digits in unit's and ten's place are 'x' and 'y' respectively.

6.3 SOLUTION OF A LINEAR EQUATION IN TWO VARIABLES

You know that linear equation in one variable has a unique solution.

What is the solution of the equation $3x - 4 = 8$?

Consider the equation $3x - 2y = 5$.

What can we say about the solution of this linear equation in two variables? Do we have only one value in the solution or do we have more ? Let us exaplain.

Can you say $x = 3$ is a solution of this equation?

Let us check, if we substitute $x = 3$ in the equation

We get $3\,(3) - 2y = 5$

$$9 - 2y = 5$$

i.e., Still we cannot find the solution of the given equation. So, to know the solution, besides the value of 'x' we also need the value of 'y'. we can get value of y from the above equation $9 - 2y = 5$. $\Rightarrow$ $2y = 4$ or $y = 2$

The values of x and y which satisfy the equation $3x - 2y = 5$, are $x = 3$ and $y = 2$. Thus to statisfy, a linear equation in two variables we need two values, one value for 'x' and one value for y.

Therefore any pair of values of 'x' and 'y' which satisfy the linear equation in two variables is called its solution.

We observed that $x = 3, y = 2$ is a solution of $3x - 2y = 5$. This solution is written as an ordered pair (3, 2), first writing the value for 'x' and then the value for 'y'. Are there any other solutions for the equation? Pick a value of your choice say $x = 4$ and substitute it in the equation $3x - 2y = 5$. Then the equation reduces to $12 - 2y = 5$. Which is an equation in one variable. On solving this we get.

$$y = \frac{12-5}{2} = \frac{7}{2}, \quad \text{so} \quad \left(4, \frac{7}{2}\right) \text{ is another solution, of } 3x - 2y = 5$$

Do you find some more solutions for $3x - 2y = 5$? Check if $(1, -1)$ is another solution?

Thus for a linear equation in two variables we can find many solutions.

Note : An easy way of getting two solutions is put $x = 0$ and get the corresponding value of 'y'. Similarly we can put $y = 0$ and obtain the corresponding value of 'x'.

TRY THIS

Find 5 more pairs of values that are solutions for the above equation.

Example 6. Find four different solutions of $4x + y = 9$. (Complete the table wherever necessary)

Solution :

S.No.	Choice of a value for variable x or y	Simplification		Solution
1.	$x = 0$	$4x + y = 9$	$\Rightarrow 4 \times 0 + y = 9$ $\Rightarrow y = 9$	(0,9)
2.	$y = 0$	$4x + y = 9$	$\Rightarrow 4x + 0 = 9$ $\Rightarrow 4x = 9$ $\Rightarrow x = 9/4$	$\left(\frac{9}{4}, 0\right)$
3.	$x = 1$	$4x + y = 9$	$\Rightarrow 4 \times 1 + y = 9$ $\Rightarrow 4 + y = 9$ $\Rightarrow y = 5$	——
4.	$x = -1$		———	$(-1, 13)$

$\therefore$ (0, 9), $\left(\frac{9}{4}, 0\right)$, (1, 5) and $(-1, 13)$ are some of the solutions for the above equation.

Example-7. Check which of the following are solutions of an equation $x + 2y = 4$? (Complete the table wherever necessary)

i) $(0, 2)$ ii) $(2, 0)$ iii) $(4, 0)$ (iv) $(\sqrt{2}, -3\sqrt{2})$

v) $(1, 1)$ vi) $(-2, 3)$

Solution : We know that if we get LHS = RHS when we substitute a pair in the given equation, then it is a solution.

The given equation is $x + 2y = 4$

S. No	Pair of Values	Value of LHS	Value of RHS	Relation between LHS and RHS	Solution/ not Solution
1.	$(0, 2)$	$x + 2y = 0 + (2 \times 2)$ $= 0 + 4 = 4$	4	$\therefore$ LHS=RHS	$\therefore (0, 2)$ is a Solution
2.	$(2, 0)$	$x + 2y = 2 + (2 \times 0)$ $= 2 + 0 = 2$	4	......	$(0, 2)$ is a Not a Solution
3.	$(4, 0)$	$x + 2y = 4 + (2 \times 0)$ $= 4 + 0 = 4$	4	LHS = RHS	___
4.	$(\sqrt{2}, -3\sqrt{2})$	$x + 2y = \sqrt{2} + 2(-3\sqrt{2})$ $= \sqrt{2} - 6\sqrt{2}$ $= -5\sqrt{2}$	___	LHS $\neq$ RHS	$(\sqrt{2}, -3\sqrt{2})$ Not a Solution
5.	$(1, 1)$	___	4	LHS $\neq$ RHS	$(1, 1)$ Not a Solution
6.	___	$x + 2y = -2 + (2 \times 3)$ $= -2 + 6 = 4$	4	LHS = RHS	$(-2, 3)$ is a Solution

Example-8. If $x = 3, y = 2$ is a solution of the equation $5x - 7y = k$, find the value of k and write the resultant equation.

Solution : If $x = 3, y = 2$ is a solution of the equation

$$5x - 7y = k \text{ then } 5 \times 3 - 7 \times 2 = k$$

$$\Rightarrow 15 - 14 = k$$

$$\Rightarrow 1 = k$$

$$\therefore k = 1$$

The resultant equation is $5x - 7y = 1$.

Example-9. If $x = 2k + 1$ and $y = k$ is a solutions of the equation $5x + 3y - 7 = 0$, find the value of k.

Solution : It is given that $x = 2k + 1$ and $y = k$ is a solution of the equation $5x + 3y - 7 = 0$ by substituting the value of x and y in the equation we get.

$$\Rightarrow 5(2k + 1) + 3k - 7 = 0$$

$$\Rightarrow 10k + 5 + 3k - 7 = 0$$

$\Rightarrow 13k - 2 = 0$ (this is the linear equation in one variable).

$$\Rightarrow 13k = 2$$

$$\therefore k = \frac{2}{13}$$

EXERCISE - 6.2

1. Find three different solutions of the each of the following equations.

   i) $3x + 4y = 7$ ii) $y = 6x$ iii) $2x - y = 7$

   iv) $13x - 12y = 25$ v) $10x + 11y = 21$ vi) $x + y = 0$

2. If (0, a) and (b, 0) are the solutions of the following linear equations. Find 'a' and 'b'.

   i) $8x - y = 34$ ii) $3x = 7y - 21$ iii) $5x - 2y + 3 = 0$

3. Check which of the following is solution of the equation $2x - 5y = 10$

   i) (0, 2) ii) (0, –2) iii) (5, 0) iv) $(2\sqrt{3}, -\sqrt{3})$ v) $\left(\frac{1}{2}, 2\right)$

4. Find the value of k, if $x = 2, y = 1$ is a solution of the equation $2x + 3y = k$. Find two more solutions of the resultant equation.

5. If $x = 2 - \alpha$ and $y = 2 + \alpha$ is a solution of the equation $3x - 2y + 6 = 0$ find the value of 'α'. Find three more solutions of the resultant equation.
6. If $x = 1, y = 1$ is a solution of the equation $3x + ay = 6$, find the value of 'a'.
7. Write five different linear equations in two variables and find three solutions for each of them?

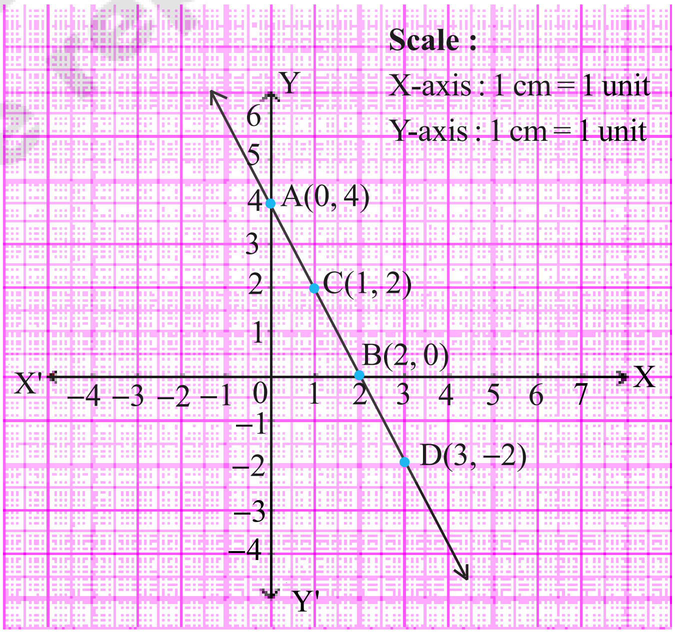
6.4 GRAPH OF A LINEAR EQUATION IN TWO VARIABLES

We have learnt that each linear equation in two variables has many solutions. If we take possible solutions of a linear equation, can we represent them on the graph? We know each solution is a pair of real numbers that can be expressed as a point in the graph.

Consider the linear equation in two variables $4 = 2x + y$. It can also be expressed as $y = 4 - 2x$. For this equation we can find the value of 'y' for a particular value of x. For example if $x = 2$ then $y = 0$. Therefore (2, 0) is a solution. In this way we find as many solutions as we can. Write all these solutions in the following table by writing the value of 'y' below the corresponding value of x.

Table of solutions:

x	$y = 4 - 2x$	(x, y)
0	$y = 4 - 2(0) = 4$	(0, 4)
2	$y = 4 - 2(2) = 0$	(2, 0)
1	$y = 4 - 2(1) = 2$	(1, 2)
3	$y = 4 - 2(3) = -2$	(3, –2)



If $x = 0$;
$y = 4 - 2x = 4 - 2(0) = 4$
If $x = 2$
$y = 4 - 2(2) = 0$

We see for each value of x there is one value of y. Let us take the value of 'x' along the X-axis. and take the value of y along the Y-axis. Let us plot the points (0, 4), (2, 0), (1, 2) and (3, -2) on the graph. If we join any of these two points we obtain a straight line AD.

Do all the other solutions also lie on the line AB?

Now pick any other point on the line say (4,–4). Is this a solution?

Pick up any other point on this line AD and check if its coordinates satisfy the equation or not?

Now take any point not on the line AD say (1, 1). Is it satisfy the equation?

Can you find any point that is not on the line AD but satisfies the equation?

Let us list our observations:

1. Every solution of the linear equation represents a point on the line of the equation.
2. Every point on this line is a solution of the linear equation.
3. Any point that does not lie on this line is not a solution of the equation and vice a versa.
4. The collection of points that give the solution of the linear equation is the graph of the linear equation.

We notice that the graphical representation of a linear equation in two variables is a straight line. Thus, $ax + by + c = 0$(a and b are not simultaneously zero) is called a linear equation in two variables.

6.4.1 How to draw the graph of a linear equation

Steps :

1. Write the linear equation.
2. Put $x = 0$ in the given equation and find the corresponding value of y.
3. Put $y = 0$ in the given equation and find the corresponding value of 'x'.
4. Write the values of x and its corresponding value of y as coordinates of x and y respectively as (x, y) form.
5. Plot the points on the graph paper.
6. Join these points.

Thus line drawn is the graph of linear equation in two variables. However to check the correctness of the line it is better to take more than two points. To find more solutions take different values for 'x' substitute them in the given equation and find the corresponding values of 'y'.

TRY THESE

Take a graph paper, plot the point (2, 4), and draw a line passing through it. Now answer the following questions.

1. Can you draw another line that passes through the point (2, 4).
2. How many such lines can be drawn?
3. How many linear equations in two variables exist for which (2, 4) is a solution?

Example-10. Draw the graph of the equation $y - 2x = 4$ and then answer the following.

(i) Does the point (2, 8) lie on the line? Is (2, 8) a solution of the equation? Check by substituting (2, 8) in the equation.

(ii) Does the point (4, 2) lie on the line? Is (4, 2) a solution of the equation? Check algebraically also.

(iii) From the graph find three more solutions of the equation and also three more which are not solutions.

Solution : Given $y - 2x = 4 \Rightarrow y = 2x + 4$

Table of Solutions

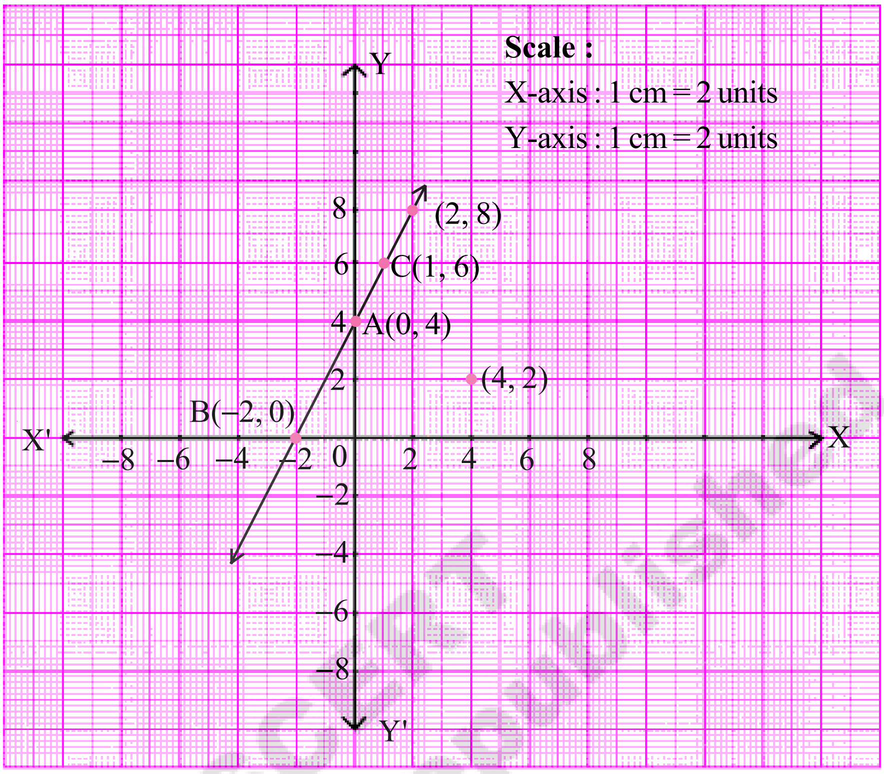
x	y = 2x + 4	(x, y)	Point
0	$y = 2(0) + 4 = 4$	(0, 4)	A(0, 4)
2	$y = 2(-2)+4 = 0$	(–2, 0)	B(–2, 0)
1	$y = 2(1) + 4 = 6$	(1, 6)	C(1, 6)

Plotting the points A, B and C on the graph paper and join them to get the straight line BC as shown in graph sheet. This line is the required graph of the equation $y - 2x = 4$.

(i) Plot the point (2, 8) on the graph paper. From the graph it is clear that the point (2, 8) lies on the line.

Checking algebraically: On substituting (2, 8) in the given equation, we get

LHS = $y - 2x = 8 - 2x2 = 8 - 4 = 4$ = RHS, So (2, 8) is a solution



(ii) Plot the point (4, 2) on the graph paper. You find that (4, 2) does not lie on the line.

Checking algebraically: By substituting (4, 2) in the given equation we have

$$\text{LHS} = y - 2x = 2 - 2\times4 = 2 - 8 = -6 \neq \text{RHS, so } (4, 2) \text{ is not a solution.}$$

(iii) We know that every point on the line is a solution of the given equation. So, we can take any three points on the line as solutions of the given equation. Eg:(-4, -4). And we also know that the point which is not on the line is not a solution of the given equation. So we can take any three points which are not on the line as not solutions of $y - 2x = 4$.
eg : (i) (1, 5); ....; ....

Example-11. Draw the graph of the equation $x - 2y = 3$.

From the graph find (i) The solution (x, y) where $x = -5$

(ii) The solution (x, y) where $y = 0$

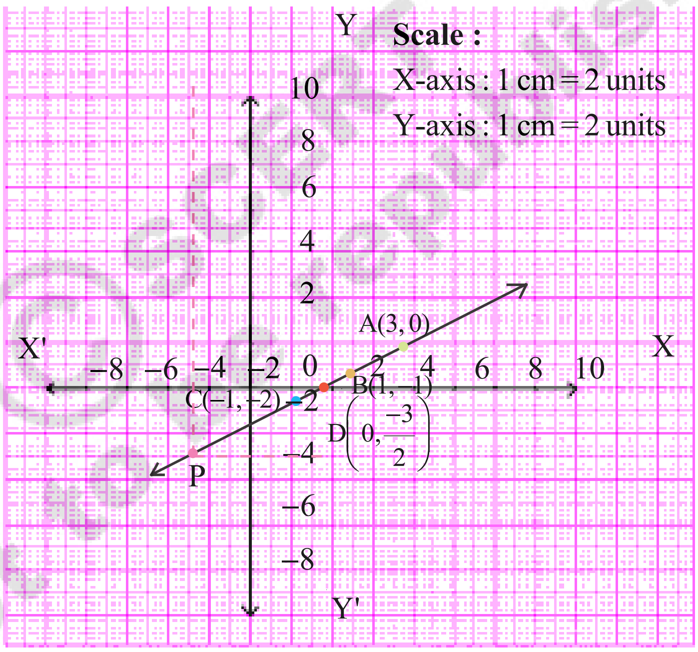
(iii) The solution (x, y) where $x = 0$

Solution : We have $x - 2y = 3 \Rightarrow y = \dfrac{x-3}{2}$

Table of Solutions

x	$y = \frac{x-3}{2}$	(x, y)	Point
3	$y = \frac{3-3}{2} = 0$	(3, 0)	A
1	$y = \frac{1-3}{2} = -1$	(1, – 1)	B
– 1	$y = \frac{-1-3}{2} = -2$	(– 1, – 2)	C

Plotting the points A, B, C on the graph paper and on joining them we get a straight line as shown in the following figure. This line is the required graph of the equation $x - 2y = 3$



(i) We have to find a solution (x, y) where $x = -5$, that is we have to find a point which lies on the straight line and whose x-coordinate is '– 5'. To find such a point we draw a line parallel to y-axis at $x = -5$. (in the graph it is shown as dotted line). This line meets the graph at 'P' from there we draw another line parallel to X-axis meeting the Y-axis at $y = -4$.

The coordinates of P = (– 5, – 4)

Since P(– 5, – 4) lies on the straight line $x - 2y = 3$, it is a solution of $x - 2y = 3$.

(ii) We have to find a solution (x, y) where $y = 0$.

Since $y = 0$, this point (x, 0) lies on the X-axis. Therefore we have to find a point that lies on the X-axis and on the graph of $x - 2y = 3$.

From the graph it is clear that (3, 0) is the required point.

Therefore, the solution is (3, 0).

(iii) We have to find a solution (x, y) where x = 0.

Since x = 0 this point (0, y) lies on the Y-axis. Therefore we have to find a point that lies on the Y-axis and on the graph of $x - 2y = 3$.

From the graph it is clear that $\left(0, \frac{-3}{2}\right)$ is this point.

Therefore, the solution is $\left(0, \frac{-3}{2}\right)$.

Example-12. 25% of the students in a school are girls and others are boys. Form an equation and draw a graph for this. By observing the graph, answer the following :

(i) Find the number of boys, if the number of girls is 25.

(ii) Find the number of girls, if the number of boys is 45.

(iii) Take three different values for number of boys and find the number of girls. Similarly take three different values for number of girls and find the number of boys?

Solution : Let the number of girls be 'x' and number of boys be 'y', then

Total number of students = x + y

According to the given information

Number of girls = 25% of the students

$x = 25\%$ of $(x + y)$

$= \frac{25}{100}$ of $(x + y) = \frac{1}{4}(x + y)$

$$x = \frac{1}{4}(x + y)$$

$$4x = x + y$$

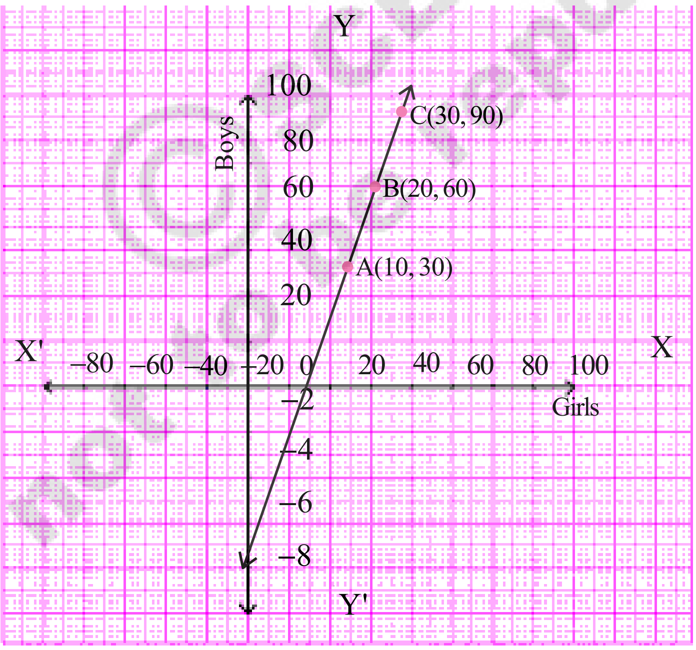
$$3x = y$$

The required equation is $3x = y$ or $3x - y = 0$.

Table of Solutions

x	y = 3x	(x, y)	Point
10	30	(10, 30)	A
20	60	(20, 60)	B
30	90	(30, 90)	C

Plotting points A, B and C on the graph and on joining them we get the straight line as shown in the following figure.



Scale :

X-axis : 1 cm = 20 units

Y-axis : 1 cm = 20 units

From the graph we find that

(i) If the number of girls is 25 then the number of boys is 75.

(ii) If the number of boys is 45, then the number of girls is 15.

(iii) Choose the number you want for girls and find the corresponding number of boys. Similarly choose the numbers you want for boys and find the corresponding number of girl. Here do you observe the graph and equation. The line is passing through the origin and if the equation which is in the form $y = mx$ where m is a real number the line passes through the origin.

Example-13. For each graph given below, four linear equations are given. Out of these find the equation that represents the given graph.

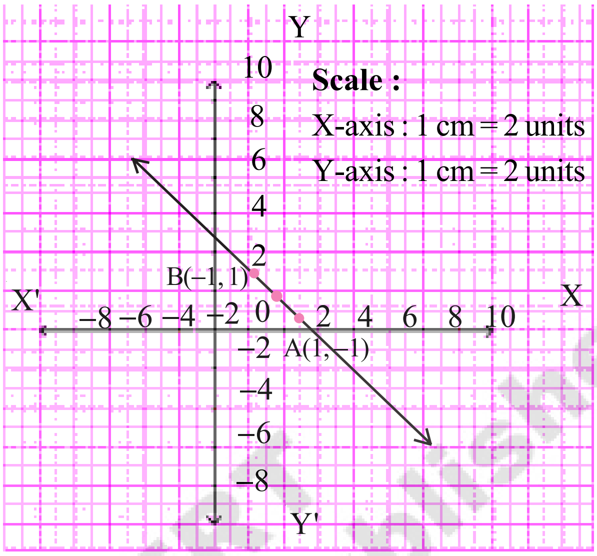
(i) Equations are

A) $y = x$

B) $x + y = 0$

C) $y = 2x$

D) $2 + 3y = 7x$



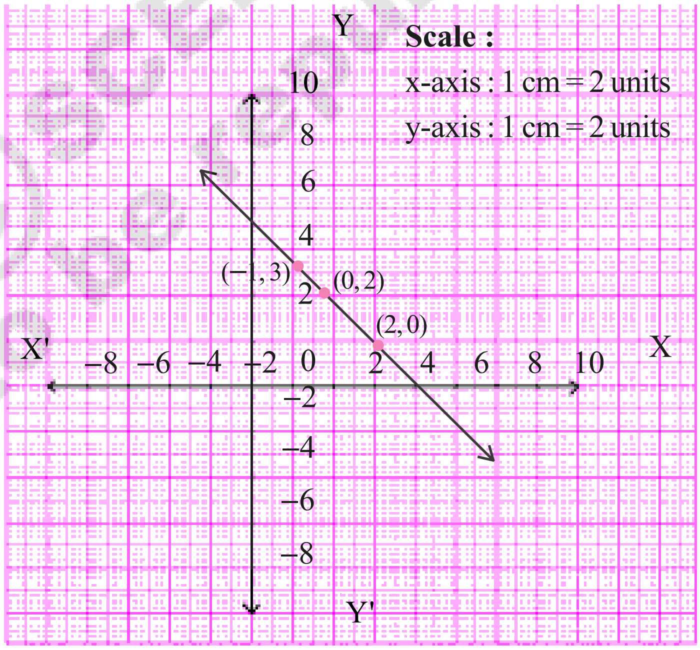
(ii) Equations are

A) $y = x + 2$

B) $y = x - 2$

C) $y = -x + 2$

D) $x + 2y = 6$



Solution :

(i) From the graph we see $(1, -1)$ $(0, 0)$ $(-1, 1)$ lie on the same line. So these are the solutions of the required equation i.e. if we substitute these points in the required equation it should be satisfied. So, we have to find an equation that shoul be satisfied by these pairs. If we substitute $(1, -1)$ in the first equation $y = x$ it is not satisfied. So $y = x$ is not the required equation.

Putting $(1, -1)$ in $x + y = 0$ we find that it satisfies the equation. In fact all the three points satisfy the second equation. So $x + y = 0$ is the required equation.

We now check whether $y = 2x$ and $2 + 3y = 7x$ are also satisfied by $(1, -1)$ $(0, 0)$ and $(-1, 1)$. We find they are not satisfied by even one of the pairs, leave alone all three. So, they are not the required equations.

(ii) The points on the line are $(2, 0)$, $(0, 2)$ and $(-1, 3)$. All these points don't satisfy the first and second equation. Let us take the third equation $y = -x + 2$. If we substitute the above three points in the equation, it is satisfied. So required equation is $y = -x + 2$. Check whether these points satisfies the equation $x + 2y = 6$.

EXERCISE - 6.3

1. Draw the graph of each of the following linear equations.

   i) $2y = -x + 1$ ii) $-x + y = 6$ iii) $3x + 5y = 15$ iv) $\frac{x}{2} - \frac{y}{3} = 3$

2. Draw the graph of each of the following linear equations and answer the following question.

   i) $y = x$ ii) $y = 2x$ iii) $y = -2x$ iv) $y = 3x$ v) $y = -3x$

   i) Are all these equations of the form $y = mx$, where m is a real number?

   ii) Are all these graphs passing through the origin?

   iii) What can you conclude about these graphs?

3. Draw the graph of the equation $2x + 3y = 11$. Find from the graph value of y when $x = 1$

4. Draw the graph of the equation $y - x = 2$. Find from the graph

   i) the value of y when $x = 4$

   ii) the value of x when $y = -3$

5. Draw the graph of the equation $2x+3y=12$. Find the solutions from the graph

   i) Whose y-coordinate is 3

   ii) Whose x-coordinate is -3

6. Draw the graph of each of the equations given below and also find the coordinates of the points where the graph cuts the coordinate axes

   i) $6x - 3y = 12$ ii) $-x + 4y = 8$ iii) $3x + 2y + 6 = 0$

7. Rajiya and Preethi two students of Class IX together collected ₹ 1000 for the Prime Minister Relief Fund for victims of natural calamities. Write a linear equation and draw a graph to depict the statement.

8. Gopaiah sowed wheat and paddy in two fields of total area 5000 square meters. Write a linear equation and draw a graph to represent the same?

9. The force applied on a body of mass 6 kg. is directly proportional to the acceleration produced in the body. Write an equation to express this observation and draw the graph of the equation.

10. A stone is falling from a mountain. The velocity of the stone is given by $V = 9.8t$. Draw its graph and find the velocity of the stone '4' seconds after start.

11. In a election 60% of voters cast their votes. Form an equation and draw the graph for this data. Find the following from the graph.

(i) The total number of voters, if 1200 voters cast their votes

(ii) The number votes cast, if the total number of voters are 800

[**Hint:** If the number of voters who cast their votes be 'x' and the total number of voters be 'y' then x = 60% of y.]

12. When Rupa was born, his father was 25 years old. Form an equation and draw a graph for this data. From the graph find

(i) The age of the father when Rupa is 25 years old.

(ii) Rupa's age when her father is 40 years old.

13. An auto charges ₹ 15 for first kilometer and ₹ 8 each for each subsequent kilometer. For a distance of 'x' km. an amount of ₹ 'y' is paid.
Write the linear equation representing this information and draw the graph. With the help of graph find the distance travelled if the fare paid is ₹ 55? How much would have to be paid for 7 kilometers?

14. A lending library has fixed charge for the first three days and an additional charges for each day thereafter. John paid ₹ 27 for a book kept for seven days. If the fixed charges be ₹ x and subsequent per day charges be ₹ y, then write the linear equation representing the above information and draw the graph of the same. From the graph if the fixed the subsequent per day charge ? and if the per day charge is ₹ 4/- find the 'fixed' charge ? charge is ₹ 7

15. The parking charges of a car in Hyderabad Railway station for first two hours is ₹ 50 and ₹10 for each subsequent hour. Write down an equation and draw the graph. Find the following charges from the graph

    (i) For three hours (ii) For six hours

    (iii) How many hours did Rekha park her car if she paid ₹ 80 as parking charges?

16. Sameera was driving a car with uniform speed of 60 kmph. Draw distance-time graph. From the graph find the distance travelled by Sameera in

    (i) $1\frac{1}{2}$ hours (ii) 2 hours (iii) $3\frac{1}{2}$ hours

17. The ratio of molecular weight of Hydrogen and Oxygen in water is 1:8. Set up an equation between Hydrogen and Oxygen and draw its graph. From the graph find the quantity of Hydrogen if Oxygen is 12 grams. And quantity of oxygen if hydrogen is $\frac{3}{2}$ gms.?

    [**Hint :** If the quantities of hydrogen and oxygen or 'x' and 'y' respectively, then $x : y = 1:8 \Rightarrow 8x = y$]

18. In a mixture of 28 litres, the ratio of milk and water is 5:2. Set up the equation between the mixture and milk. Draw its graph. By observing the graph find the quantity of milk in the mixture.

    [**Hint:** Ratio between mixture and milk $= 5 + 2 : 5 = 7 : 5$]

19. In countries like USA and Canada temperature is measured in Fahrenheit where as in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius $F = \left(\frac{9}{5}\right) C + 32$

    (i) Draw the graph of the above linear equation having Celsius on x-axis and Fahrenheit on Y-axis.

    (ii) If the temperature is 30°C, what is the temperature in Fahrenheit?

    (iii) If the temperature is 95°F, what is the temperature in Celsius?

    (iv) Is there a temperature that has numerically the same value in both Fahrenheit and Celsius? If yes find it?

6.5 Equation of lines parallel to X-axis and Y-axis

Consider the equation x = 3. If this is treated as an equation in one variable x, then it has the unique solution x = 3 which is a point on the number line

$$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$$

However when treated as an equation in two variables and plotted on the coordinate plane it can be expressed as x + 0.y – 3 = 0

This has infinitely many solutions, let us find some of them. Here the coefficient of y is zero. So for all values of y, x becomes 3.

Table of solutions

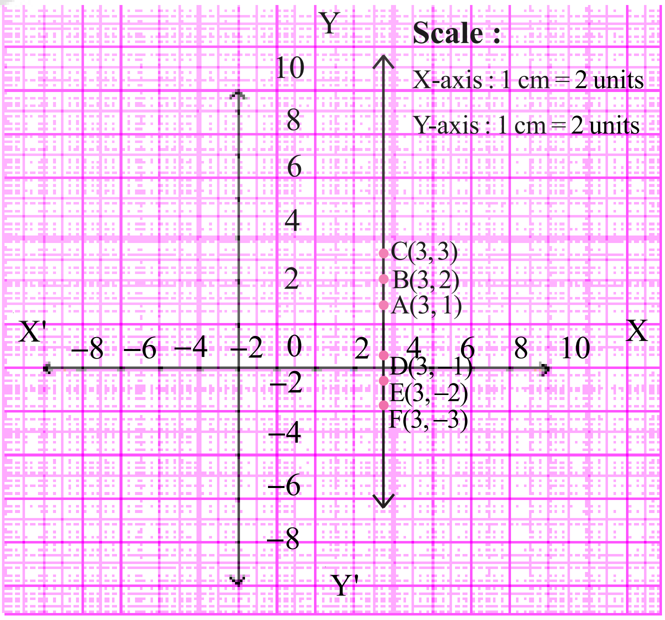
x	3	3	3	3	3	3	…...
y	1	2	3	–1	–2	–3	…..
(x, y)	(3, 1)	(3, 2)	(3, 3)	(3, –1)	(3, –2)	(3, –3)	…..
Points	A	B	C	D	E	F	…..

From the table it is clear that this equation has infinitely many solutions of the form (3, a) where a is any real number.

Now draw the graph using the above solutions. What do you notice from the graph?

Is it a straight line? Whether it is any line or axes? The line drawn is a straight line and is parallel to Y-axis?

What is the distance of this line from the y-axis?



Thus the graph of x = 3 is a line parallel to the y-axis at a distance of 3 units to the right of it.

DO THIS

1. i) Draw the graph of following equations.

   a) $x = 2$ b) $x = -2$ c) $x = 4$ d) $x = -4$

   ii) Are the graphs of all these equations parallel to Y-axis?

   iii) Find the distance between the graph and the Y-axis in each case

2. i) Draw the graph of the following equations

   a) $y = 2$ b) $y = -2$ c) $y = 3$ d) $y = -3$

   ii) Are all these parallel to the X-axis?

   iii) Find the distance between the graph and the X-axis in each case

From the above observations we can conclude the following:

1. The graph of $x = k$ is a line parallel to Y-axis at a distance of k units and passing through the point (k, 0)
2. The graph of $y = k$ is a line parallel to X-axis at a distance of k units and passing through the point (0, k)

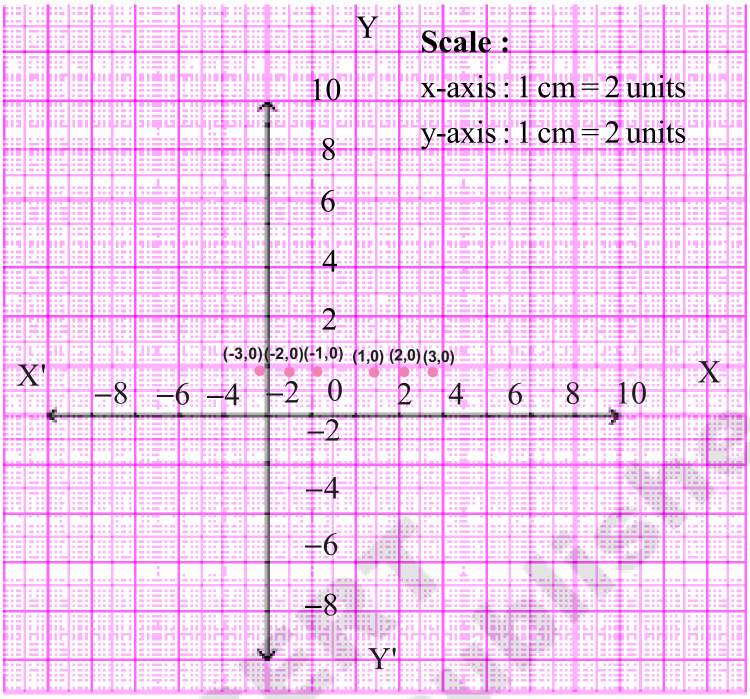
6.5.1 Equation of the X-axis and the Y-axis:

Consider the equation $y = 0$. It can be written as $0.x + y = 0$. Let us draw the graph of this equation.

Table of solutions

x	1	2	3	–1	–2	……
y	0	0	0	0	0	…..
(x, y)	(1, 0)	(2, 0)	(3, 0)	(–1, 0)	(–2, 0)	…..
Points	A	B	C	D	E	…..

By plotting all these points on the graph paper, we get the following figure. From the graph what do we notice?



We notice that all these points lie on the X-axis and y-coordinate of all these points is ‘0’.

Therefore the equation $y = 0$ represents X-axis. In other words the equation of the X-axis is $y = 0$.

TRY THESE

Find the equation of the y-axis.

EXERCISE - 6.4

1. Give the graphical representation of the following equation.

   a) On the number line and b) On the Cartesian plane

   i) $x = 3$ ii) $y + 3 = 0$ iii) $y = 4$ iv) $2x - 9 = 0$

   v) $3x + 5 = 0$

2. Give the graphical representation of $2x - 11 = 0$ as an equation in

   i) one variable ii) two variables

3. Solve the equation $3x + 2 = 8x - 8$ and represent the solution on

   i) the number line   ii) the Cartesian plane

4. Write the equation of the line parallel to X-axis, and passing through the point

   i) $(0, -3)$ ii) $(0, 4)$ iii) $(2, -5)$ iv) $(3, 4)$

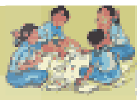
5. Write the equation of the line parallel to Y-axis and passing through the point

   i) $(-4, 0)$ ii) $(2, 0)$ iii) $(3, 5)$ iv) $(-4, -3)$

6. Write the equation of three lines that are

   (i) parallel to the X-axis   (ii) parallel to the Y-axis.

WHAT WE HAVE DISCUSSED



1. If a linear equation has two variables then it is called linear equation in two variables.
2. Any pair of values of 'x' and 'y' which satisfy the linear equation in two variables is called its solution.
3. A linear equation in the two variables has many solutions.
4. The graph of every linear equation in two variables is a straight line.
5. An equation of the type $y = mx$ represents a line passing through the origin.
6. The graph of $x = k$ is a line parallel to Y - axis at a distance of k units and passing through the point (k, 0).
7. The graph of y = k is a line parallel to X-axis at a distance of k units and passing through the point (0, k).
8. Equation of X-axis is y = 0.
9. Equation of Y-axis is x = 0.