CBSE SAMPLE PAPER
Class XI Mathematics
Paper 1(Answers)

1. Solution:

We have,

$\sin(\sin^{-1}\frac{3}{5}+\cos^{-1}x)=1$

$\Rightarrow \sin^{-1}\frac{3}{5}+\cos^{-1}x=\sin^{-1}(1)$ $[\because \sin\theta = x \ \Rightarrow \theta=\sin^{-1}x]$

$\Rightarrow \sin^{-1}\frac{3}{5}+\cos^{-1}x=\sin^{-1}\left(\sin\frac{\pi}{2}\right)$ $\left[\because \sin\frac{\pi}{2}=1\right]$

$\Rightarrow \sin^{-1}\frac{3}{5}+\cos^{-1}x=\frac{\pi}{2}$ ... (1)

We know that,

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2},\ x\,\epsilon\,[-1,1]\ \ldots(2)$

Equating equations (1) and (2), we get

LHS of both equations are equal.

$\sin^{-1}\frac{3}{5}+\cos^{-1}x=\sin^{-1}x+\cos^{-1}x$

$\Rightarrow \sin^{-1}\frac{3}{5}=\sin^{-1}x$

Hence, $x=\frac{3}{5}$.

2. Solution:

We have, $(3+7i)^2$

$(3+7i)^2=3^2+(7i)^2+2(3)(7i)$

$=9+49i^2+42i$

$=9-49+42i\ [\because i^2=-1]$

$=-40+42i$

3. Solution:

Let $A = \begin{vmatrix} \cos 18° & \sin 18° \\ \sin 72° & \cos 72° \end{vmatrix}$

On expanding, we get

$A = (\cos 18° \cdot \cos 72° - \sin 18° \cdot \sin 72°)$

$= \cos(18° + 72°)$ [$\because \cos A \cdot \cos B - \sin A \cdot \sin B = \cos(A + B)$]

$= \cos 90° = 0$ [$\because \cos 90° = 0$]

OR

Solution:

We have, $\begin{vmatrix} x & 10 \\ 5 & 2x \end{vmatrix} = 0$

On expanding, we get

$\Rightarrow x \cdot 2x - 5 \cdot 10 = 0$

$\Rightarrow 2x^2 - 50 = 0$

$\Rightarrow 2x^2 = 50$

$\Rightarrow x^2 = 25$

$\Rightarrow x = \pm 5$

Hence, $x = \pm 5$.

4. Solution:

We have, $\lim\limits_{x \to 1} \frac{x^{45} - 1}{x^{40} - 1}$

$= \lim\limits_{x \to 1} \left\{ \left(\frac{x^{45} - 1}{x - 1} \right) \div \left(\frac{x^{40} - 1}{x - 1} \right) \right\}$

$= \lim\limits_{x \to 1} \left(\frac{x^{45} - 1}{x - 1} \right) \div \lim\limits_{x \to 1} \left(\frac{x^{40} - 1}{x - 1} \right)$

$= (45 \times 1^{44}) \div (40 \times 1^{39}) = \frac{45}{40} = \frac{9}{8}$

Hence, $\lim_{x\to 1}\frac{x^{45}-1}{x^{40}-1}=\frac{9}{8}$.

SECTION B

5. Solution:

We have,

$\tan^{-1}\frac{1}{\sqrt{3}}+\cot^{-1}x=\frac{\pi}{2}$ ... (1)

We know that,

$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2},\ \ x \epsilon R\ \ \ldots(2)$

Equating equations (1) and (2), we get

$\tan^{-1}\frac{1}{\sqrt{3}}+\cot^{-1}x=\tan^{-1}x+\cot^{-1}x$

$\Rightarrow \tan^{-1}\frac{1}{\sqrt{3}}=\tan^{-1}x$

Hence, $x=\frac{1}{\sqrt{3}}$.

6. Solution:

We have,

$\sec\theta\begin{bmatrix}\sec\theta & \tan\theta\\ \tan\theta & \sec\theta\end{bmatrix}-\tan\theta\begin{bmatrix}\tan\theta & \sec\theta\\ \sec\theta & \tan\theta\end{bmatrix}$

$=\begin{bmatrix}\sec^2\theta & \sec\theta\cdot\tan\theta\\ \sec\theta\cdot\tan\theta & \sec^2\theta\end{bmatrix}-\begin{bmatrix}\tan^2\theta & \sec\theta\cdot\tan\theta\\ \sec\theta\cdot\tan\theta & \tan^2\theta\end{bmatrix}$

$=\begin{bmatrix}\sec^2\theta-\tan^2\theta & \sec\theta\cdot\tan\theta-\sec\theta\cdot\tan\theta\\ \sec\theta\cdot\tan\theta-\sec\theta\cdot\tan\theta & \sec^2\theta-\tan^2\theta\end{bmatrix}$

$=\begin{bmatrix}\sec^2\theta-\tan^2\theta & 0\\ 0 & \sec^2\theta-\tan^2\theta\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I=$ Unit matrix $[\because \sec^2\theta-\tan^2\theta=1]$

OR

Solution:

Since, $|A^3| = 512$

$$\Rightarrow |A^3| = 8^3$$

$\therefore A = \begin{bmatrix} \alpha & 1 \\ 1 & \alpha \end{bmatrix}$

$\Rightarrow \alpha^2 - 1 = 8$

$\Rightarrow \alpha^2 = 8 + 1$

$\Rightarrow \alpha^2 = 9$

$\Rightarrow \alpha = \pm 3$

Hence, $\alpha = \pm 3$.

7. Solution:

Here, $a = 32, d = 36 - 32 = 4$ and $a_n = 320$

Let n be the number of terms.

Now, $a_n = 320 \Leftrightarrow a + (n-1)d = 320$

$\Leftrightarrow 32 + (n-1)(4) = 320$ [$\because a = 32, d = 4$]

$\Leftrightarrow 32 + 4n - 4 = 320$

$\Leftrightarrow 4n + 28 = 320 \Leftrightarrow 4n = 320 - 28 = 292$

$\Leftrightarrow n = \frac{292}{4} = 73$

Hence, the given AP contains 73 terms.

OR

Solution:

We have,

$(n+2)(n+1) \times n! = 90 \times n!$

$\Rightarrow (n+2)(n+1) = 90$

$\Rightarrow n^2 + 2n + n + 2 = 90$

$\Rightarrow n^2 + 3n - 88 = 0$

$\Rightarrow n^2 + 11n - 8n - 88 = 0$

$\Rightarrow n(n+11) - 8(n+11) = 0$

$\Rightarrow (n+11)(n-8) = 0$

$\Rightarrow n = 8$

$[\therefore n \geq 0]$

$Hence, n = 8.$

8. Solution:

We have, $\frac{6+\sqrt{5}i}{1-\sqrt{5}i}$

$\frac{6+\sqrt{5}i}{1-\sqrt{5}i} = \frac{6+\sqrt{5}i}{1-\sqrt{5}i} \times \frac{1+\sqrt{5}i}{1+\sqrt{5}i}$

$= \frac{6+6\sqrt{5}i+\sqrt{5}i+(\sqrt{5}i)^2}{1^2-(\sqrt{5}i)^2}$ $[\because a^2 - b^2 = (a+b)(a-b)]$

$= \frac{6+7\sqrt{5}i+5i^2}{1-5i^2} = \frac{6+7\sqrt{5}i-5}{1+5}$ $[\because i^2 = -1]$

$= \frac{1+7\sqrt{5}i}{6} = \frac{1}{6} + \frac{7\sqrt{5}}{6}i$

Hence, $\frac{6+\sqrt{5}i}{1-\sqrt{5}i} = \frac{1}{6} + \frac{7\sqrt{5}}{6}i.$

9. Solution:

Given, $f(x) = 256x^4$ and $g(x) = x^{\frac{1}{4}}$

$\therefore gof(x) = g[f(x)] = g(256x^4)$

$= (256x^4)^{\frac{1}{4}} = (256)^{\frac{1}{4}} \cdot (x^4)^{\frac{1}{4}}$

$= (4^4)^{\frac{1}{4}} \cdot (x^4)^{\frac{1}{4}} = 4^{4^{\times\frac{1}{4}}} \cdot x^{4^{\times\frac{1}{4}}} = 4x$

Hence, $gof(x) = 4x$.

OR

Solution:

Here, g is the inverse of $f(x)$.

$\Rightarrow fog(x) = x$

On differentiating with respect to x, we get

$f'\{g(x)\} \times g'(x) = 1$

$\Longrightarrow g'(x) = \frac{1}{f'\{g(x)\}}$

$= \frac{1}{\frac{1}{1+\{g(x)\}^{19}}}$ $[\because f'(x) = \frac{1}{1+x^{19}}]$

$\Longrightarrow g'(x) = 1 + \{g(x)\}^{19}$

Hence, the value of $g'(x)$ is $1 + \{g(x)\}^{19}$.

10. Solution:

The given equation of the line is $\frac{5-x}{7} = \frac{y}{4} = \frac{3-z}{4}$.

It can be rewritten in standard form

$\frac{x-5}{-7} = \frac{y}{4} = \frac{z-3}{-4}$.

1. Direction ratio of the line is $(-7, 4, -4)$.

Now, $\sqrt{(-7)^2 + (4)^2 + (-4)^2}$

$= \sqrt{49 + 16 + 16} = \sqrt{81} = 9$ Units

2. Direction cosine of the line is $\left(-\frac{7}{9}, \frac{4}{9}, -\frac{4}{9}\right)$.

11. Solution:

We have, $(x^3 + 4y)^4$

Using the binomial theorem, we get

$$(x^3 + 4y)^4 = C_0^4(x^3)^4 + C_1^4(x^3)^3(4y) + C_2^4(x^3)^2(4y)^2 + C_3^4(x^3)^1(4y)^3 + C_4^4(4y)^4$$

$$= 1 \cdot x^{12} + 4 \cdot x^9 \cdot (4y) + 6 \cdot x^6 \cdot 16 \cdot y^2 + 4 \cdot x^3 \cdot 64 \cdot y^3 + 1 \cdot 256 \cdot y^4$$

$$= x^{12} + 36 \cdot x^9 \cdot y + 96 \cdot x^6 \cdot y^2 + 256 \cdot x^3 \cdot y^3 + 256 \cdot y^4$$

Hence, $(x^3 + 4y)^4 = x^{12} + 36x^9y + 96x^6y^2 + 256x^3y^3 + 256y^4$

12. Solution:

We know that the equation of a line with slope m and $x - intercept$ d is given by:

$y = m(x - d)$.

Here, $\tan\theta = \frac{1}{5}$ and $d = 6$.

Hence, the required equation of the line is:

$y = \frac{1}{5}(x - 6)$

$\Rightarrow 5y = x - 6$

$\Rightarrow x - 5y - 6 = 0$

13. Solution:

Clearly, $f : R \rightarrow R$ is a one - one function.

So, it is invertible.

Let $(x) = y$.

Then, $11x - 13 = y \Rightarrow 11x = y + 13$

$\therefore x = \frac{y + 13}{11}$.

$$\Rightarrow f^{-1}(y) = \frac{y + 13}{11}$$

Hence, $f^{-1}(x) = \frac{x+13}{11}$.

14. Solution:

The given function is:

$f(x) = \frac{6}{4}x^4 - 2x^3 - 6x^2 + 32$

On differentiating both sides $w.r.t. x$, we get

$f'(x) = \frac{6}{4} \cdot 4 \cdot x^3 - 2 \cdot 3 \cdot x^2 - 6 \cdot 2 \cdot x + 0$

$\Rightarrow f'(x) = 6x^3 - 6x^2 - 12x$

For strictly increasing or strictly decreasing,

Put $f'(x) = 0$, we get

$6x^3 - 6x^2 - 12x = 0$

$\Rightarrow 6x(x^2 - x - 2) = 0$

$\Rightarrow 6x(x^2 - 2x + x - 2) = 0$

$\Rightarrow 6x(x + 1)(x - 2) = 0$

$\Rightarrow x = 0, -1 \text{ } or \text{ } 2$

Now, we find intervals in which $f(x)$ is strictly increasing or strictly decreasing.

Interval	$f'(x) = 6x(x+1)(x-2)$	Sign of $f'(x)$
$x < -1$	$(-)(-)(-)$	$-ve$
$-1 < x < 0$	$(-)(+)(-)$	$+ve$
$0 < x < 2$	$(+)(+)(-)$	$-ve$
$x > 2$	$(+)(+)(+)$	$+ve$

We know that, a function $f(x)$ is said to be strictly increasing, if $f'(x) > 0$ and it is said to be strictly increasing, if$f'(x) < 0$. So, the given function $f(x)$ is

1. Strictly increasing on the interval $(-1,0)$ and $(2,\infty)$.

2. Strictly decreasing on the interval $(-\infty, -1)$ and (0,2).

15. Solution:

There are 5 letters in the word 'PUNAM', of which all are each of its own kind.

After fixing P at first place and M at last place, we have 3 letters out of which all are its own kind.

So, total number of words

$= 3! = 3 \times 2 \times 1 = 6$

After fixing P at first place, we have 4 letters out of which are all the each of its own kind

So, total number of words

$= 4! = 4 \times 3 \times 2 \times 1 = 24$

∴Number of words begin with P and does not end with M = Number of words begin with P - Number of words begin with P and end with M

$= 24 - 6 = 18$

16. Solution:

We have,

$(1+3x)^6(1-x)^7 = [1 + C_1^6(3x) + C_2^6(3x)^2 + C_3^6(3x)^3 + C_4^6(3x)^4 + C_5^6(3x)^5 + C_6^6(3x)^6]$

$\times [1 - C_1^7(x) + C_2^7(x)^2 - C_3^7(x)^3 + C_4^7(x)^4 - C_5^7(x)^5 + C_6^7(x)^6 - C_7^7(x)^7]$

$= [1 + 7 \times 3x + 15 \times 9x^2 + 20 \times 27x^3 + 15 \times 81x^4 + 6 \times 243x^5 + 1 \times 729x^6]$

$\times [1 - 7 \times x + 21 \times x^2 - 35 \times x^3 + 35 \times x^4 - 21 \times x^5 + 7 \times x^6 - 1 \times x^7]$

$= [1 + 21x + 135x^2 + 540x^3 + 1215x^4 + 1458x^5 + 729x^6]$

$\times [1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7]$

∴ Co-efficient of the x^4 in the product

$= 1 \times 35 + 21 \times (-35) + 135 \times 21 + 540 \times (-7) + 1215 \times 1$

$= 35 - 735 + 2835 - 3780 + 1215 = -430$

Hence, the co-efficient of the product of x^4 in the given expansion is -430.

17. Solution:

The equation of a line passing through the points $(1, -3, 6)$ and parallel to $x = y = z$ is

$\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-6}{1} = \lambda$ (Let)

Thus, any point on this line is of the form $(\lambda + 1, \lambda - 3, \lambda + 6)$.

Now, if $\mathrm{P}(\lambda + 1, \lambda - 3, \lambda + 6)$ is the point of intersection of line and plane, then

$\lambda + 1 - (\lambda - 3) + \lambda + 6 = 6$

$\therefore \lambda + 1 = -4 + 1 = -3, \lambda - 3 = -4 - 3 = -7, \lambda + 6 = -4 + 6 = 2$

$\therefore$Coordinates of point P are $(-3, -7, 2)$.

Hence, required distance

$= \sqrt{(-3-1)^2 + (-7+3)^2 + (2-6)^2}$

$= \sqrt{4^2 + 4^2 + 4^2}$

$= \sqrt{16 + 16 + 16} = \sqrt{48} = 4\sqrt{3}$ units

18. Solution:

$\because$ The vertices of the ellipse lie on the $y - axis$, it is a vertical ellipse.

Let the required equation be $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where $a^2 > b^2$.

Its vertices are $(0, \pm a)$ and therefore, $a = 5$.

Let $c^2 = a^2 - b^2$

Then, $e = \frac{c}{a} \Rightarrow c = ae = 5 \times \frac{3}{5} = 3$

Now, $c^2 = a^2 - b^2 \Leftrightarrow b^2 = a^2 - c^2 = 5^2 - 3^2 = 25 - 9 = 16$

$\therefore a^2 = 5^2 = 25$ and $b^2 = 16$

Hence, the required equation is $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

19. Solution:

$$L.H.S. = \begin{vmatrix} x+\lambda & 6x & 6x \\ 6x & x+\lambda & 6x \\ 6x & 6x & x+\lambda \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$L.H.S. = \begin{vmatrix} 13x+\lambda & 6x & 6x \\ 13x+\lambda & x+\lambda & 6x \\ 13x+\lambda & 6x & x+\lambda \end{vmatrix}$$

$$L.H.S. = (13x+\lambda)\begin{vmatrix} 1 & 6x & 6x \\ 1 & x+\lambda & 6x \\ 1 & 6x & x+\lambda \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$$L.H.S. = (13x+\lambda)\begin{vmatrix} 1 & 6x & 6x \\ 0 & \lambda-5x & 0 \\ 0 & 0 & \lambda-5x \end{vmatrix}$$

$$L.H.S. = (13x+\lambda)(\lambda-5x)^2 = R.H.S.$$

Hence, it is proved.

OR

Solution:

We have, $A = \begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 4+36 & 12+48 \\ 12+48 & 36+64 \end{bmatrix} = \begin{bmatrix} 40 & 60 \\ 60 & 100 \end{bmatrix}$$

$$\therefore A^2 - 20I_2 = \begin{bmatrix} 40 & 60 \\ 60 & 100 \end{bmatrix} - 20\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 40 & 60 \\ 60 & 100 \end{bmatrix} - \begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$$

$$\therefore A^2 - 20I_2 = \begin{bmatrix} 20 & 60 \\ 60 & 80 \end{bmatrix} = 10\begin{bmatrix} 2 & 6 \\ 6 & 8 \end{bmatrix} = 10A$$

$\Rightarrow A^2 - 20I_2 = 10A$

$\because kA = 10A$

$\Rightarrow k = 10$

Hence, $k = 10$.

20. Solution:

Let $\cos^{-1}\frac{12}{13} = \theta$. Then, $\cos\theta = \frac{12}{13}$.

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{\sqrt{1-\frac{144}{169}}}{\frac{12}{13}} = \frac{5}{13} \times \frac{13}{12} = \frac{5}{12}$

$\therefore \theta = \tan^{-1}\frac{5}{12}$

$L.H.S. = \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{4}{3}$

$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\cdot\frac{4}{3}}\right)$ $\left(\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right)$

$= \tan^{-1}\left(\frac{\frac{5+16}{12}}{\frac{36-20}{36}}\right) = \tan^{-1}\left(\frac{21}{12} \times \frac{36}{16}\right) = \tan^{-1}\left(\frac{7}{4} \times \frac{9}{4}\right)$

$= \tan^{-1}\left(\frac{63}{16}\right) = R.H.S.$

OR

Solution:

Given, $\tan^{-1}(x+3) + \tan^{-1}(x-3) = \tan^{-1}\frac{2}{3}, x > 0$

$\Rightarrow \tan^{-1}\left(\frac{(x+3)+(x-3)}{1-(x+3)\cdot(x-3)}\right) = \tan^{-1}\frac{2}{3}$

$\left(\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right)$

$\Rightarrow \tan^{-1}\left(\frac{x+3+x-3}{1-(x^2-9)}\right) = \tan^{-1}\frac{2}{3}$

$\Rightarrow \tan^{-1}\left(\frac{2x}{1-x^2+9}\right) = \tan^{-1}\frac{2}{3}$

$\Rightarrow \frac{2x}{10-x^2} = \frac{2}{3}$

$\Rightarrow 6x = 20 - 2x^2$

$\Rightarrow 2x^2 + 6x - 20 = 0$

$\Rightarrow x^2 + 3x - 10 = 0$

$\Rightarrow x^2 + 5x - 2x - 10 = 0$

$\Rightarrow x(x+5) - 2(x+5) = 0$

$\Rightarrow (x+5)(x-2) = 0$

$\Rightarrow x = 2 \; or - 5$

But it is given that, $x > 0$.

$\therefore x = 2$.

21. Solution:

$1 + 21 + 41 + 61 + \cdots + x = 622500$

The given series is an AP

Here, first term $(a) = 1$, Common difference $(d) = 21 - 1 = 20$ and

x be the number of term

$S_x = 622500$

Using formula,

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

$S_x = \frac{x}{2}\{2 \cdot 1 + (x-1)20\} = 622500$

$\Rightarrow \frac{x}{2}\{20x - 18\} = 622500$

$\Rightarrow 10x^2 - 9x - 622500 = 0$

$\therefore x = \frac{-(-9)\pm\sqrt{(-9)^2-4\cdot10\cdot(-622500)}}{2\cdot10} = \frac{9\pm\sqrt{81+40\times622500}}{20}$

$= \frac{9\pm\sqrt{24900081}}{20} = \frac{9\pm4990}{20}$

$= \frac{9+4990}{20}, \frac{9-4990}{20}$

$= 249, -249.5$ (Negative sign neglected)

Hence, $x = 249$.

OR

Solution:

We have, $x^2 + 3x + 3 = 0$

Here, $a = 1, b = 3\ and\ c = 3$

$\therefore D = b^2 - 4ac$

$= 3^2 - 4 \cdot 1 \cdot 3$

$= 9 - 12 = -3 < 0$

So, the given equation has complex roots.

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-3\pm\sqrt{-3}}{2\cdot1}$

$= \frac{-3\pm i\sqrt{3}}{2}$ $[\because i = \sqrt{-1}]$

$\therefore$ Solution set $= \left\{\frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}$

22. Solution:

We have, $\sim (m \vee n) \vee (\sim m \wedge n)$

$\sim (m \vee n) \vee (\sim m \wedge n) \equiv (\sim m \wedge \sim n) \vee \ (\sim m \wedge n)$

[$\because$ De-Morgan's law $\sim (x \vee y) = (\sim x \wedge \sim y)$]

$\equiv \sim m \wedge (\sim m \vee n)$ [$\because$ Distributive law $\sim x \vee y = t)$]

$\equiv \sim m \wedge t$

$\equiv \sim m$

23. Solution:

Let the probability that Raju can solve a problem be denoted by P(R).

So, $P(R) \ = \frac{1}{3}$

Let the probability that Akash can solve a problem be denoted by P(A).

So, $P(A) \ = \frac{3}{4}$

Also,$P(X \cap Y) \ = \ P(X) \cdot P(Y)$

Hence,

$P(R \cap A) \ = \ P(R) \cdot P(A) \ = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$

Also,$P(X \cup Y) \ = \ P(X) \ + \ P(Y) - P(X \cap Y)$

$P(R \cup A)$ represents that both of them will solve the problem.

$\therefore P(R \cup A) \ = \ P(R) \ + \ P(A) - P(R \cap A)$

$P(R \cup A) \ = \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$

$= \frac{4+9-3}{12} = \frac{10}{12} = \frac{5}{6}$

Hence, required answer is $\frac{5}{6}$

24. Solution:

We have, $f(x) = |\log 11 - \sin x|$ and $(x) = f(f(x))$, $x \in R$

Note that $x \to 0, \log 11 > \sin x$

$$\therefore f(x) = \log 11 - \sin x$$

$\Rightarrow g(x) = \log 11 - \sin f(x)) = \log 11 - \sin(\log 11 - \sin x)$

Clearly, $g(x)$ is differentiable at $x = 0$ as $\sin x$ is differentiable.

Now,
$g'(x) = 0 - \cos(\log 11 - \sin x)(-\cos x)$

$= \cos x . \cos(\log 11 - \sin x)$

$\Rightarrow g'(0) = \cos 0 \cdot \cos(\log 11 - \sin 0) = 1 \cdot \cos(\log 11)$

Hence, $g'(0) = \cos(\log 11)$.

OR

Solution:

Given,

$$f(x) = \begin{cases} (1 + |sin\theta|^{\frac{a}{|sin\theta|}}, -\frac{\pi}{6} < \theta < 0 \\ b \quad , \theta < 0 \\ e^{tan\,7\theta/tan\,8\theta}, 0 < \theta < \frac{\pi}{6} \end{cases}$$

Since, $f(x)$ is continuous at $x = 0$, therefore

$\lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^+} f(x)$

$\Rightarrow \lim_{x\to 0^-} 1 + |sin\theta|^{\frac{a}{|sin\theta|}} = b = \lim_{x\to 0^+} e^{tan\,7\theta/tan\,8\theta}$

$\Rightarrow e^a = b = e^{\frac{7}{8}}$

$\Rightarrow a = \frac{7}{8} and\ a = \log_e b$

Hence, $a = \frac{7}{8} and\ a = \log_e b$.

25. Solution:

Let a be the first term and r be the common ratio.

$a + ar + ar^2 + \cdots + \infty = 8$

$\Rightarrow \frac{a}{1-r} = 8$

Squaring both sides, we get

$\frac{a^2}{(1-r)^2} = 64$

$\Rightarrow a^2 = 64(1-r)^2 \quad \ldots\ldots(1)$

Also, $a^2 + a^2r^2 + a^2r^4 + \cdots + \infty = 4$

$\frac{a^2}{1-r^2} = 4 \;\; \ldots\ldots(2)$

Putting the value of (1) in (2), we get

$\frac{64(1-r)^2}{1-r^2} = 4$

$\Rightarrow \frac{16(1-r)^2}{(1+r)(1-r)} = 1$

$\Rightarrow \frac{16(1-r)}{(1+r)} = 1$

$\Rightarrow 16 - 16r = 1 + r \;\; \Rightarrow 17r = 15$

$\therefore r = \frac{15}{17}$

Putting $r = \frac{15}{17}$ in $\frac{a}{1-r} = 8$, we get

$\frac{a}{1-\frac{15}{17}} = 8 \Rightarrow a = \frac{16}{17}$

Hence, first term $(a) = \frac{16}{17}$ and common difference $(r) = \frac{15}{17}$.

26. Solution:

Given matrix is $A = \begin{bmatrix} -3 & 3 & 6 \\ 3 & 6 & 9 \\ 9 & 3 & 3 \end{bmatrix}$

Let $A \ = IA$

$$\Rightarrow \begin{bmatrix} -3 & 3 & 6 \\ 3 & 6 & 9 \\ 9 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + R_1, R_3 \to R_3 + 3R_1$, we get

$$\Rightarrow \begin{bmatrix} -3 & 3 & 6 \\ 0 & 9 & 15 \\ 0 & 12 & 21 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to (-1)R_1$, we get

$$\Rightarrow \begin{bmatrix} 3 & -3 & -6 \\ 0 & 9 & 15 \\ 0 & 12 & 21 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - R_3$, we get

$$\Rightarrow \begin{bmatrix} 3 & -3 & -6 \\ 0 & -3 & -6 \\ 0 & 12 & 21 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & -1 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2, R_3 \to R_3 + 4R_2$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & -6 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -2 & 1 & -1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_2 \to (-1)R_2$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 6 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 1 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 2R_3$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ -5 & 4 & -3 \end{bmatrix} A$$

Applying $R_3 \to (-1)R_3$, we get

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

$$\Rightarrow (27) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

$$\Rightarrow A^{-1} = \frac{1}{27} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$$

27. Solution:

We have, $(\tan x)^y = (\tan y)^x$,

On taking log both sides, we get

$y \log(\tan x) = x \log(\tan y) \quad [\because \log m^n = n \log m]$ ... (1)

On differentiating both sides of (1) w.r.t. x, we get

$$y \cdot \frac{\mathrm{d}}{\mathrm{dx}}(\log(\tan x)) + \log(\tan x) \cdot \frac{\mathrm{d}}{\mathrm{dx}}(y) = x \cdot \frac{\mathrm{d}}{\mathrm{dx}}(\log(\tan y)) + \log(\tan x) \cdot \frac{\mathrm{d}}{\mathrm{dx}}(x)$$

$$\Rightarrow y \cdot \frac{1}{(\tan x)} \cdot \frac{\mathrm{d}}{\mathrm{dx}}(\tan x) + \log(\tan x) \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = x \cdot \frac{1}{(\tan y)} \cdot \frac{d}{dx}((\tan y)) + \log(\tan y) \cdot 1$$

$$\Rightarrow y \cdot \frac{1}{(\tan x)} \cdot \sec^2 x + \log(\tan x) \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = x \cdot \frac{1}{(\tan y)} \cdot \sec^2 x \cdot \frac{\mathrm{dy}}{\mathrm{dx}} + \log(\tan y)$$

$$\Rightarrow \log(\tan x) \cdot \frac{\mathrm{dy}}{\mathrm{dx}} - x \cdot \frac{1}{(\tan y)} \cdot \sec^2 x \cdot \frac{\mathrm{dy}}{\mathrm{dx}} = \log(\tan y) - y \cdot \frac{1}{(\tan x)} \cdot \sec^2 x$$

$$\Rightarrow \left(\log(\tan x) - x \cdot \frac{1}{(\tan y)} \cdot \sec^2 x \right) \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{(\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x}{(\tan x)}$$

$$\Rightarrow \left(\frac{(\tan y) \cdot \log(\tan x) - x \cdot \sec^2 x}{(\tan y)} \right) \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{(\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x}{(\tan x)}$$

$$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{(\tan y)\left((\tan x) \cdot \log(\tan y) - y \cdot \sec^2 x\right)}{(\tan x)\left((\tan y) \cdot \log(\tan x) - x \cdot \sec^2 x\right)}$$

Hence, it is proved.

OR

Solution:

Let $P(n)$: $51^n - 14^n$, $\forall n \in N$

For $n = 1$, the given expression becomes

$51^1 - 14^1 = 51 - 14 = 37$, which is multiple of 37.

So, the given statement is true for $1, i.e.$, $P(1)$ is true.

Let $P(k)$ be true.

Then, $P(k)$: $51^k - 14^k$ is multiple of 37.

$\Rightarrow 51^k - 14^k = 37m$ for some natural number m. $\quad \ldots(1)$

Now, $51^{k+1} - 14^{k+1} = (51^{k+1} - 51 \cdot 14^k) + (51 \cdot 14^k - 14^{k+1})$

[On subtracting and adding $51 \cdot 14^k$]

$= 51(51^k - 14^k) + 14^k(51 - 14)$

$= 51 \cdot 37m + 37 \cdot 14^k$ [Using(1)]

$= 37(51m + 14^k)$, Which is multiple of 37.

$\therefore$ $P(k+1)$: $51^{k+1} - 14^{k+1}$ is multiple of 37.

$\Rightarrow P(k+1)$ is true, whenever $P(k)$ be true.

Thus, $P(1)$ is true and $P(k+1)$ is true, whenever $P(k)$ be true.

Hence, by principle of mathematical induction, $P(n)$ is true for all $n \epsilon N$.

28. Solution:

The given function is

$f(x) = x^3 - 2x + 9$

On differentiating both sides $w.r.t.x$, we get

$f'(x) = 3x - 2$

On putting $f'(x) = 0$, we get

$\Rightarrow 3x - 2 = 0$

$\Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$

Now, we find intervals in which $f(x)$ is strictly increasing or strictly decreasing.

Interval	$f'(x) = 3x - 2$	Sign of $f'(x)$
$x < 1$	$(-)$	$-ve$
$x > 1$	$(+)$	$+ve$

We know that, a function $f(x)$ is said to be strictly increasing, if $f'(x) > 0$ and it is said to be strictly increasing, if $f'(x) < 0$. So, the given function $f(x)$ is:

1. Strictly increasing on the interval $\left(\frac{2}{3}, \infty\right)$.

2. Strictly decreasing on the interval $\left(-\infty, \frac{2}{3}\right)$.

Hence, $f(x)$ increasing nor decreasing in $(-1,1)$.

29. Solution:

Given curves are $y = \sqrt{x}$ $\ldots\ldots(1)$

and $y - x + 2 = 0$ $\ldots\ldots(2)$

On solving (1) and (2), we get

$\sqrt{x} - \sqrt{x}^2 + 2 = 0$

$\Rightarrow \sqrt{x}^2 - \sqrt{x} - 2 = 0$

$\Rightarrow \sqrt{x}^2 - 2\sqrt{x} + \sqrt{x} - 2 = 0$

$\Rightarrow \sqrt{x}(\sqrt{x} - 2) + 1(\sqrt{x} - 2) = 0$

$\Rightarrow (\sqrt{x} - 2)(\sqrt{x} + 1) = 0$

$\Rightarrow \sqrt{x} = 2$ [$\because \sqrt{x} = -1$ is not possible]

Hence, required area $= \int_0^2 (x_2 - x_1)dy = \int_0^2 (y + 2)dy - \int_0^2 y^2\, dy$

$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_0^2$

$= 2 + 4 - \frac{8}{3} = \frac{10}{3}$ sq. units

OR

Solution:

Given equations of lines are:

$\frac{x-1}{2} = \frac{y-3}{\lambda} = \frac{z+1}{-1}$ and $\frac{x+1}{\lambda} = \frac{y-1}{2} = \frac{z-2}{2}$.

The given lines are parallel to the vectors $\overrightarrow{b_1} = 2\hat{\imath} + \lambda\hat{\jmath} - \hat{k}$ and $\overrightarrow{b_2} = \lambda\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ respectively. The lines are perpendicular if $\overrightarrow{b_1} \cdot \overrightarrow{b_2} = 0$

$\Rightarrow (2\hat{\imath} + \lambda\hat{\jmath} - \hat{k})(\lambda\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) = 0$

$\Rightarrow 2 \times \lambda + \lambda \times 2 + (-1) \times 2 = 0$

$\Rightarrow 4\lambda - 2 = 0$

$\Rightarrow 4\lambda = 2$

Hence, $\lambda = \frac{1}{2}$.